Ensemble of Data Assimilations and Hybrid Gain EnDA

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Contributions from:
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Outline

- KF, RR-KF, EnKF
- Hybrid Var-EnKF methods
- Ensemble of Data Assimilations (EDA)
- Hybrid Gain Ensemble Data Assimilation

The EDA method

Question: "How can we set up an ensemble data assimilation system for a large dimensional system without using an Ensemble Kalman Filter?"

The EDA (Ensemble of Data Assimilations, Isaksen et al., 2007) is one possible answer.

The EDA method

For a linear system (linear model M, linear observation operator H) the data assimilation update is:

$$\mathbf{x}_t^a = \mathbf{x}_t^b + \mathbf{K}\left(\mathbf{y}_t - \mathbf{H}\mathbf{x}_t^b\right)$$
$$\mathbf{x}_{t+1|}^b = \mathbf{M}\mathbf{x}_t^a \qquad (1)$$

Assuming background (**P**^{b}), observation (**R**) and model errors (**Q**) to be statistically independent, the evolution of the system error covariances is given by:

$$\mathbf{P}_t^a = \left(\mathbf{I} - \mathbf{KH}\right)\mathbf{P}_t^b\left(\mathbf{I} - \mathbf{KH}\right)^{\mathrm{T}} + \mathbf{KRK}^{\mathrm{T}}$$
$$\mathbf{P}_{t+1}^b = \mathbf{M}\mathbf{P}_t^a\mathbf{M}^{\mathrm{T}} + \mathbf{Q} \qquad (2)$$

The EDA method

Consider now the evolution of the same system if we perturb the observations and the forecast model with random noise drawn from the respective error covariances:

$$\begin{aligned} \tilde{\mathbf{x}}_t^a &= \tilde{\mathbf{x}}_t^b + \mathbf{K}\left(\mathbf{y} + \boldsymbol{\eta} - \mathbf{H}\tilde{\mathbf{x}}_t^b\right) \\ \tilde{\mathbf{x}}_{t+1}^b &= \mathbf{M}\tilde{\mathbf{x}}_t^a + \boldsymbol{\zeta} \end{aligned} \qquad (3)$$

where $\boldsymbol{\eta} \sim \mathcal{N}(\mathbf{0},\mathbf{R})$, $\boldsymbol{\zeta} \sim \mathcal{N}(\mathbf{0},\mathbf{Q})$.

If we define the differences between the perturbed and unperturbed state $\boldsymbol{\varepsilon}_a \equiv \tilde{\mathbf{x}}_a - \mathbf{x}_a$ and $\boldsymbol{\varepsilon}_b \equiv \tilde{\mathbf{x}}_b - \mathbf{x}_b$, their evolution is obtained by subtracting the unperturbed state evolution equations from the perturbed ones, i.e. (3)-(1):

$$\begin{aligned} \boldsymbol{\varepsilon}_t^a &= \boldsymbol{\varepsilon}_t^b + \mathbf{K}\left(\boldsymbol{\eta} - \mathbf{H}\boldsymbol{\varepsilon}_t^b\right) \\ \boldsymbol{\varepsilon}_{t+1}^b &= \mathbf{M}\boldsymbol{\varepsilon}_t^a + \boldsymbol{\zeta} \end{aligned} \qquad (4)$$

The EDA method

$$\boldsymbol{\varepsilon}_t^a = \boldsymbol{\varepsilon}_t^b + \mathbf{K}\left(\boldsymbol{\eta} - \mathbf{H}\boldsymbol{\varepsilon}_t^b\right)$$
$$\boldsymbol{\varepsilon}_{t+1}^b = \mathbf{M}\boldsymbol{\varepsilon}_t^a + \boldsymbol{\zeta} \qquad (4)$$

- i.e., the perturbations from the control evolve with the same update equations of the state.

How do the errors evolve?

If we compute the covariance of the perturbations in (4) we obtain:

$$\left\langle \boldsymbol{\varepsilon}_t^a \left(\boldsymbol{\varepsilon}_t^a\right)^{\mathrm{T}} \right\rangle = \left(\mathbf{I} - \mathbf{KH}\right)\left\langle \boldsymbol{\varepsilon}_t^b \left(\boldsymbol{\varepsilon}_t^b\right)^{\mathrm{T}} \right\rangle \left(\mathbf{I} - \mathbf{KH}\right)^{\mathrm{T}} + \mathbf{KRK}^{\mathrm{T}}$$
$$\left\langle \boldsymbol{\varepsilon}_{k+1}^b \left(\boldsymbol{\varepsilon}_{t+1}^b\right)^{\mathrm{T}} \right\rangle = \mathbf{M}\left\langle \boldsymbol{\varepsilon}_t^a \left(\boldsymbol{\varepsilon}_t^a\right)^{\mathrm{T}} \right\rangle \mathbf{M}^{\mathrm{T}} + \mathbf{Q} \qquad (5)$$

The EDA method

$$\left\langle \boldsymbol{\varepsilon}_t^a \left(\boldsymbol{\varepsilon}_t^a\right)^{\mathrm{T}} \right\rangle = \left(\mathbf{I} - \mathbf{KH}\right)\left\langle \boldsymbol{\varepsilon}_t^b \left(\boldsymbol{\varepsilon}_t^b\right)^{\mathrm{T}} \right\rangle\left(\mathbf{I} - \mathbf{KH}\right)^{\mathrm{T}} + \mathbf{KRK}^{\mathrm{T}}$$
$$\left\langle \boldsymbol{\varepsilon}_{k+1}^b \left(\boldsymbol{\varepsilon}_{t+1}^b\right)^{\mathrm{T}} \right\rangle = \mathbf{M}\left\langle \boldsymbol{\varepsilon}_t^a \left(\boldsymbol{\varepsilon}_t^a\right)^{\mathrm{T}} \right\rangle\mathbf{M}^{\mathrm{T}} + \mathbf{Q} \qquad (5)$$

- These are the same equations for the evolution of the error covariances of the control analysis:

$$\mathbf{P}_t^a = \left(\mathbf{I} - \mathbf{KH}\right)\mathbf{P}_t^b\left(\mathbf{I} - \mathbf{KH}\right)^{\mathrm{T}} + \mathbf{KRK}^{\mathrm{T}}$$
$$\mathbf{P}_{t+1}^b = \mathbf{M}\mathbf{P}_t^a\mathbf{M}^{\mathrm{T}} + \mathbf{Q} \qquad (2)$$

provided that the applied perturbations $\boldsymbol{\eta}_k$, ζ_k have the right covariances (**R**, **Q**)

The EDA method

What does all this mean in practice?

- We can use an ensemble of perturbed data assimilation cycles to simulate the errors of our reference DA cycle;

- The ensemble of perturbed DAs should be as similar as possible to the reference DA (i.e., same or similar $\mathbf{K}$ matrix, $\mathbf{M}$, $\mathbf{H}$, and resolution)

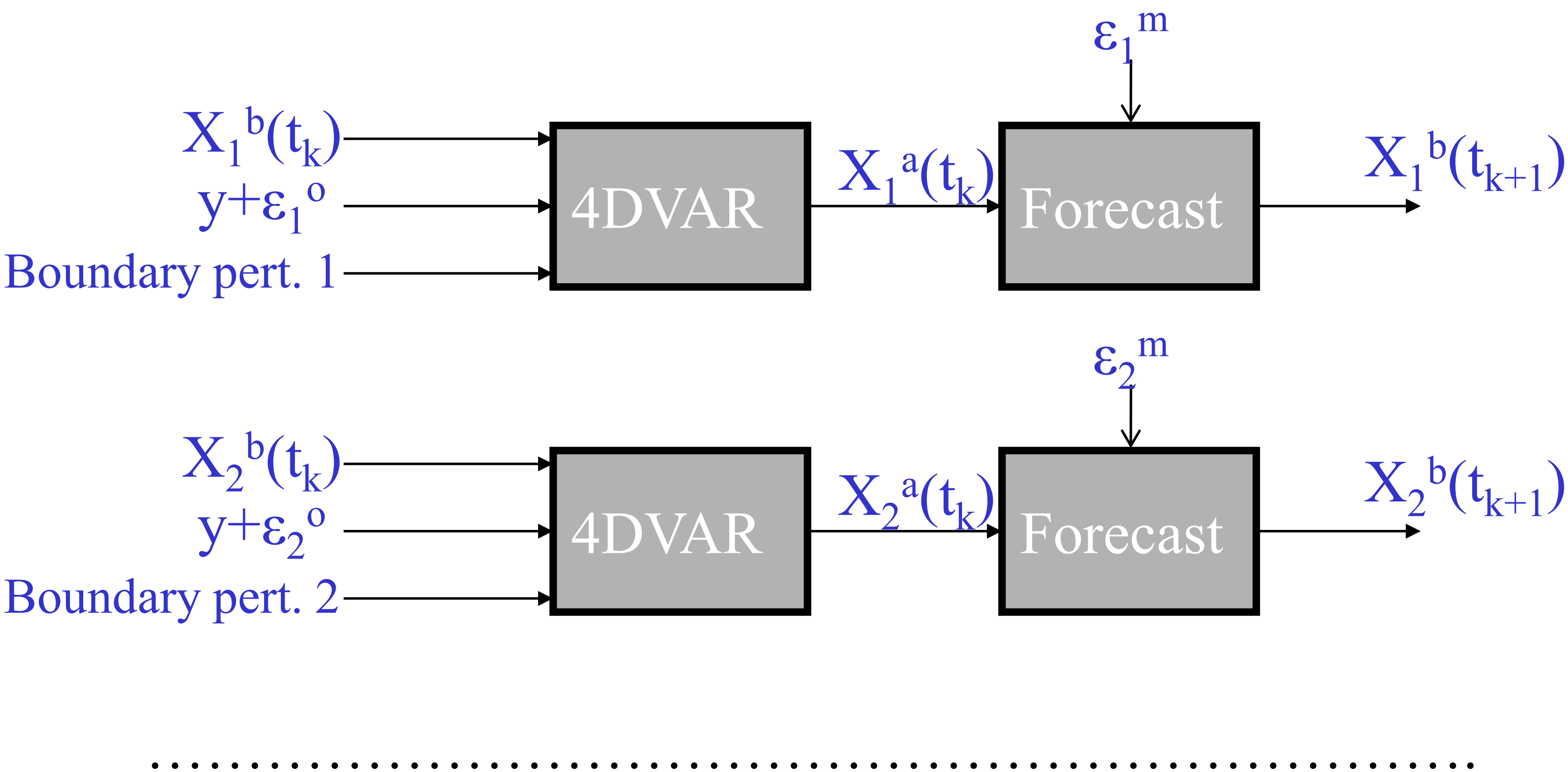
- The applied perturbations $\boldsymbol{\eta}_k$, ζ_k must have the required error covariances ($\mathbf{R}$, $\mathbf{Q}$);

- There is no need to explicitly perturb the background $\mathbf{x}_b$

The EDA method

- 25 ensemble members using 4D-Var assimilations at reduced resolution
- **T399** outer loop, T95/T159 inner loops. (Reference DA: T1279 outer loop, T159/T255/T255 inner loops). Note that this week the EDA will be upgraded to TCo639, T191/T191 resol. and the reference HRES DA at TCo1279, T255/T319/T399.
- Observations randomly perturbed according to their estimated errors
- SST perturbed with climatological error structures
- Model error represented by stochastic methods (**SPPT,** Leutbecher, 2009)

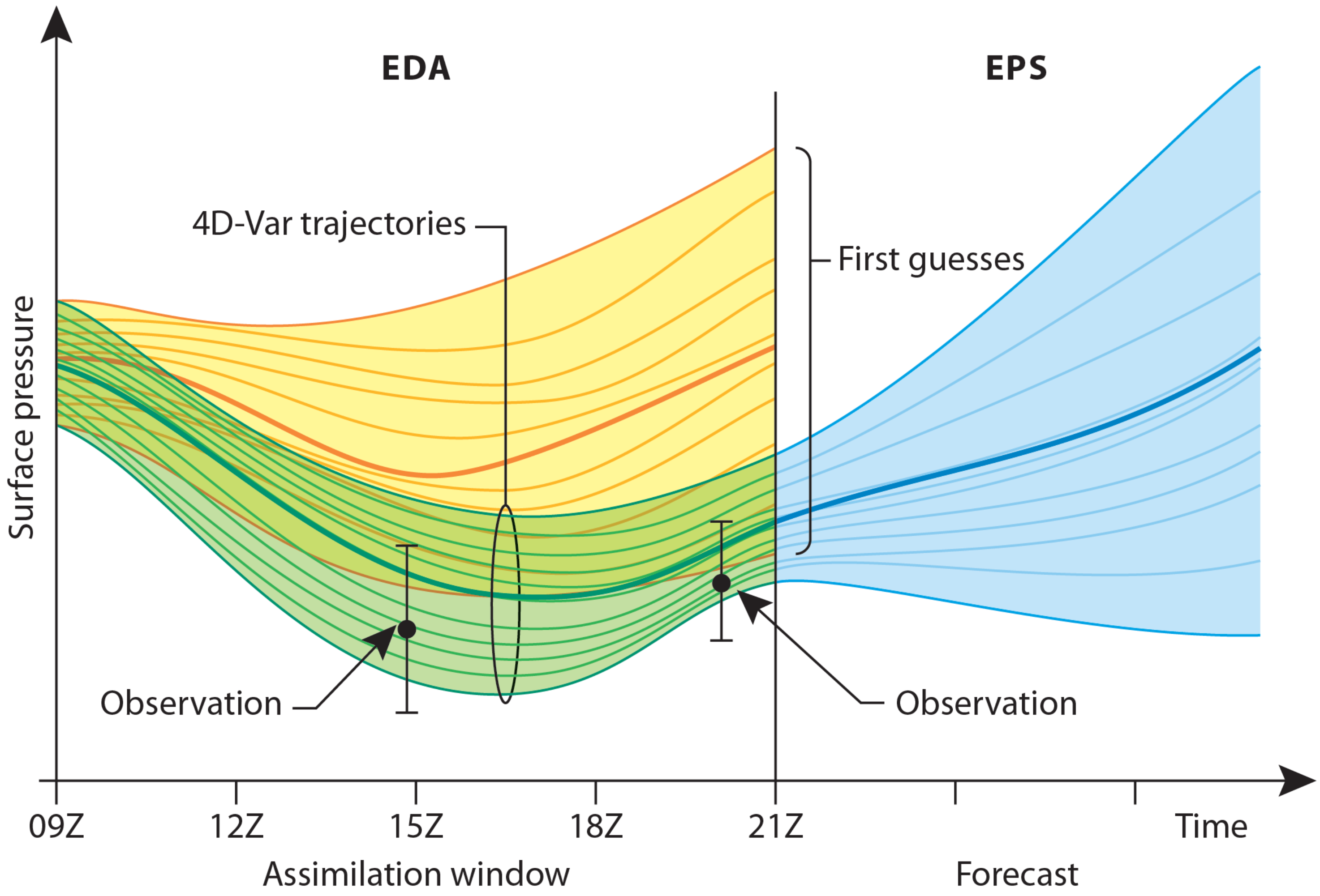
The EDA method



Applications of the EDA

- **The EDA simulates the error evolution of the 4DVar analysis cycle**. As such it has two main applications:
    1. Provide a flow-dependent estimate of analysis errors to initialize the ensemble prediction system (EPS)
    2. Provide a flow-dependent estimate of background errors for use in 4D-Var assimilation

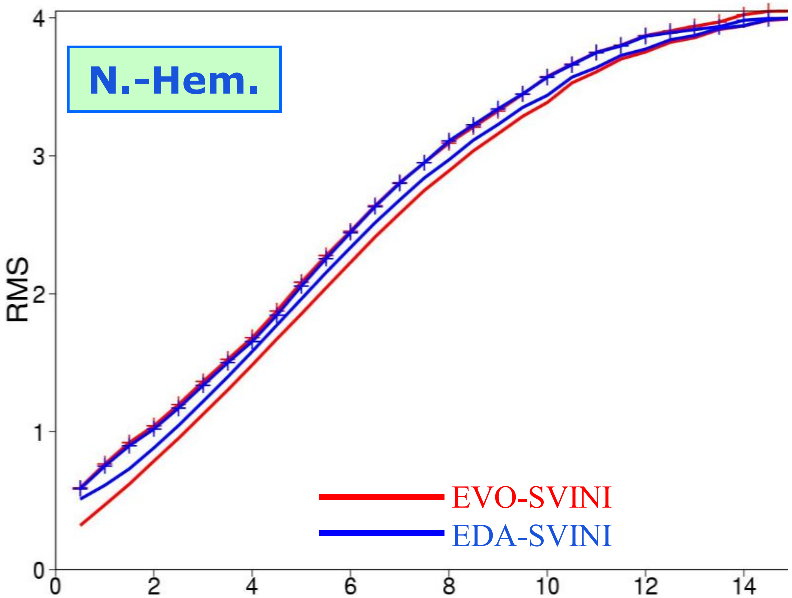
Applications of the EDA

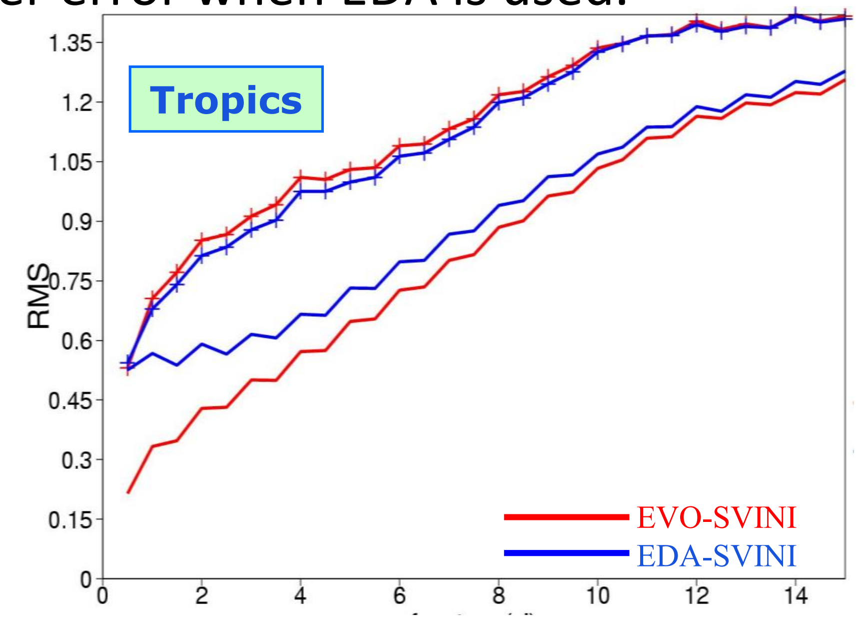


Applications of the EDA

Improving Ensemble Prediction System by including EDA perturbations for initial uncertainty (implemented June 2010)

The Ensemble Prediction System (EPS) benefits from using EDA based perturbations. Replacing evolved singular vector perturbations by EDA based perturbations improve EPS spread, especially in the tropics.
The Ensemble Mean has slightly lower error when EDA is used.

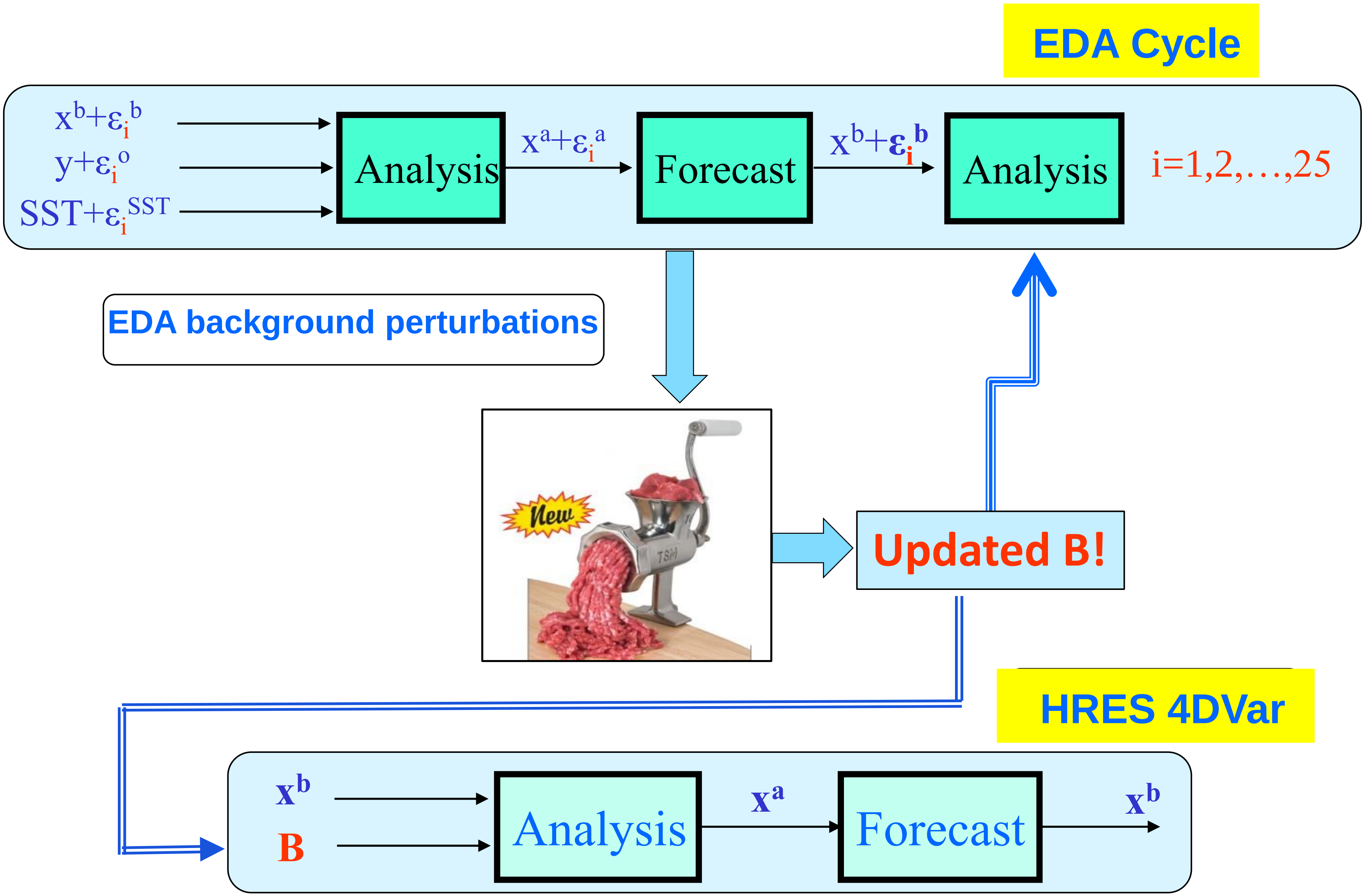




Ensemble spread and Ensemble mean RMSE for 850hPa T

Applications of the EDA

- The EDA simulates **the error evolution** of the 4DVar analysis cycle. As such it has two main applications:
  1. Provide a flow-dependent estimate of analysis errors to initialize the ensemble prediction system (EPS)
  2. Provide a flow-dependent estimate of background errors for use in 4D-Var assimilation


EDA Cycle
$x^b+\varepsilon_i^b$
$y+\varepsilon_i^o$
$SST+\varepsilon_i^{SST}$
Analysis
$x^a+\varepsilon_i^a$
Forecast
$x^b+\varepsilon_i^b$
Analysis
i=1,2,…,25
EDA background perturbations
New
Updated B!
HRES 4DVar
x^b
B
Analysis
x^a
Forecast
x^b

ECMWF

Hybrid EDA 4DVar

We have seen in the previous lecture that one way to incorporate ensemble information in 3-4DVar is to add a flow-dependent term to the model of $\mathbf{P}^{\mathbf{b}}$ (extended control variable):

$$\mathbf{B} = \beta_c^2 \mathbf{B}_c + \beta_e^2 \mathbf{P}_e \circ \mathbf{C}_{loc}$$

Another way is to sample $\mathbf{B}$ completely and for all the assimilation window from the ensemble forecast perturbations (4D-En-Var):

$$\mathbf{B}(t) = \mathbf{P}_e(t) \circ \mathbf{C}_{loc}$$

Still another way is to continuously update your $\mathbf{B}$ model using ensemble forecast perturbations (Hybrid EDA 4DVar)

Hybrid EDA 4DVar

In variational DA, the $\mathbf{B}$ matrix is usually defined implicitly in terms of a transformation from the first guess departure ($\mathbf{x}$-$\mathbf{x_b}$) to a control variable χ:

$$(\mathbf{x}-\mathbf{x_b}) = \mathbf{L}\chi$$

so that the implied $\mathbf{B}=\mathbf{L}\mathbf{L}^{\mathbf{T}}$.

In the current wavelet formulation (Fisher, 2003), the variable transform can be written as:

$$\left(\mathbf{x}-\mathbf{x}_b\right)=\mathbf{K}\boldsymbol{\Sigma}_b^{1/2}\sum_j \psi_j \otimes \left[\mathbf{C}_j^{1/2}(\lambda,\phi)\chi_j\right]$$

$\mathbf{K}$ is the balance operator, i.e. the operator that links the control variables to the model variables

$\boldsymbol{\Sigma}_\mathrm{b}$ is the grid point variance of background errors

$\mathbf{C_j}(\lambda,\varphi)$ is the vertical correlation matrix for wavelet index j

ψ_j are the set of radial basis function that define the wavelet transform.

Hybrid EDA 4DVar

$$\left(\mathbf{x}-\mathbf{x}_b\right)=\mathbf{K}\boldsymbol{\Sigma}_b^{1/2}\sum_j \psi_j \otimes\left[\mathbf{C}_j^{1/2}(\lambda,\phi)\chi_j\right]$$

$\mathbf{C}_j(\lambda,\varphi)$ are full vertical correlation matrices, function of (λ,φ). They determine both the horizontal and vertical background error *correlation structures*;

In standard 4DVar $\boldsymbol{\Sigma}_b$, $\mathbf{K}$ and $\mathbf{C}_j$ are computed off-line using a climatology of EDA perturbations.

How do we make this error covariance model flow-dependent?

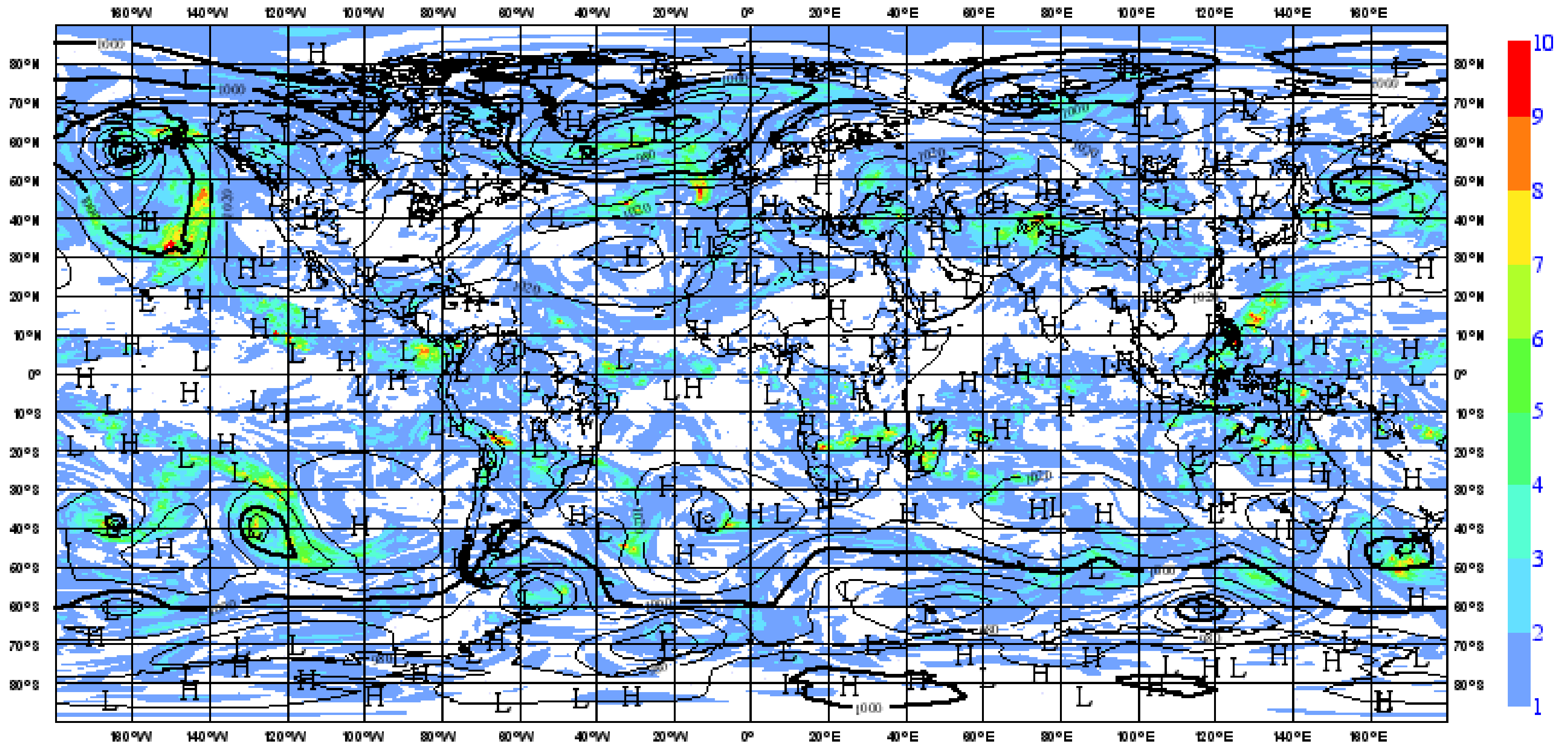
We look for flow-dependent estimates of $\boldsymbol{\Sigma}_b$ and $\mathbf{C}_j(\lambda,\varphi)$

(Bonavita et al., 2012; Bonavita et al., 2015)

Hybrid EDA 4DVar

What do raw ensemble variances look like?

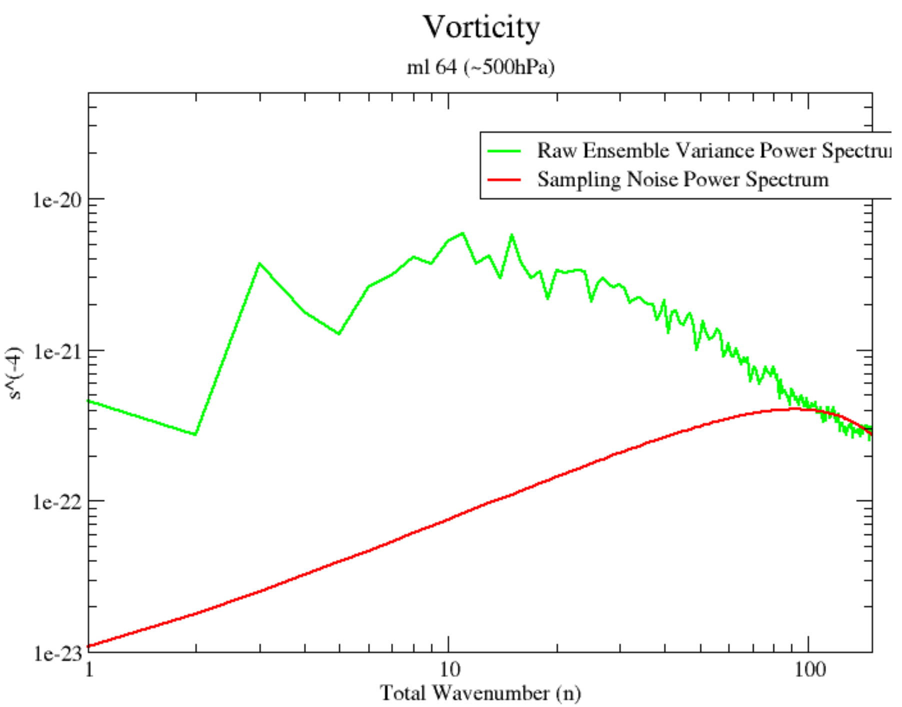
Standard Deviation of Vorticity t+9h 500hPa

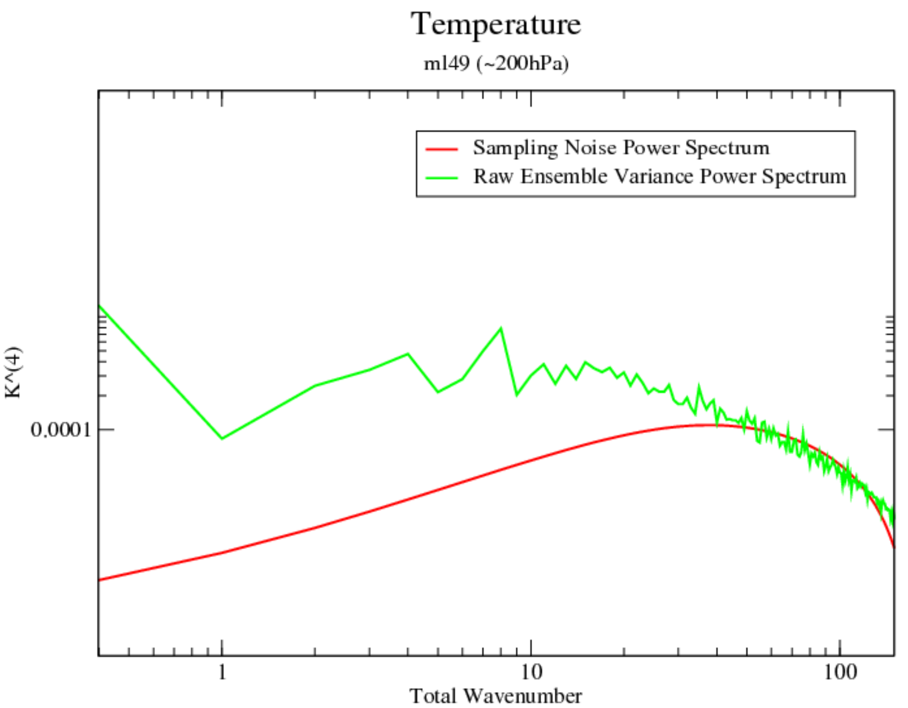


Hybrid EDA 4DVar

- Noise level is due to sampling errors: 25 member ensemble
- EDA is a stochastic system: error variance of variance estimator $\sim 1/N_{ens}$
- We need a system to effectively filter out noise from first guess ensemble forecast variances: Reduce the random component of the estimation error

Hybrid EDA 4DVar

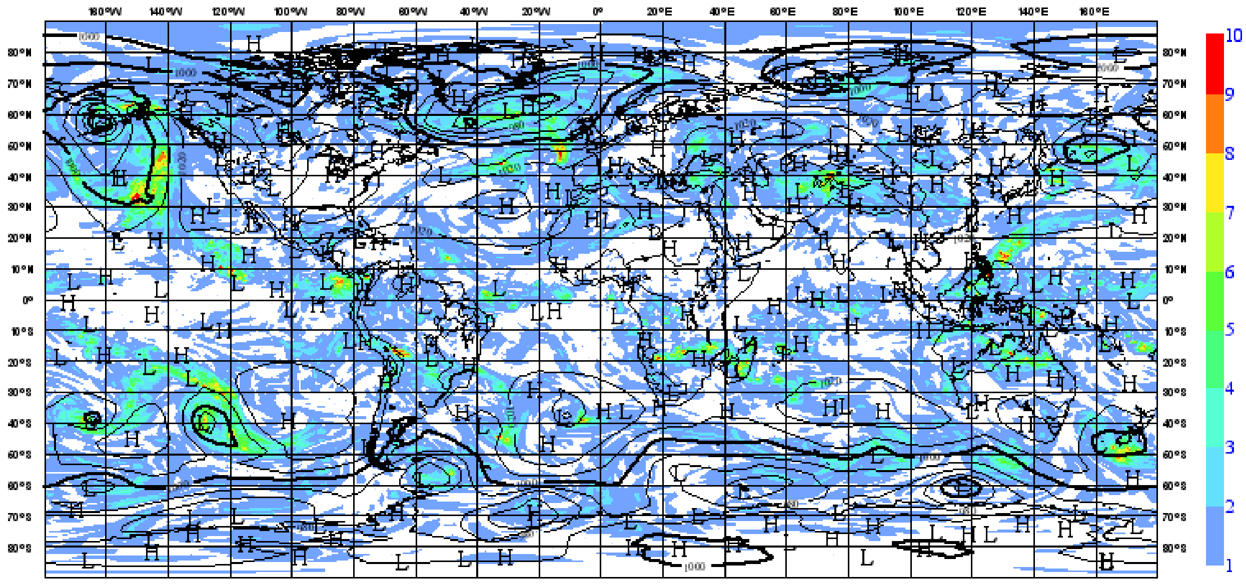




- We can use a spectral filter to disentangle noise from signal
- Truncation wavenumber is determined by **maximizing signal-to-noise** ratio of filtered variances (Raynaud *et al.*, 2009; Bonavita *et al.*, 2011)

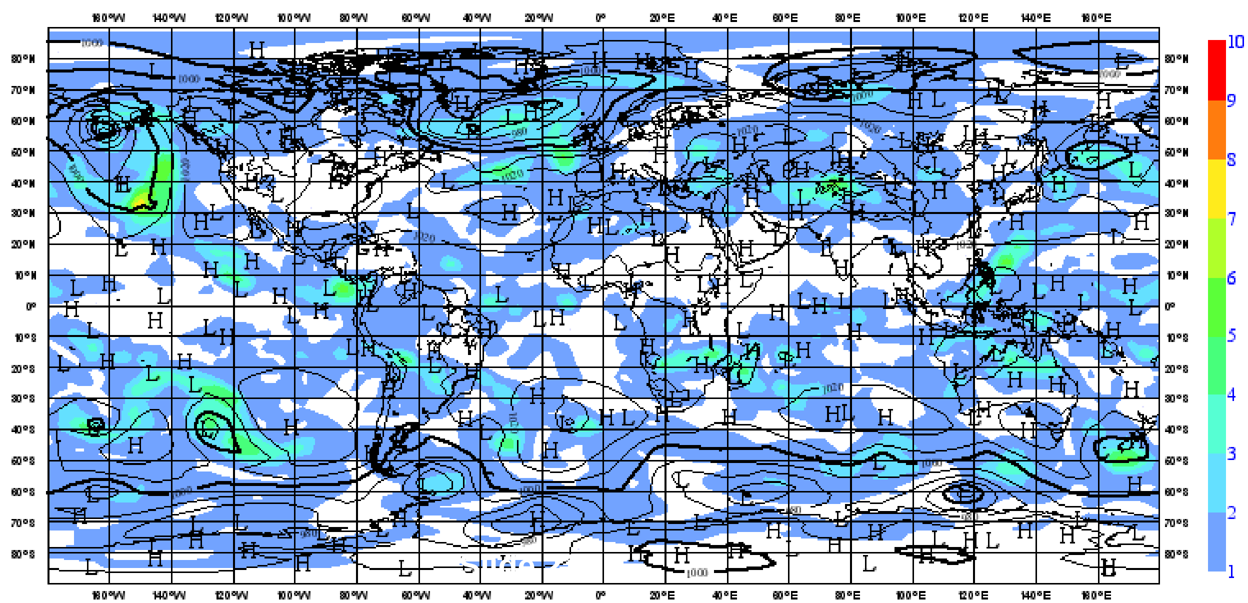
ECMWF

Hybrid EDA 4DVar

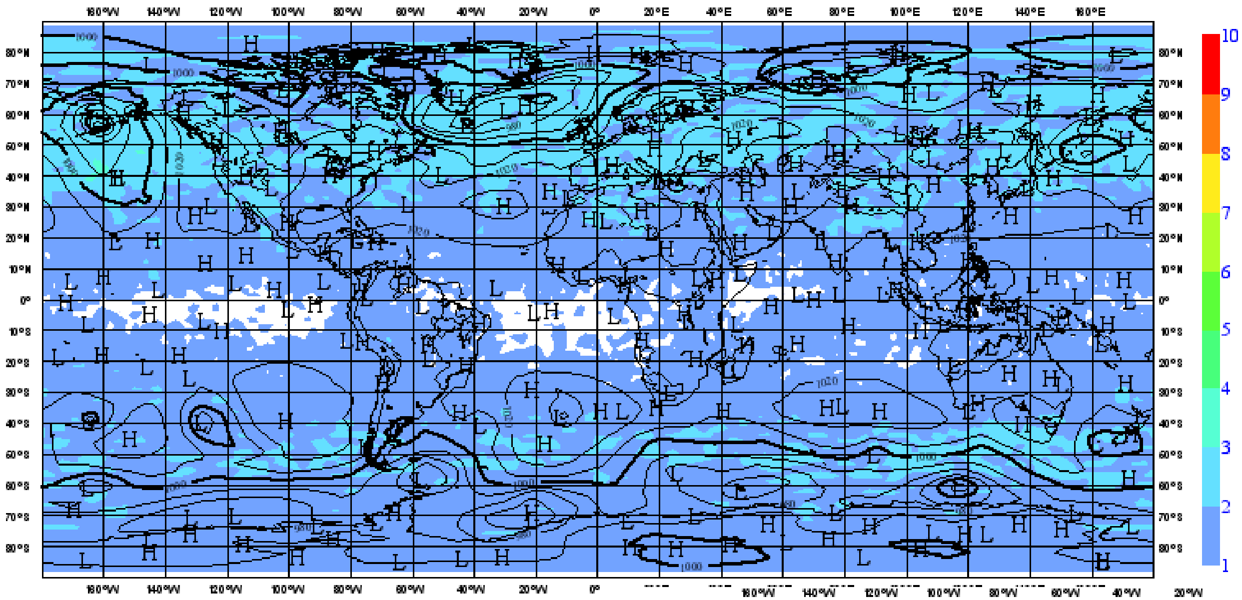


Raw EDA StDev
Vorticity 500 hPa

Filtered EDA StDev
Vorticity 500 hPa

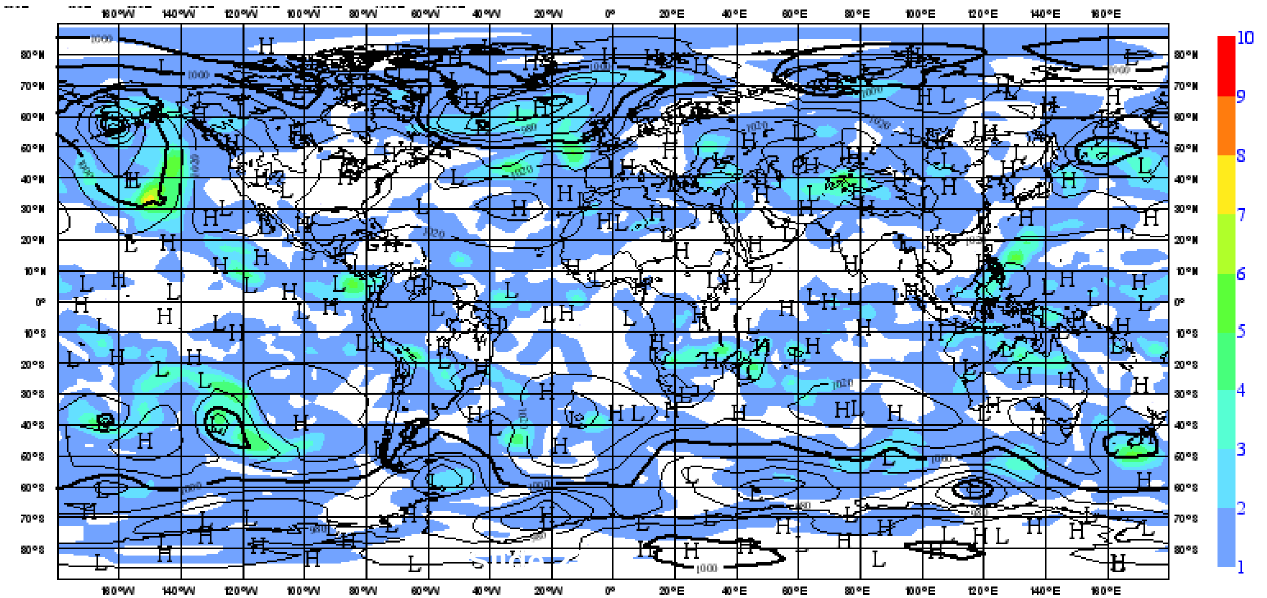


Applications of the EDA



StDev of Vorticity at 500 hPa estimated from **climat. B** Random. Method (Fisher & Courtier, 1995)

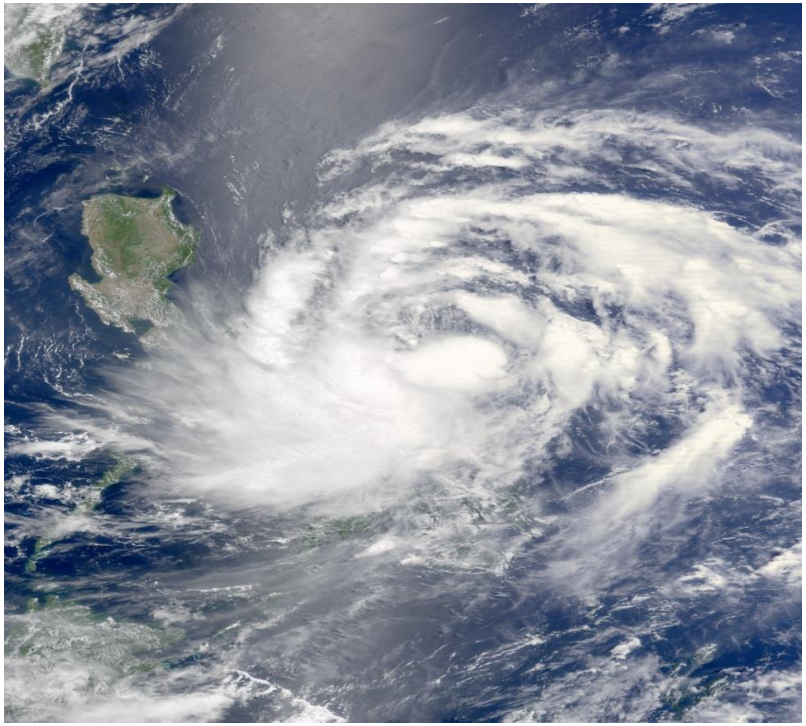
Filtered EDA estimate of StDev of Vorticity 500 hPa

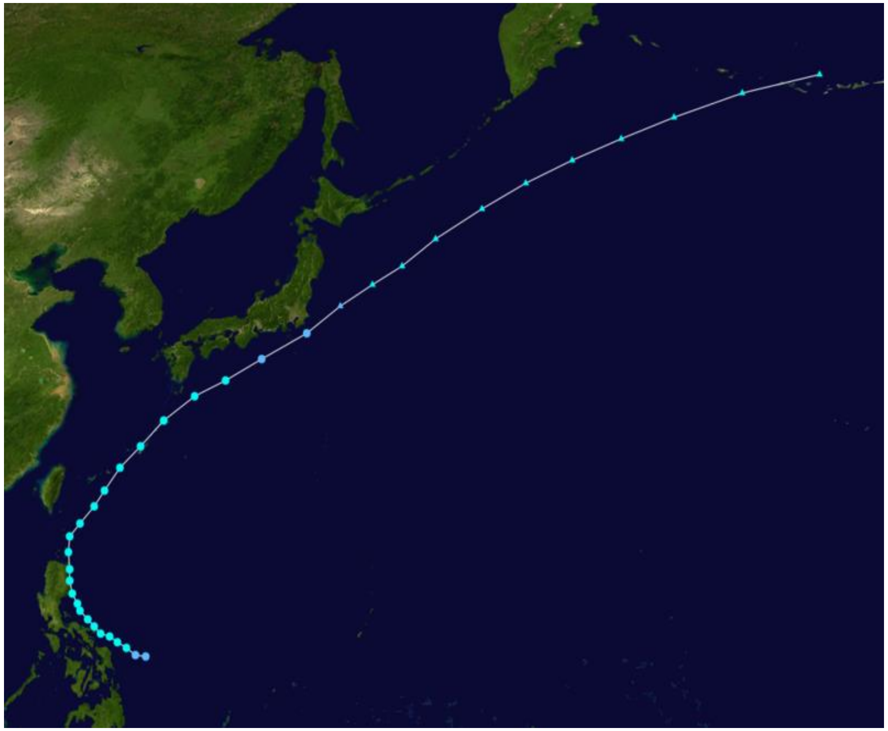


Use of EDA variances in 4DVar

1. Inside 4DVar EDA derived background error estimates change the shape and size of analysis increments

- Tropical Cyclone Aere, Philippines 8-9 May 2011.



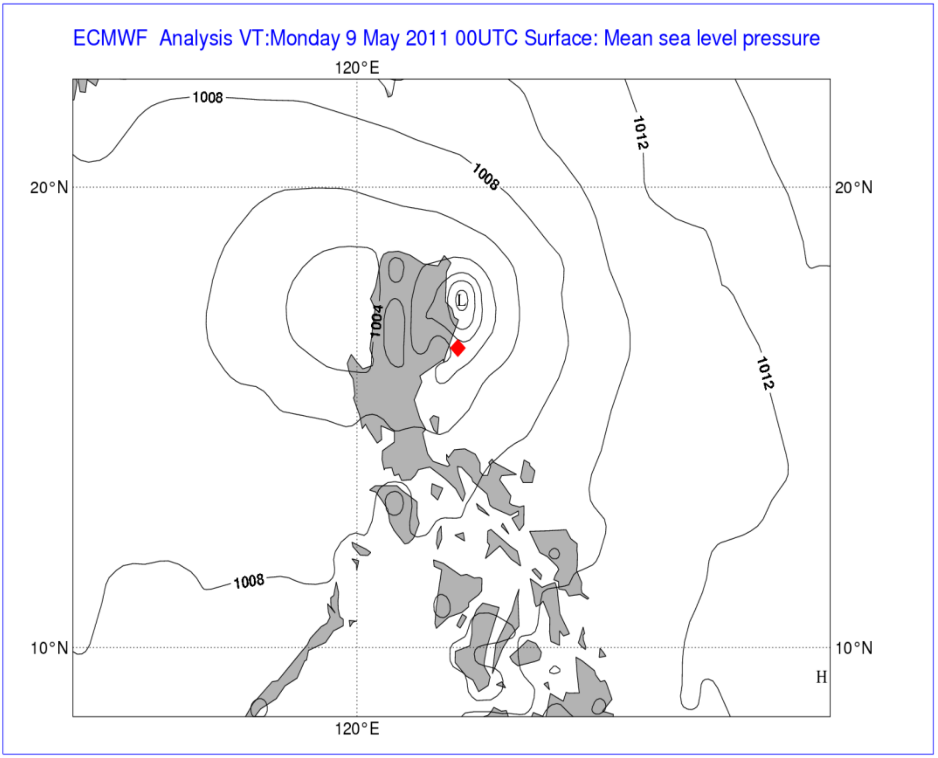


Use of EDA variances in 4DVar

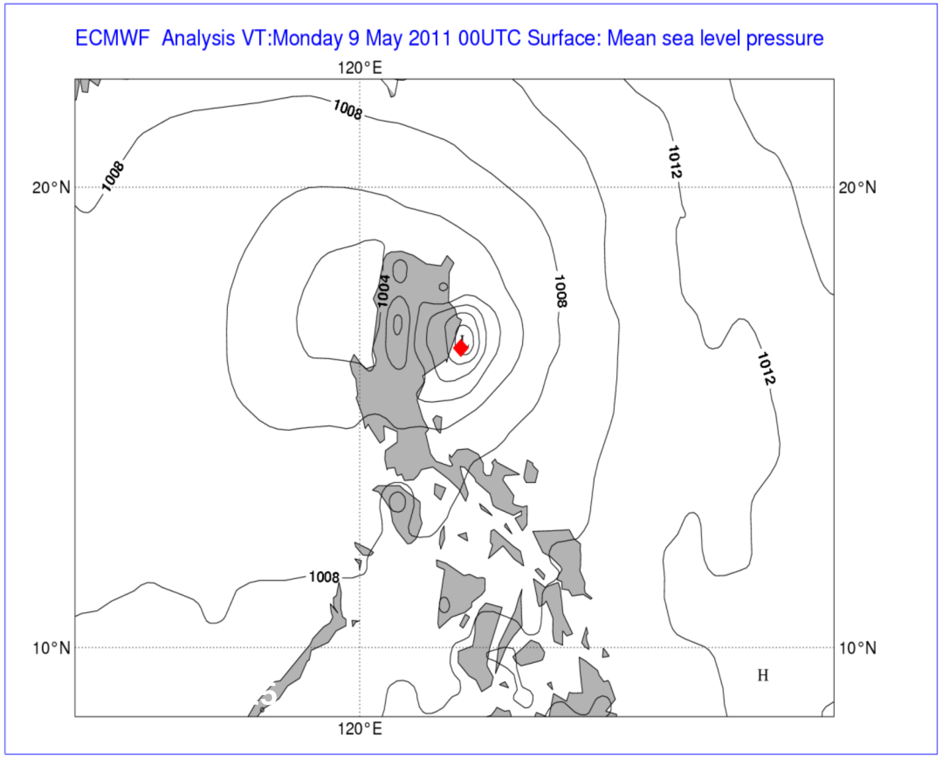
1. Inside 4DVar EDA variances change the shape and size of analysis increments

- Significant operational analysis error, corrected by 4DVar with EDA variances

4DVar with Climat. errors

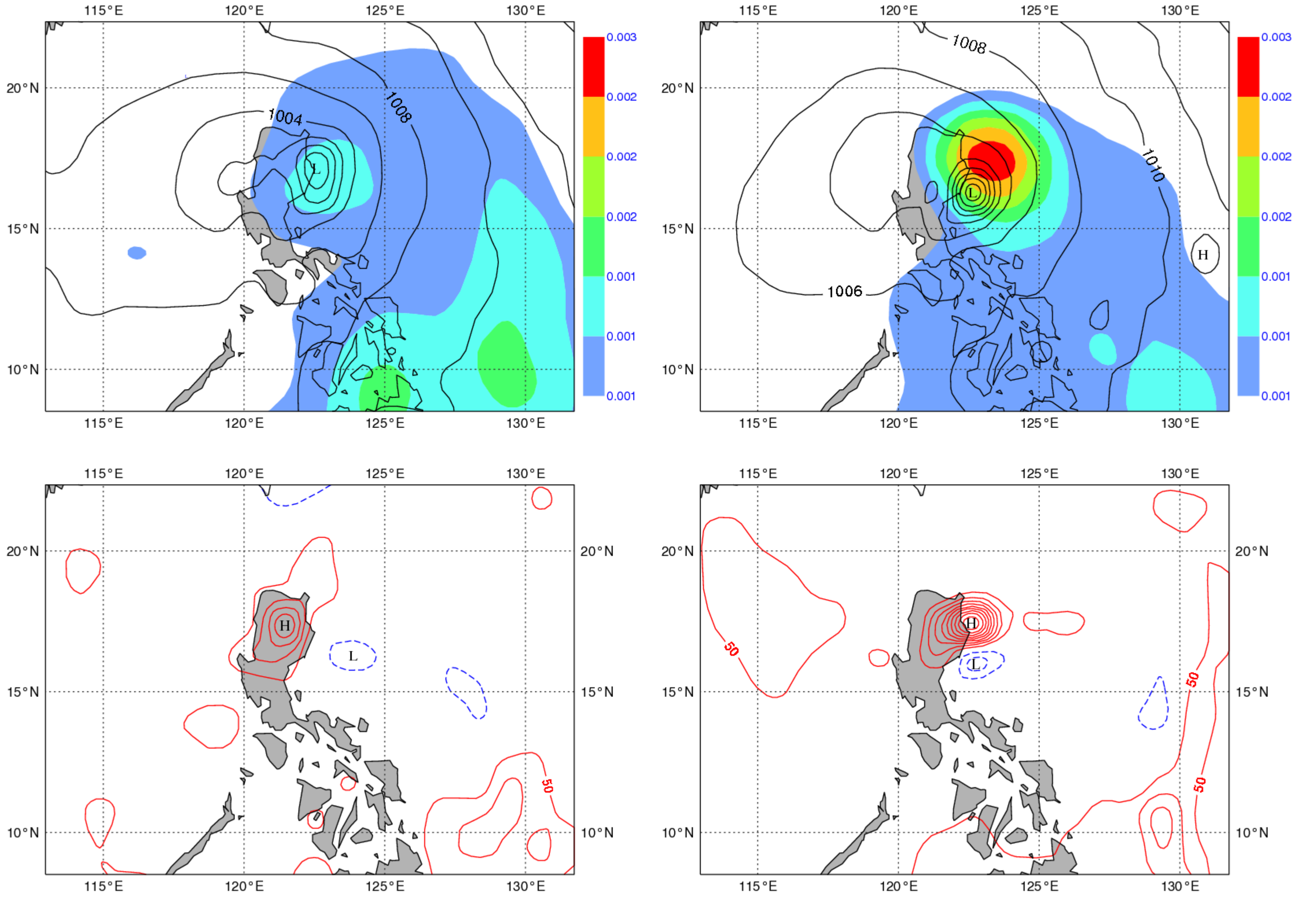


4DVar with EDA errors



$\log(P_s)$ Static errors

$\log(P_s)$ EDA errors



Static mslp ana incr.

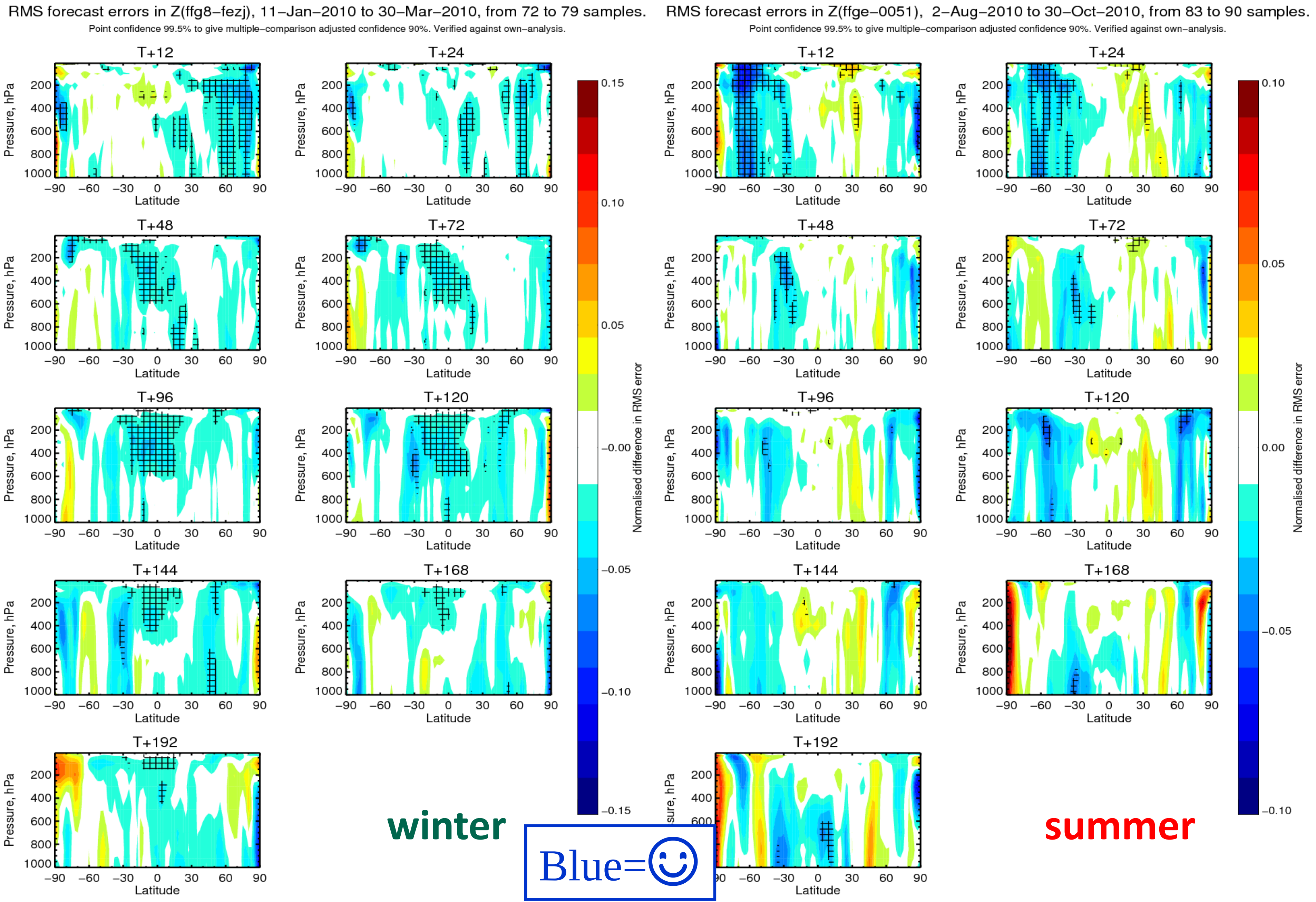
EDA mslp ana incr.

Use of EDA variances in 4DVar

- Flow-dependent EDA errors have been used operationally since May 2012 (CY37R2)
- The effect of using flow-dependent EDA estimated errors is large on average skill scores

ECMWF

Geopotential RMSE reduction



Use of EDA covariances in 4DVar

$$\left(\mathbf{x}-\mathbf{x}_b\right)=\mathbf{K}\mathbf{\Sigma}_b^{1/2}\sum_j \psi_j \otimes\left[\mathbf{C}_j^{1/2}(\lambda,\phi)\chi_j\right]$$

$\mathbf{K}$ is the balance operator

$\mathbf{\Sigma}_\mathrm{b}$ is the grid point variance of background errors

$\mathbf{C}_\mathbf{j}(\lambda,\varphi)$ is the vertical correlation matrix for wavelet index j

ψ_j are the set of radial basis function that define the wavelet transform

$\mathbf{C}_j(\lambda,\varphi)$ are fields of full vertical correlation matrices, defined for each wavelet band. They determine both the horizontal and vertical background error correlation structures.

In order to get flow-dependent estimates of error correlation structures we need flow-dependent estimates of $\mathbf{C}_j(\lambda,\varphi)$.

Flow-dependent wavelet B model

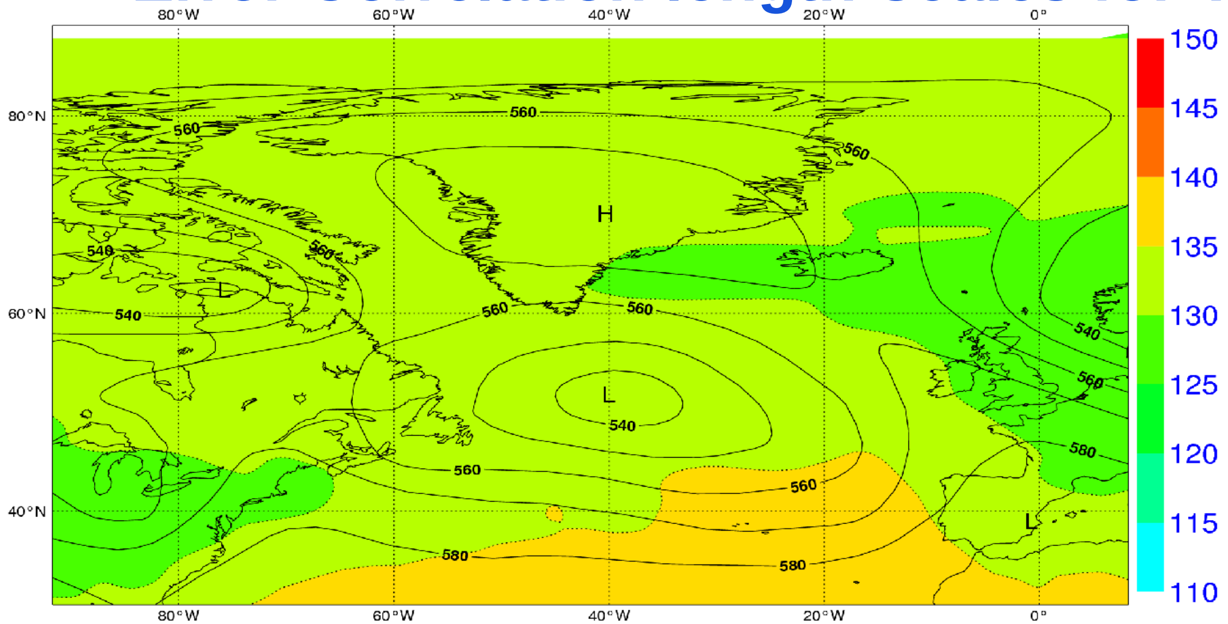
$$\left(\mathbf{x}-\mathbf{x}_b\right)=\mathbf{K}\boldsymbol{\Sigma}_b^{1/2}\sum_j \psi_j \otimes \left[\mathbf{C}_j^{1/2}(\lambda,\phi)\chi_j\right]$$

The computation of the wavelet B (i.e., the correlations ($C_j(\lambda,\varphi)$) requires considerably more EDA perturbations than those available from the latest EDA. For this reason they are estimated through a linear combination of a climatological wavelet B and perturbations from the latest EDA:

$$\mathbf{C}_{hybrid} = (1-\alpha)\mathbf{C}_{static} + \alpha\mathbf{C}_{online}$$

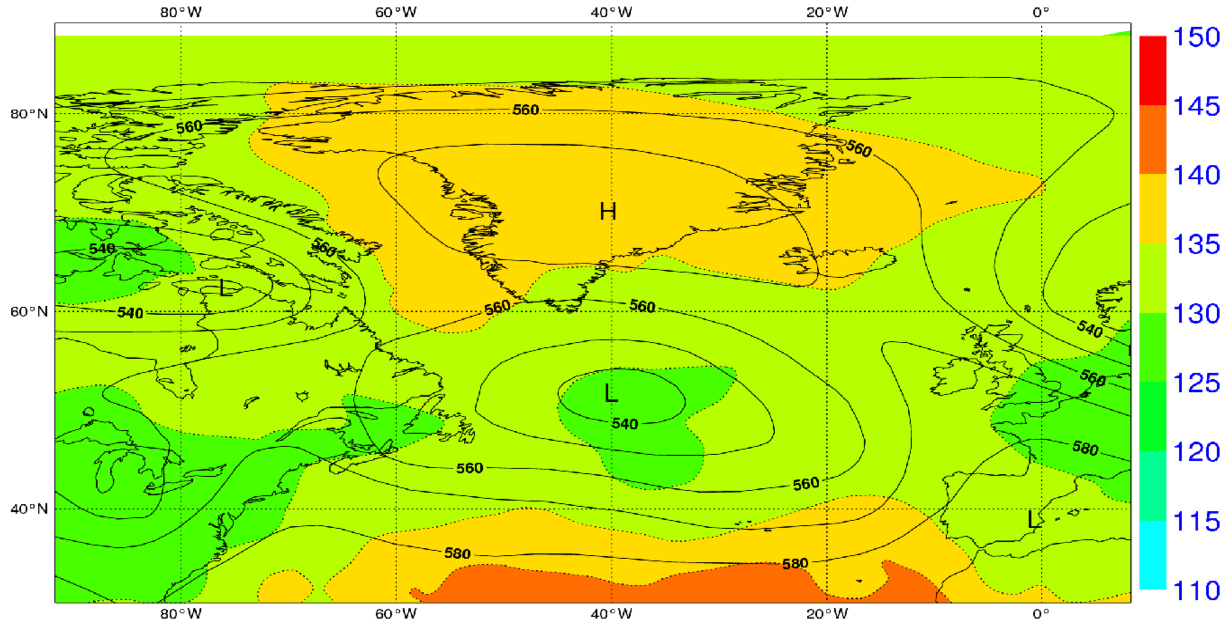
alpha is currently set at 0.3.

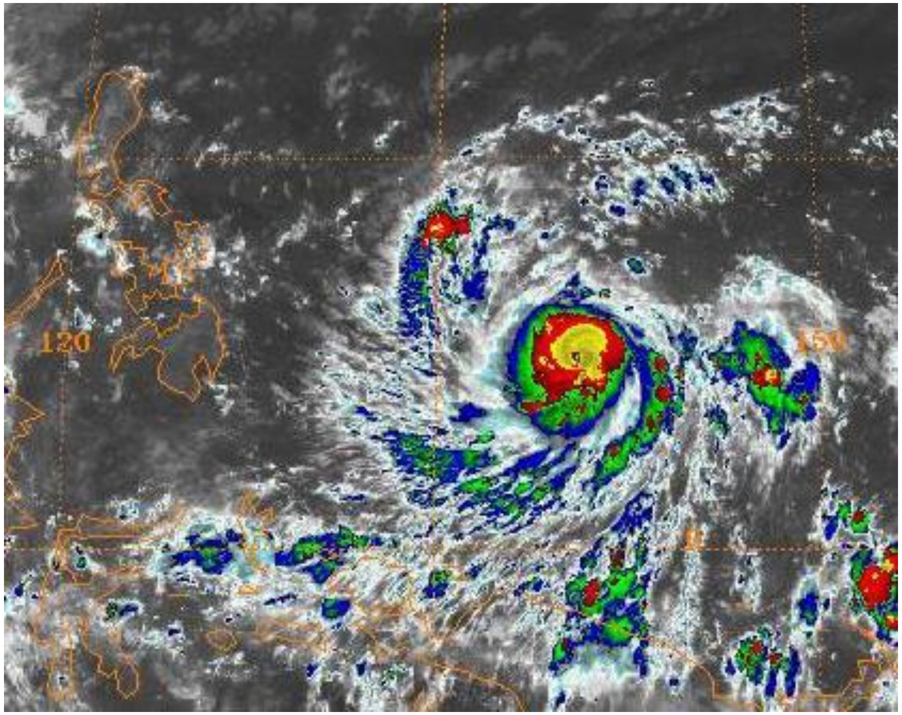
Error Correlation length-scales for Vorticity, 500 hPa



Static wavelet B

Hybrid wavelet B

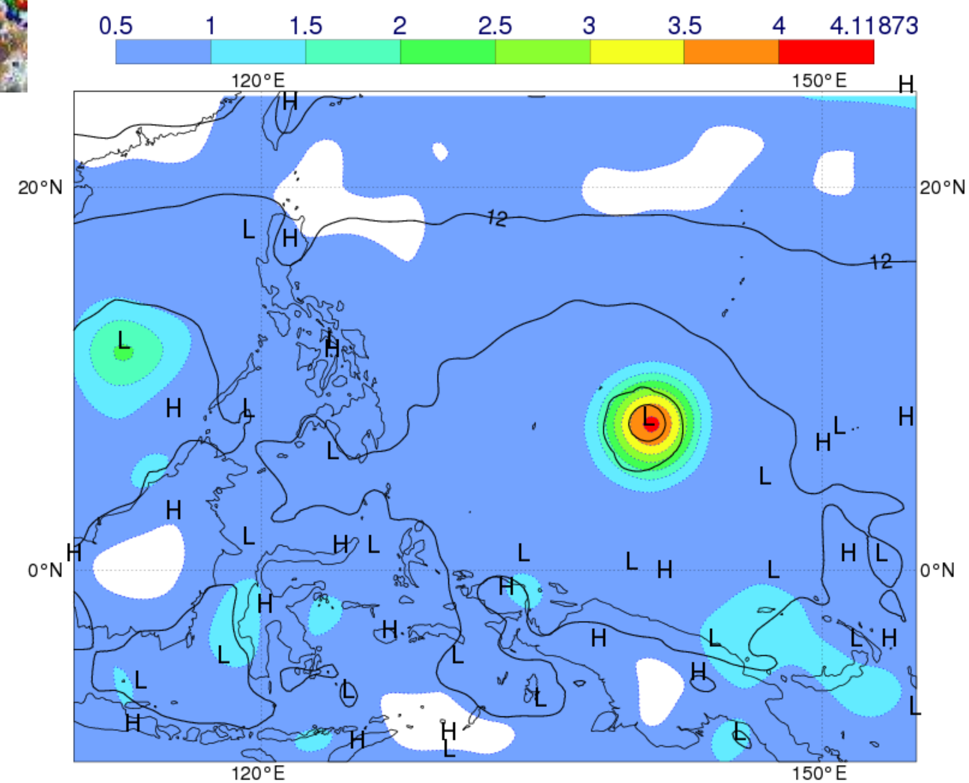




TYPHOON HAIYAN

MTSAT IR

2013-11-05 21UTC

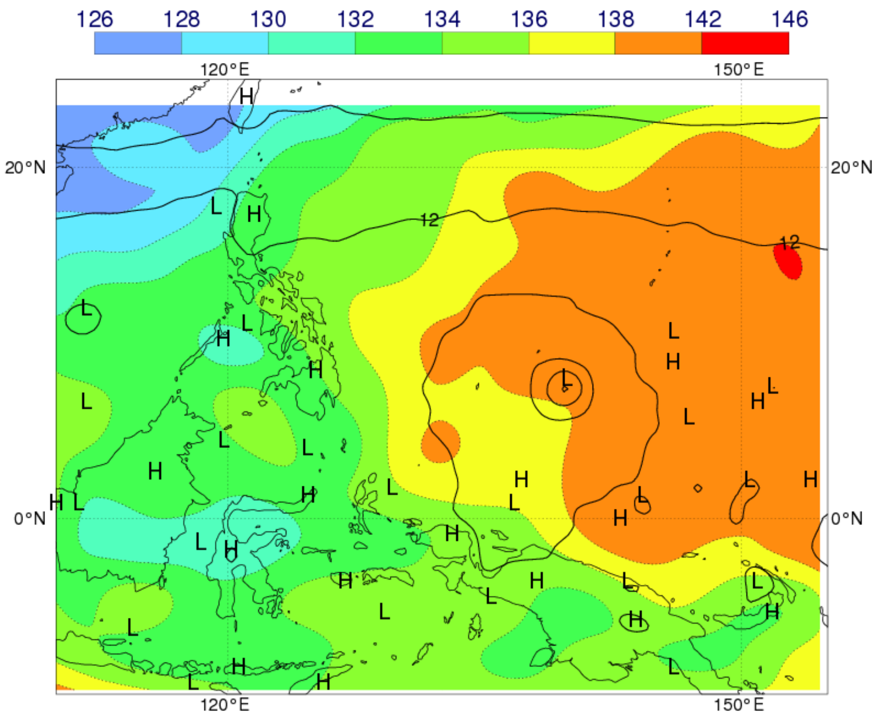


Z1000 BG (isolines)

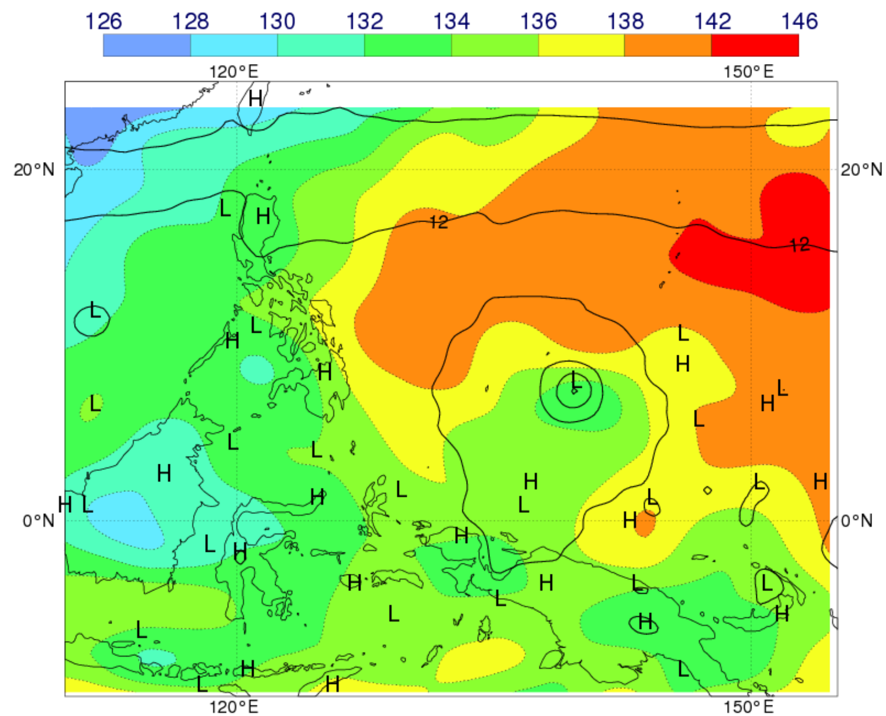
EDA Vorticity Spread (shaded) $10^{-5}s^{-1}$

valid at 2013-11-05 21UTC

Climatol. Wavelet B



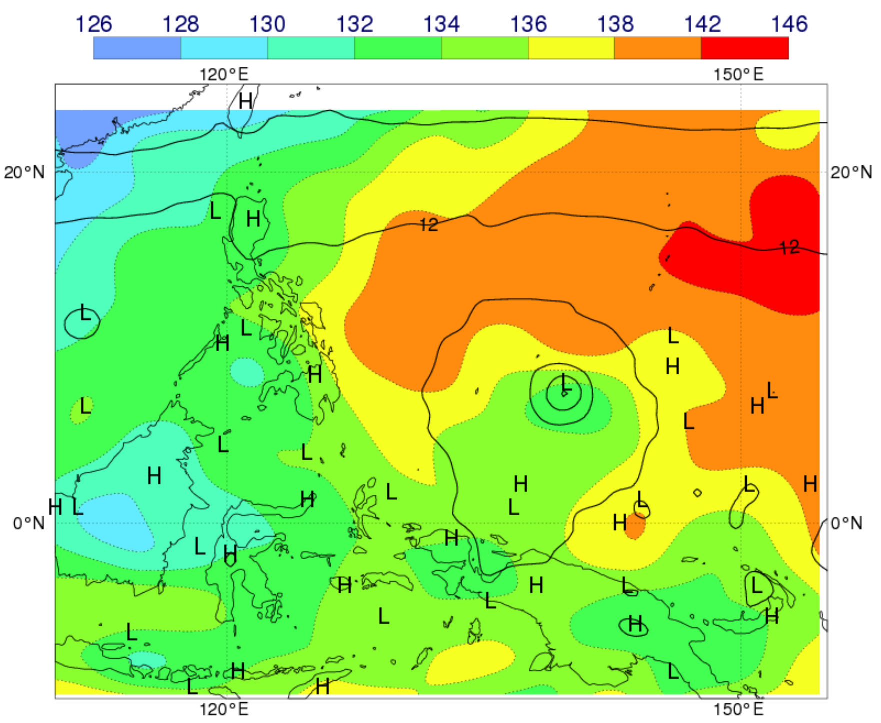
Hybrid Wavelet B
(α=0.3)



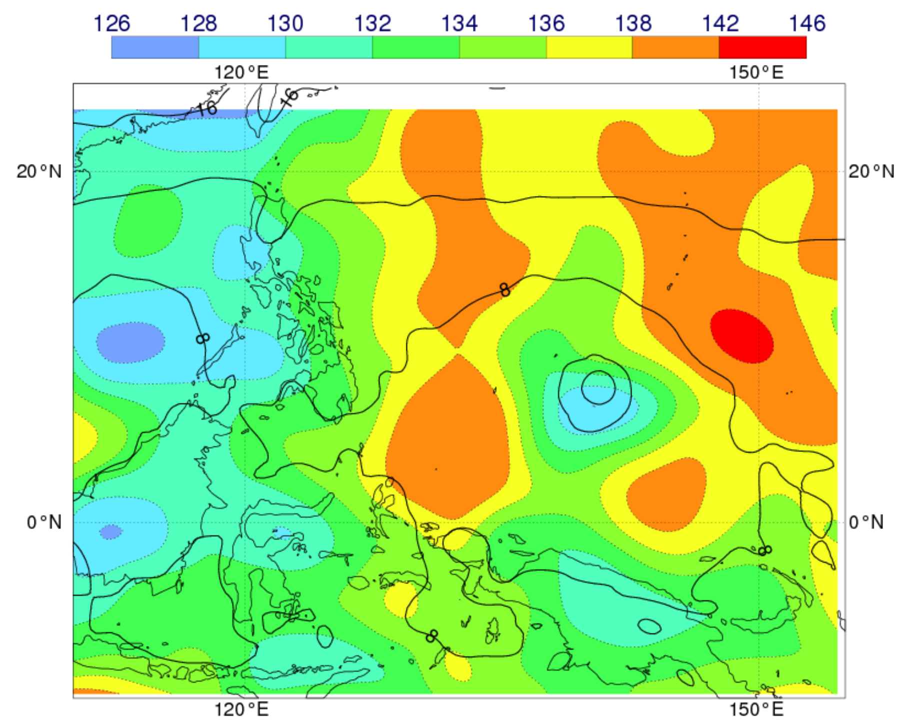
Vorticity errors length scale at the surface (shaded)
Geopotential height at 1000hPA (isolines)

“What if we had a 100 member ensemble DA?”

Hybrid B (α=0.3)



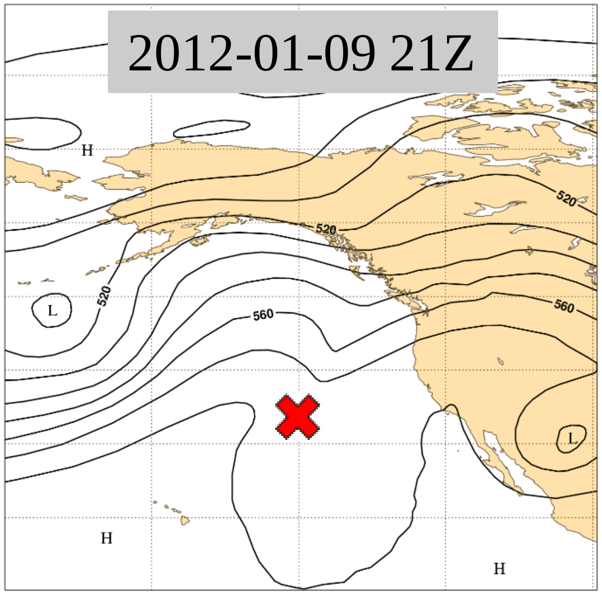
Hybrid B (α=0.7)



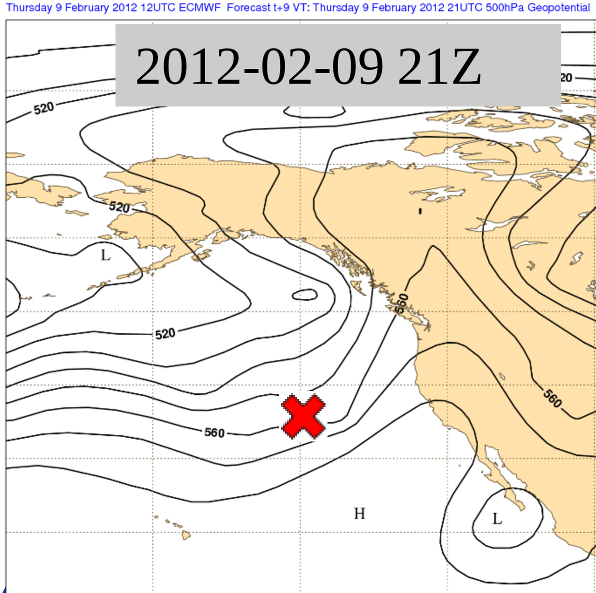
Vorticity errors length scale at the surface (shaded)
Geopotential height at 1000hPA (isolines)

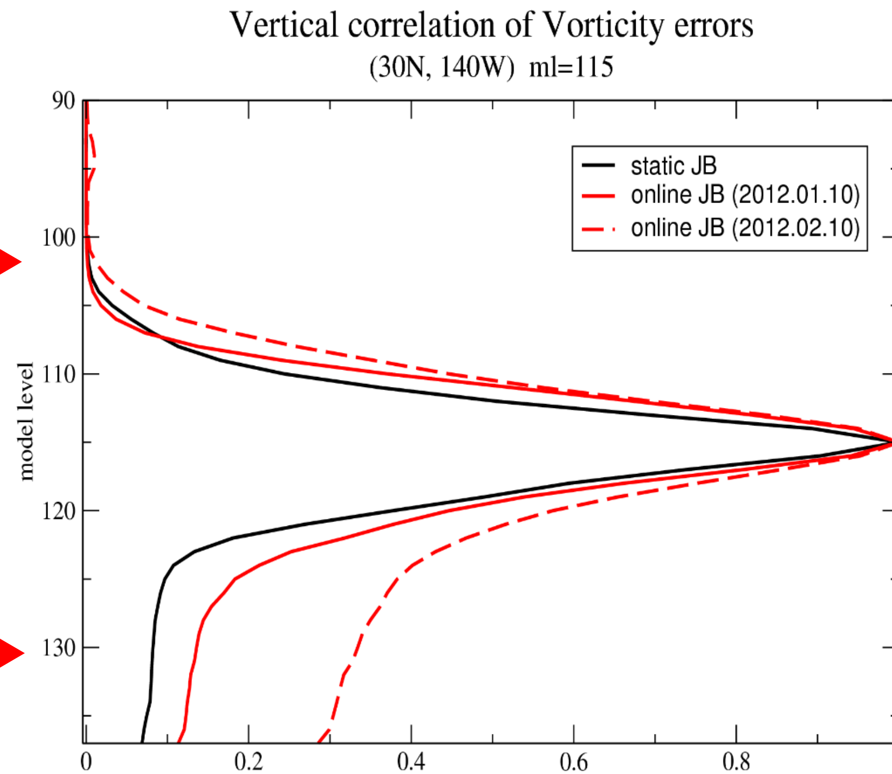
Vertical Error Correlation - Vorticity, 850 hPa

Monday 9 January 2012 12UTC ECMWF Forecast t+9 VT: Monday 9 January 2012 21UTC 500hPa Geopotential



Thursday 9 February 2012 12UTC ECMWF Forecast t+9 VT: Thursday 9 February 2012 21UTC 500hPa Geopotential

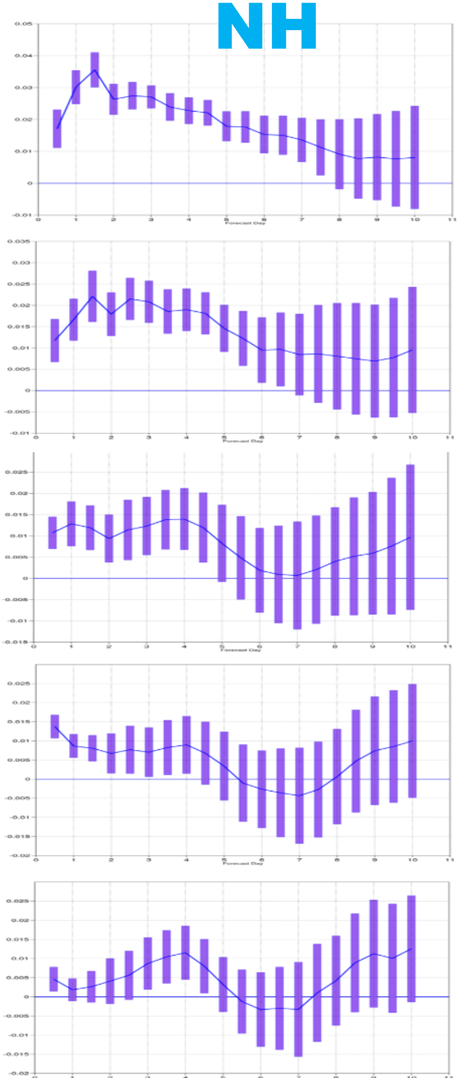




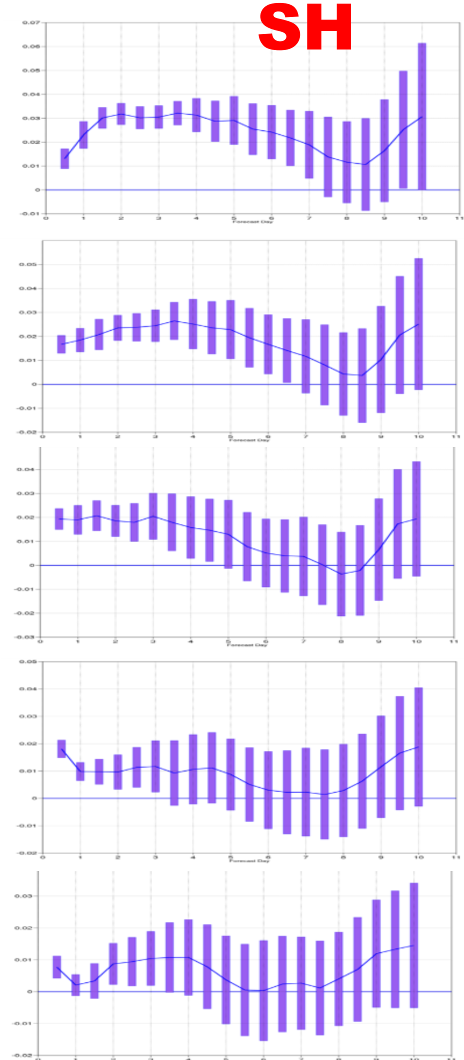
Impact of online wavelet B

Reduction in Geopotential RMSE - 95% confidence

NH



SH



50 hPa

100 hPa

200 hPa

500 hPa

1000 hPa

Period: Feb - June 2012

T511L91, 3 Outer Loops (T159/T255/T255)

Verified against operational analysis

Outline

- KF, EKF, EnKF
- Hybrid Var-EnKF methods
- The Ensemble of Data Assimilations (EDA) method
- Hybrid Gain Ensemble Data Assimilation

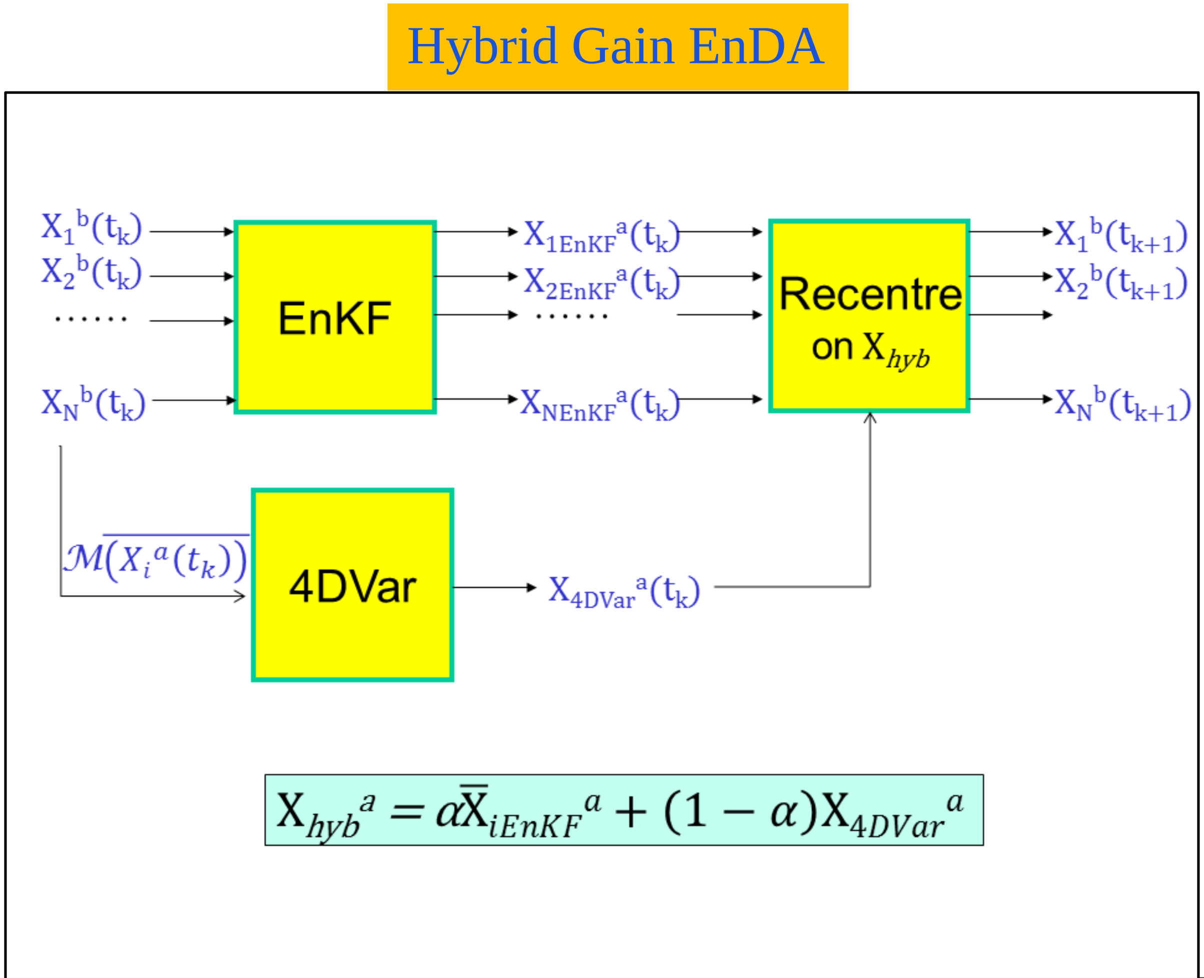
Hybrid Gain EnDA

Hybrid Gain EnDA (Hamrud et al., 2015; Bonavita et al., 2015)

- Based on ideas from Penny (2014)
- Majority of proposed Hybrid DA systems use ensemble to construct/augment/blend the $\mathbf{B}$ model used in a variational analysis update with current ensemble perturbations
- We have seen that EnKF and 4DVar (with a climatological $\mathbf{B}$) have comparable accuracy (at least at ECMWF!)
- We could just as well try blending the complete Kalman Gain matrices of the two systems (EnKF and 4DVar) in an EnKF framework

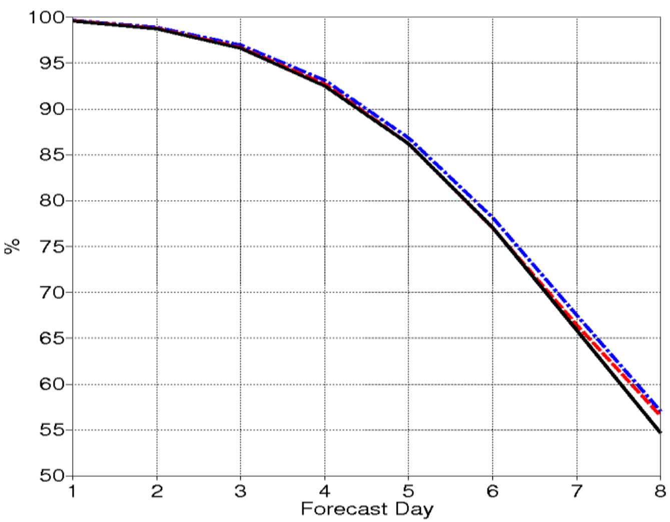
Hybrid Gain EnDA

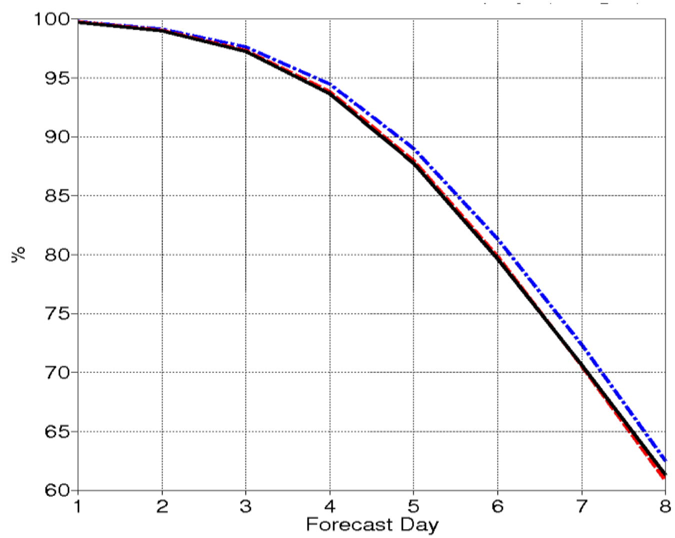
- Can we improve by blending two analysis system of similar quality inside the EnKF framework?

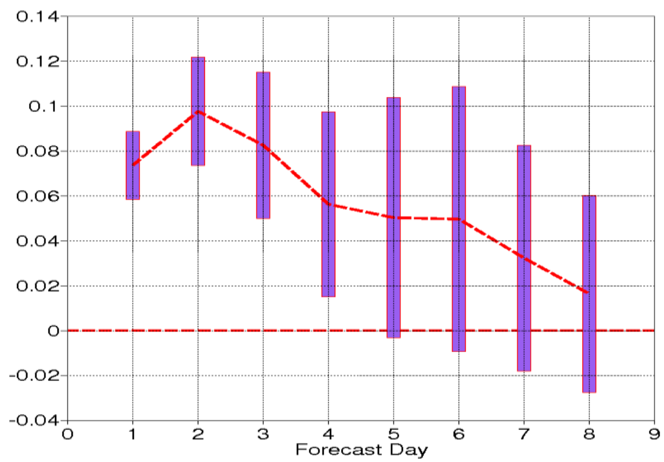


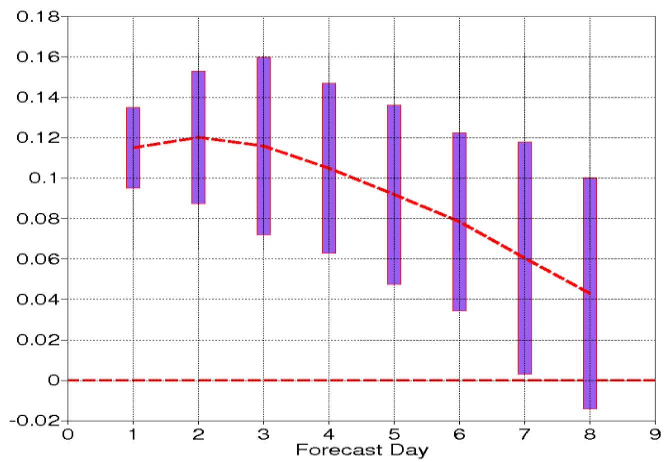
Hybrid Gain EnDA

TL399 100 member EnKF
TL399 100 member Hyb. Gain EnDA
TL399 4DVar – static B









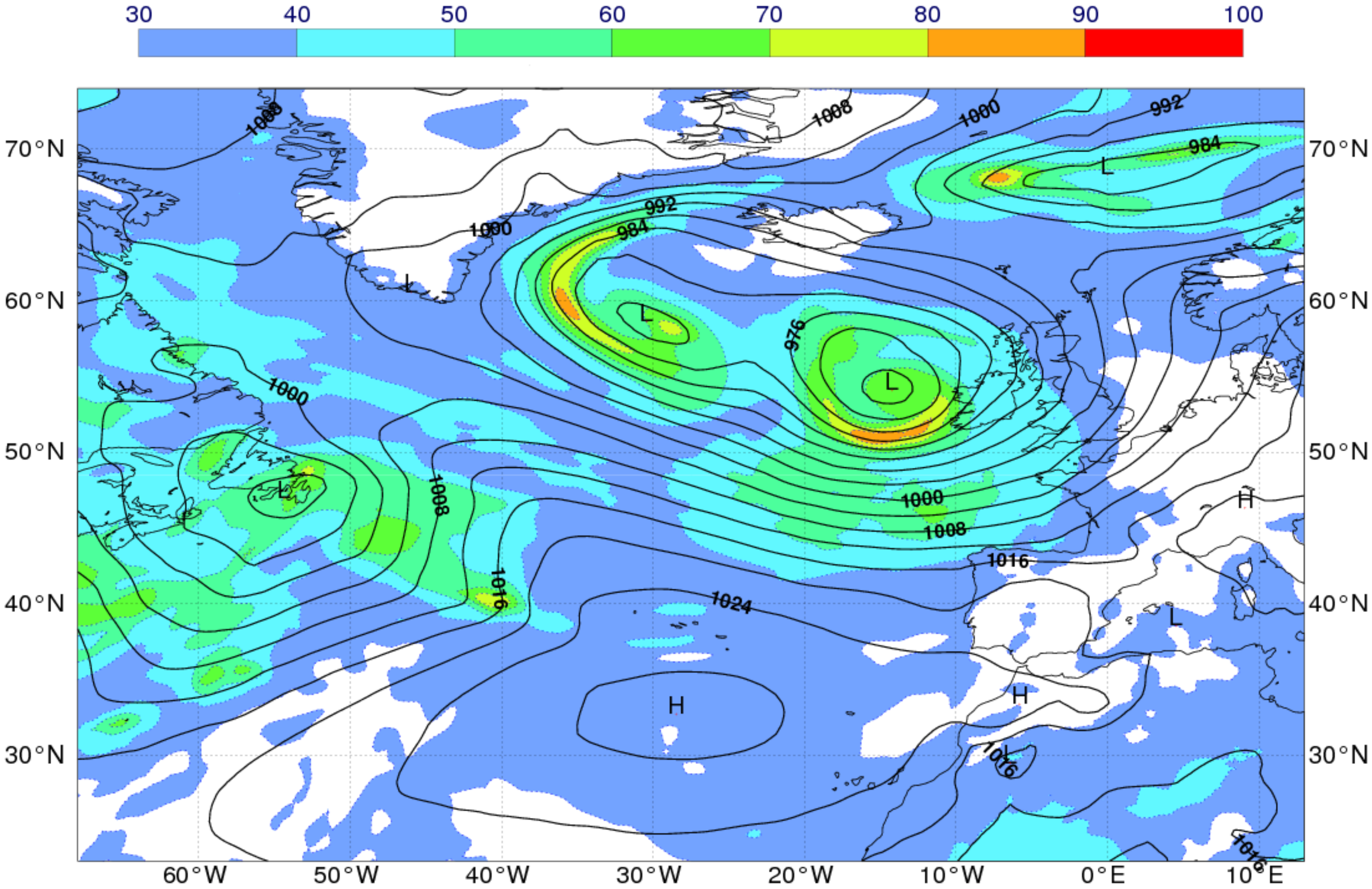
Z500 hPa AC - NHem

Z500 hPa AC - SHem

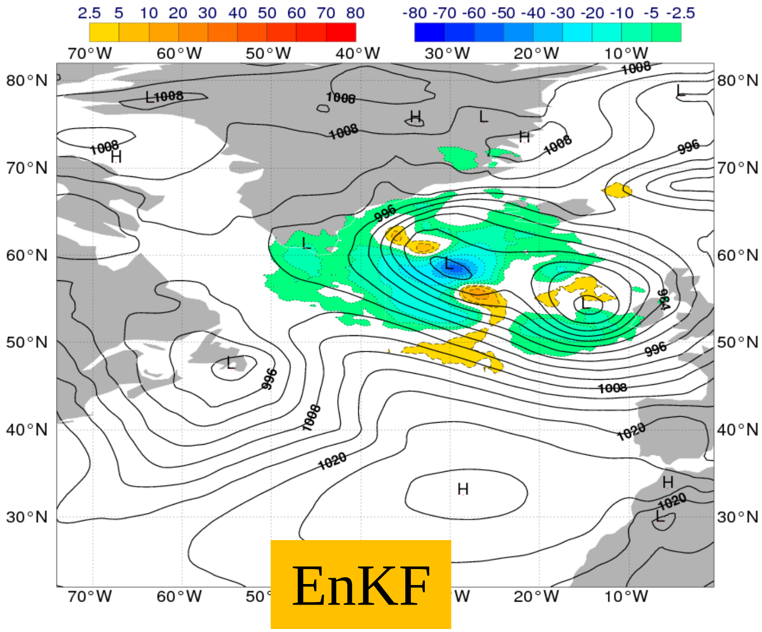
Hybrid Gain EnDA

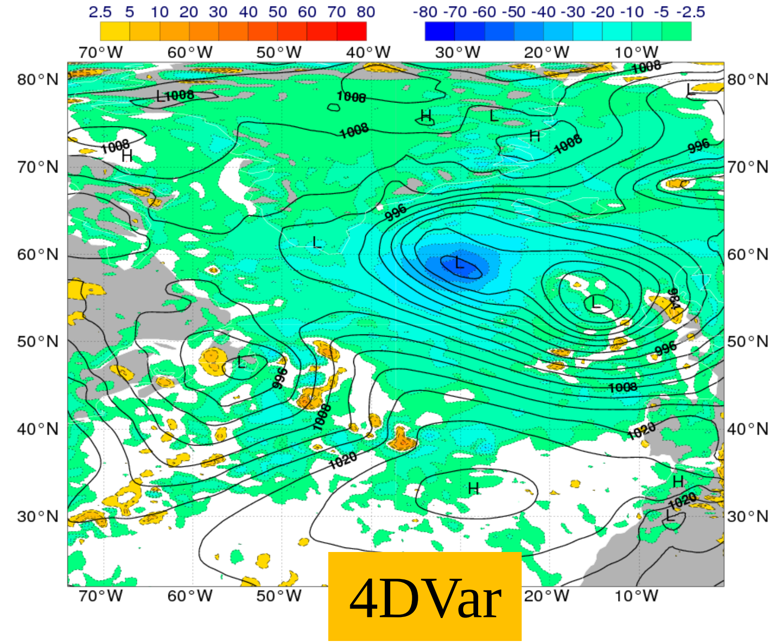
- **Hybrid Gain EnDA works surprisingly well. But why?**

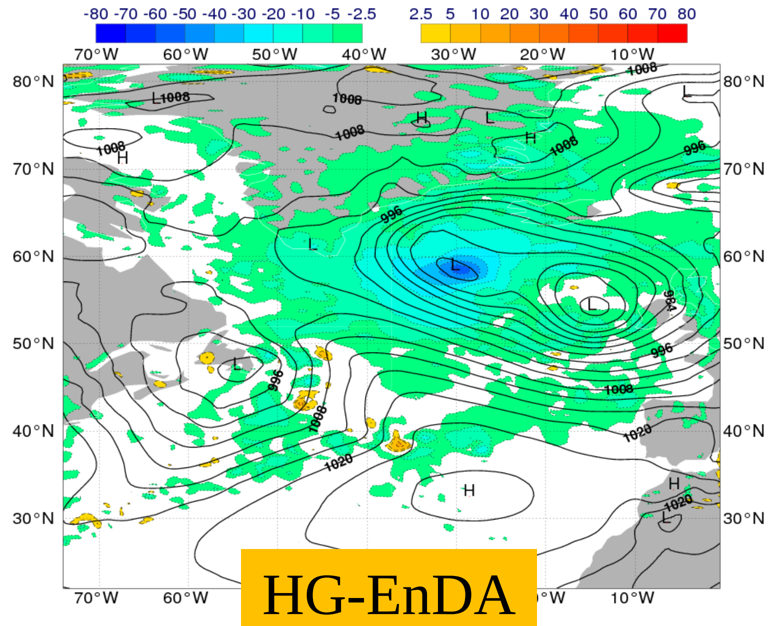
MSLP t+6h fcst and MSLP Ensemble stdev (shaded)
SP obs at (58.5N, 30.3W), middle of window, y-H(x)=-1hPa



SP obs at (58.5N, 30.3W), middle of window, y-H(x)=-1hPa







Hybrid Gain EnDA

- The positive effects of the HG-EnDA seem to originate from:
  1) **Mitigating the effects of localization in the EnKF increments**
  2) **Introducing climatological information in the EnKF covariance estimates**

  Note that similar effects have been reported elsewhere in the literature of hybrid DA systems (e.g., Huang et al., 2009)

- Ideally we would like to keep more of the EnKF flow-dependent structures near the observation location and gradually revert to the climatological covariances of 4DVar farther away: we are now looking at a scale-dependent blending of the two analyses ($\alpha = \alpha(n)$)

Summary

- Traditional view of data assimilation: provide the best (minimum variance, most likely) estimate of the initial state plus its uncertainty (error bars!)

- The Kalman Filter provides the solution to the data assimilation problem under mildly restrictive conditions for global NWP: linear model evolution over the background forecast length (3-12hours) and linear observation operators

- The standard Kalman Filter can not be implemented in large-dimensional systems (like NWP!) because it is impossible to explicitly compute and evolve $\mathbf{P}^a/\mathbf{P}^b$

ECMWF

Summary

- 4D-Var and the EnKF provide two computationally tractable approximations to the Kalman Filter

- Standard 4D-Var uses a model of $\mathbf{P}^b$ (at ECMWF the wavelet model) and does not compute $\mathbf{P}^a$. The modelled $\mathbf{P}^b$ evolves during the 4D-Var assimilation window but is not cycled: each 4D-Var analysis starts with a climatological estimate of $\mathbf{P}^b$

- The EnKF solves the dimensionality problem of the Kalman Filter by reducing the space in which $\mathbf{P}^{a/b}$ are computed to the space spanned by the ensemble perturbations (N_{ens}-1)

- Both 4D-Var and the EnKF thus introduce further approximations to the Kalman Filter

Summary

- Hybrid DA methods try to combine the strengths of standard 4D-Var and the EnKF
- Hybrid DA methods have mostly be implemented as variants on pre-existing Variational DA systems where the **B** used in the variational analysis is supplemented or completely determined by forecast perturbations from a parallel EnKF/EnDA system (extended control variable, 4D-En-Var, hybrid EDA 4DVar)
- We have seen that the symmetric approach is also feasible: supplement an EnKF-based DA with a variational component (Hybrid Gain EnDA)
- There does not seem to be any fundamental reasons to favour one hybrid over another. Practical considerations should guide your choice (computational efficiency, scalability, ease of implementation, etc.)

Summary

- Traditional view of data assimilation: provide the best (minimum variance and/or most likely) estimate of the initial state plus its uncertainty (error bars!)
- But if we accept that weather forecasting is a probabilistic exercise, then the defining task of data assimilation is to provide the "best" representation of the initial pdf of the atmospheric state
- The more this initial pdf differs from a Gaussian distribution, the more the Kalman Filter paradigm will need to be revisited and more general methods will have to be considered
- In either case, ensemble data assimilation will become ever more important as it currently is the only practical method to sample this pdf.

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Additional Slides

Spectral B model

In variational analysis the B matrix is usually defined implicitly in terms of a transformation from the departure δx in state space to a control variable χ:

$$\boldsymbol{\delta x} = \mathbf{x}-\mathbf{x_b} = \mathbf{L}\boldsymbol{\chi}$$

where $\mathbf{L}$ verifies $\mathbf{B}=\mathbf{LL}^{\mathbf{T}}$

In the spectral formulation (Derber and Bouttier, 1999), the change of variable $\mathbf{L}$ has the form:

$$\mathbf{L} = \mathbf{K}\,\mathbf{B_u}^{\mathbf{1/2}}$$

where $\mathbf{K}$ is a balance operator going from the set of “unbalanced “ variables [ζ, η_u, $(T,ps)_u$,q] (the “control vector”) to the set of state variables [ζ, η,(T,ps),q]

There is a degree of flow-dependence in $\mathbf{K}$ as the balance constraints are linearised about the first-guess trajectory

Spectral B model

$$\delta \mathbf{x} = \mathbf{x} - \mathbf{x_b} = \mathbf{L}\chi \qquad \mathbf{L} = \mathbf{K}\,\mathbf{B_u}^{1/2}$$

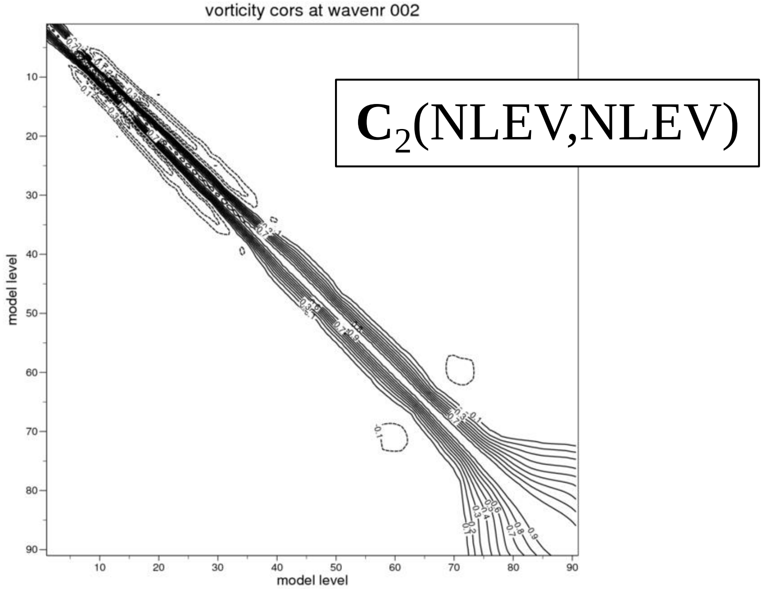
Since we assume that the balance operator accounts for all inter-variable correlations, B_u is block diagonal

$$\mathbf{B}_u = \begin{pmatrix} \mathbf{B}_\zeta & 0 & 0 & 0 \\ 0 & \mathbf{B}_{D_u} & 0 & 0 \\ 0 & 0 & \mathbf{B}_{(T,p_s)_u} & 0 \\ 0 & 0 & 0 & \mathbf{B}_q \end{pmatrix}$$

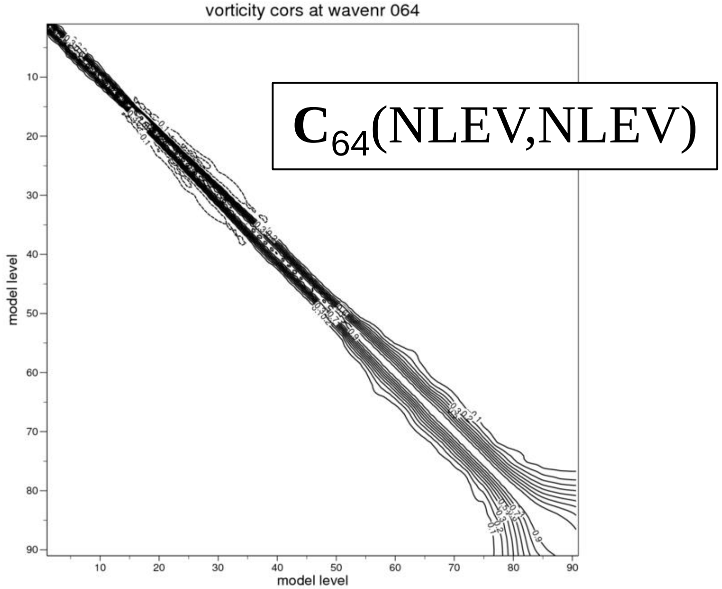
Each block in B_u is of the form $\mathbf{\Sigma^T C \Sigma}$.

$\mathbf{\Sigma}$ is the gridpoint standard deviation of background errors.

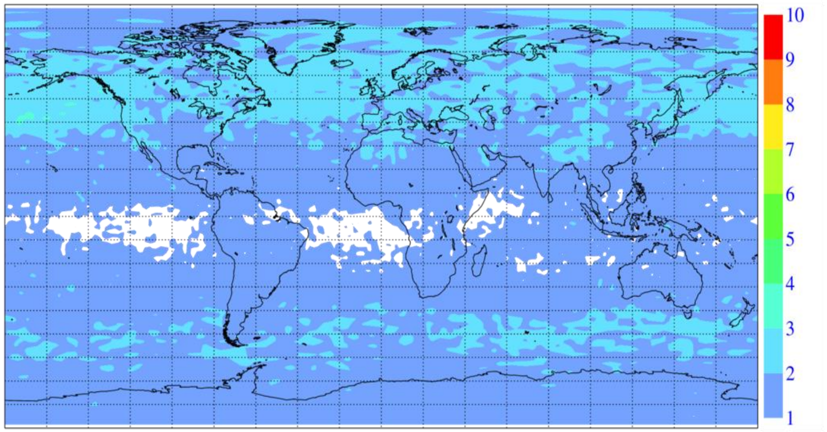
$\mathbf{C}$ models the autocorrelation of the control variables. It is block diagonal with one full vertical correlation matrix for each spectral wavenumber, i.e. C_n(NLEV,NLEV) (non-separable B model)



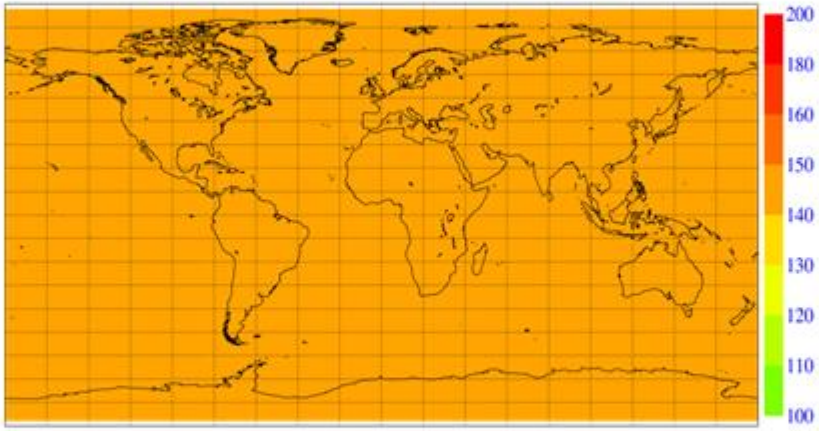
Vorticity correl. wavenum=2



Vorticity correl. wavenum=64



Vorticity bg error stdev, 500hPa



Vorticity bg error corr. Lscale, 500hPa

From Spectral to Wavelet B model

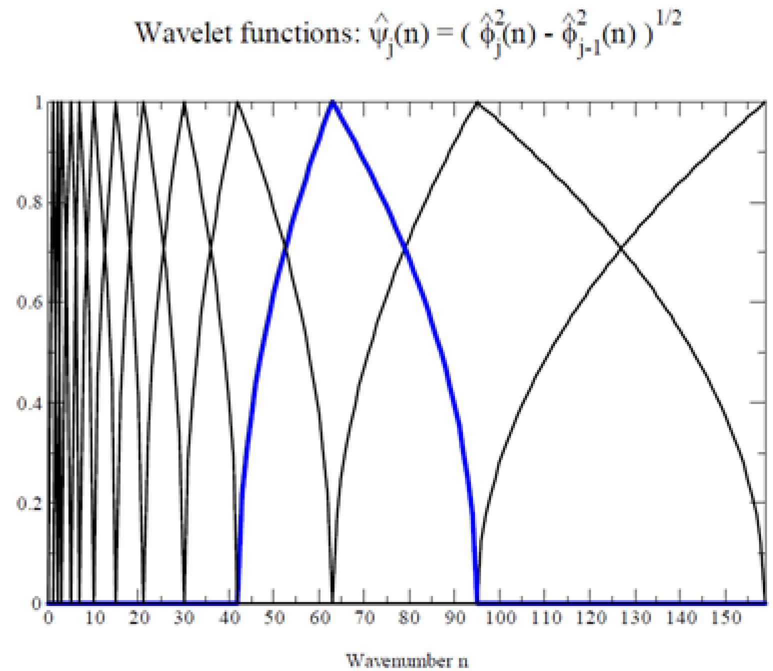
- The spectral B model is one end of the spectrum: full resolution of the variation of vertical correlation with horizontal scale, but it allows no horizontal variability of the vertical/horizontal correlations

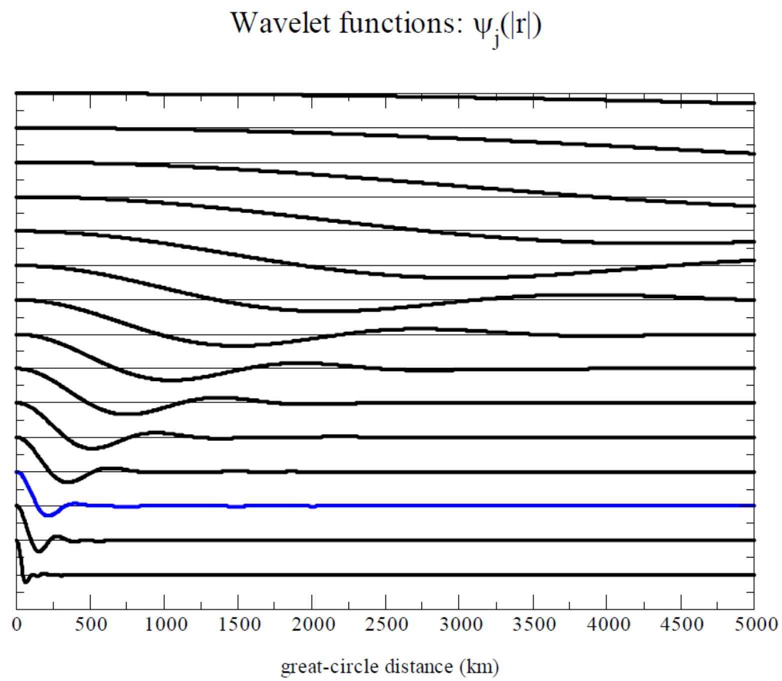
- The other end of the spectrum is represented by the separable formulation which allows full horizontal variation of the correlations (we may specify a different vertical covariance matrix for each horizontal grid point), but has no variation of vertical correlation with horizontal scale

- The wavelet B (Fisher, 2003) is a compromise between these two extremes and allows a degree of variation of correlation with both wavenumber and horizontal location

Wavelet B model

- The wavelet **B** is based on a wavelet expansion on the sphere.
- The basis functions (wavelets) are chosen to be band-limited and, to a good approximation, spatially localized

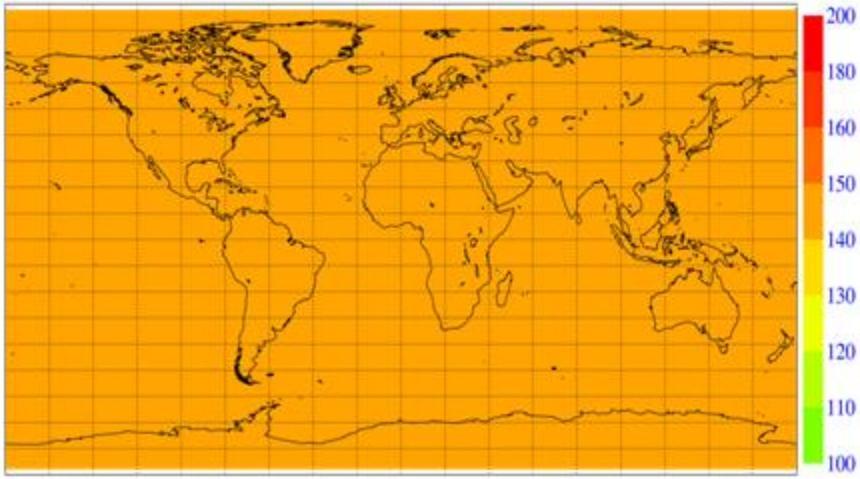




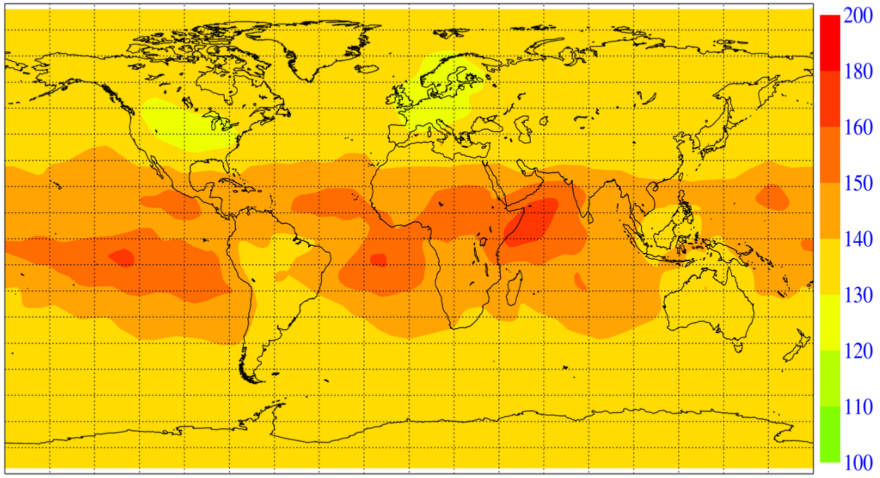
Wavelet B model

- The correlation matrices $\mathbf{C}_n[N_{lev} x N_{lev}]$ are now of the form $\mathbf{C}_j[N_{lev} x N_{lev}](\lambda,\varphi)$, where j is now the index of the wavelet component

- The choice of the wavelet bandwidths $[N_j, N_{j+1}]$ determines the trade-off between spectral and spatial resolution. If the bands are narrow, the corresponding wavelet functions are not spatially localized, and vice versa



Climat. Spectral B
Vorticity bg error corr. Lscale, 500hPa



Climat. Wavelet B
Vorticity bg error corr. Lscale, 500hPa

Flow-dependent wavelet B model

The wavelet **B** formulation:

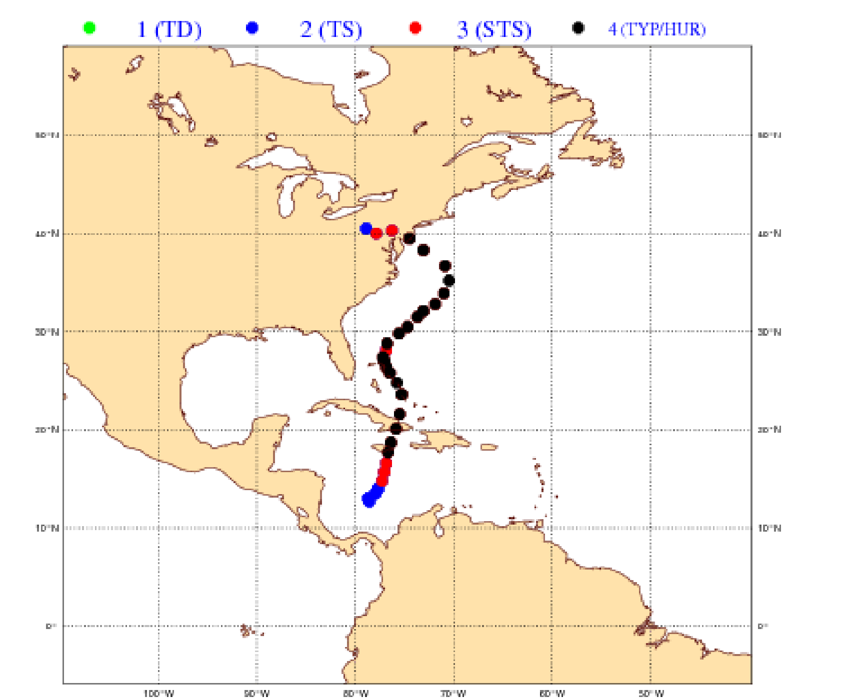
$$\left(\mathbf{x}-\mathbf{x}_b\right)=\mathbf{L}\boldsymbol{\chi}=\mathbf{K}\boldsymbol{\Sigma}_b^{1/2}\sum_j \psi_j \otimes \left[\mathbf{C}_j^{1/2}(\lambda,\phi)\chi_j\right]$$

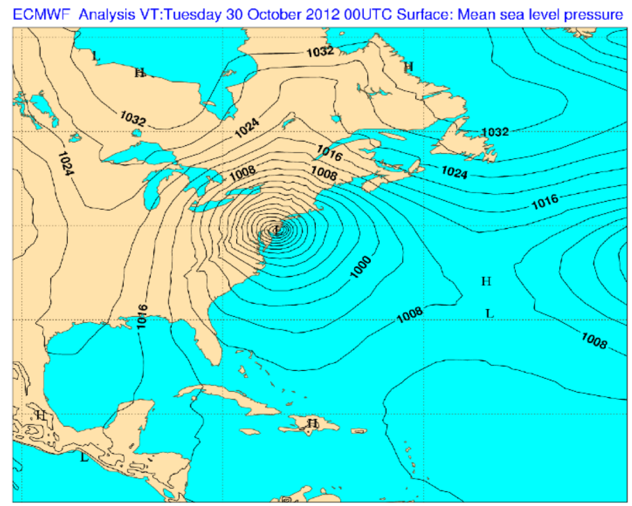
can be made flow-dependent by obtaining flow-dependent estimates of the background error variances ($\boldsymbol{\Sigma}_b$) and correlations ($\mathbf{C}_j(\lambda,\varphi)$) from the EDA background perturbations

Use of EDA variances in 4DVar

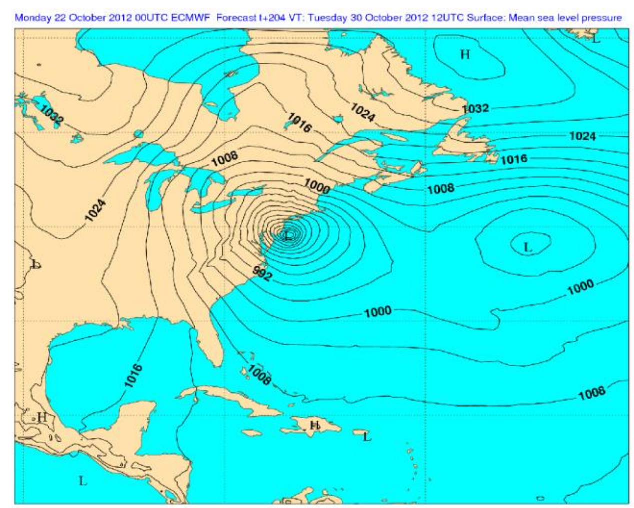
2. Before 4DVar they affect the observation quality control decisions

- Super Storm Sandy





Mslp Ana 30/10/2012 00UTC



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Use of EDA variances in 4DVar

What happens if we withhold polar-orbiters observations (i.e., approx. 90% of obs. counts)?

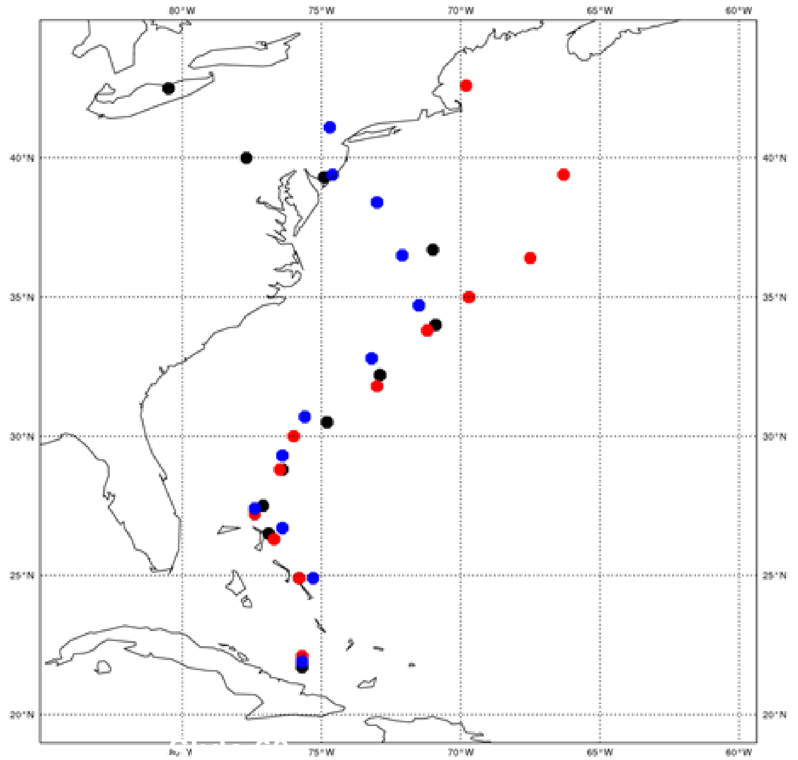
The forecast performance is obviously degraded, and only 5 days before landfall the system recovers the correct track

Sandy's forecast tracks 25 Oct 2013 00UTC

Operational forecast

Forecast from HRES assimilation cycle without polar orbiters and errors from operational EDA

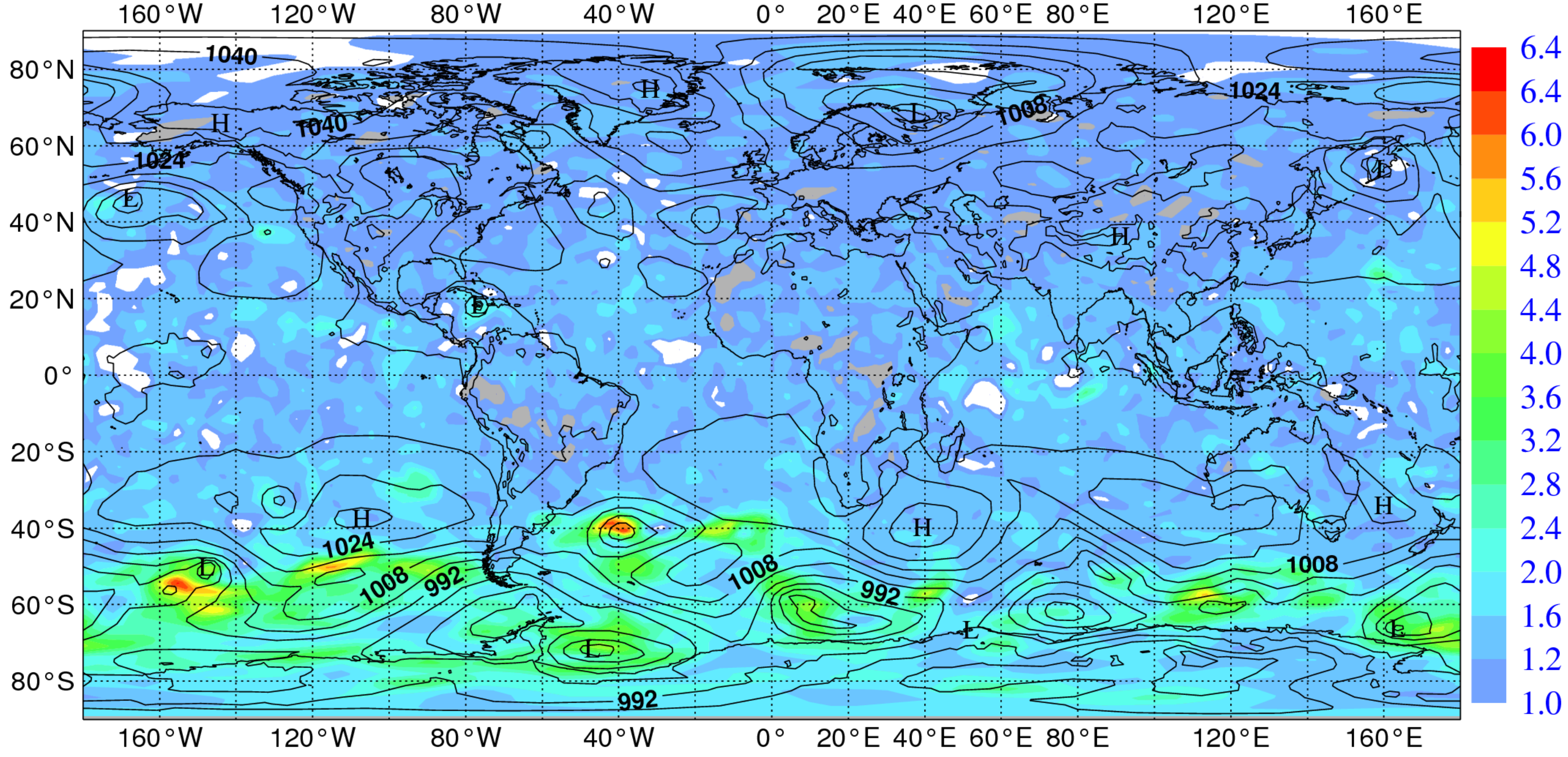
Forecast from HRES assimilation cycle and EDA both without polar orbiters data



Use of EDA variances in 4DVar

EDA without polar orbiters' data has larger spread than operational EDA

EDA spread for u-wind component at 850 hPa: No-Polar/Oper ratio



ECMWF

Use of EDA variances in 4DVar

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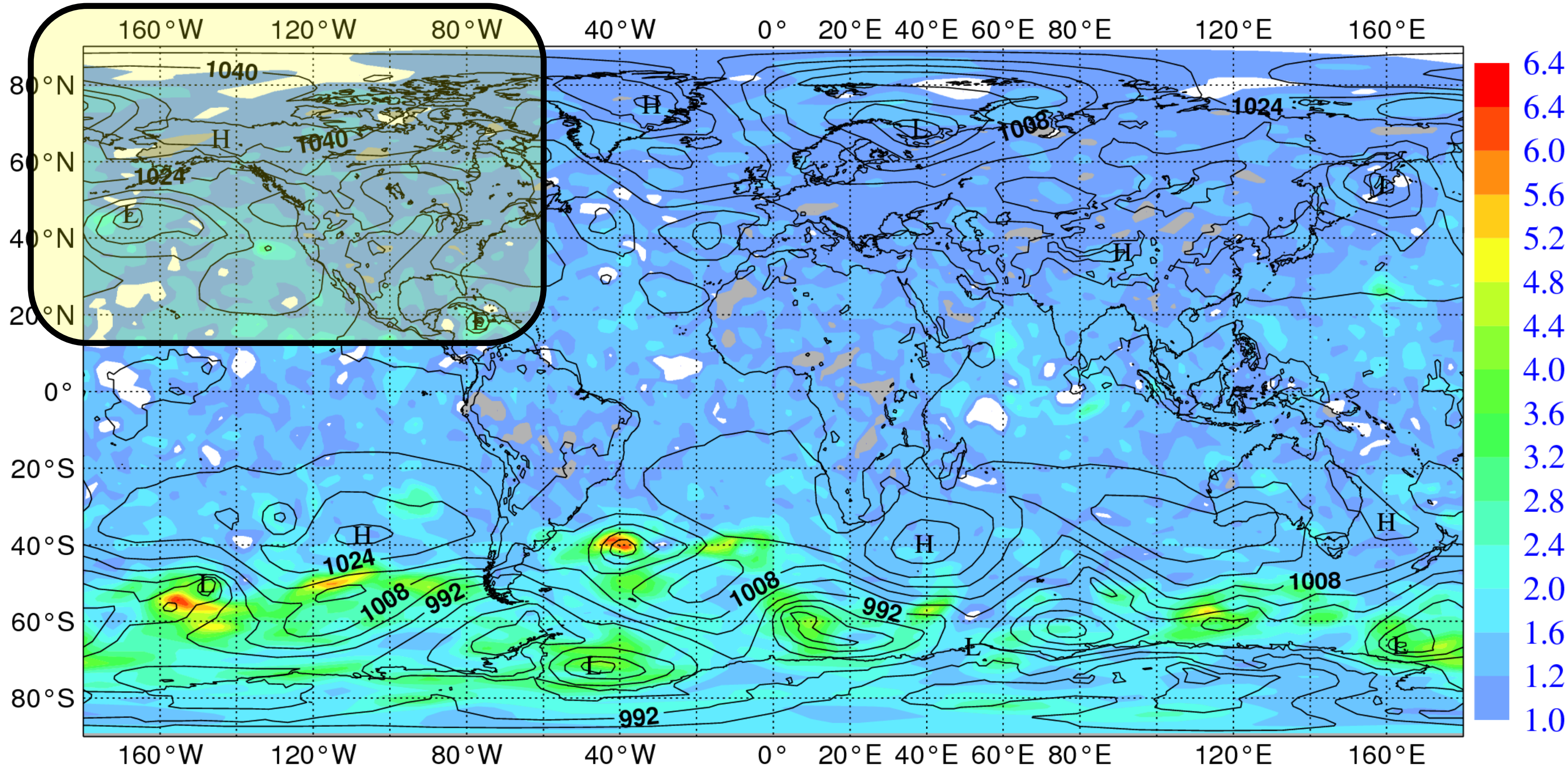

ECMWF

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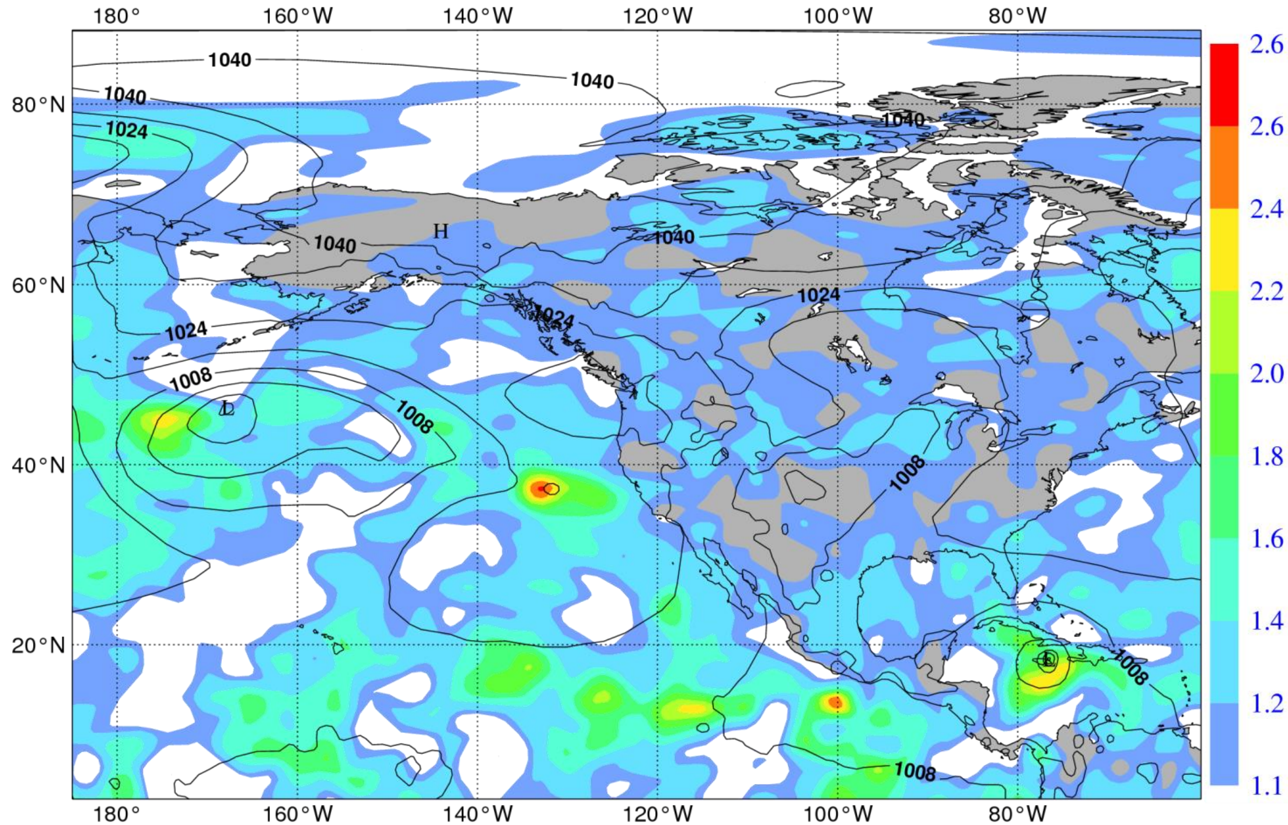
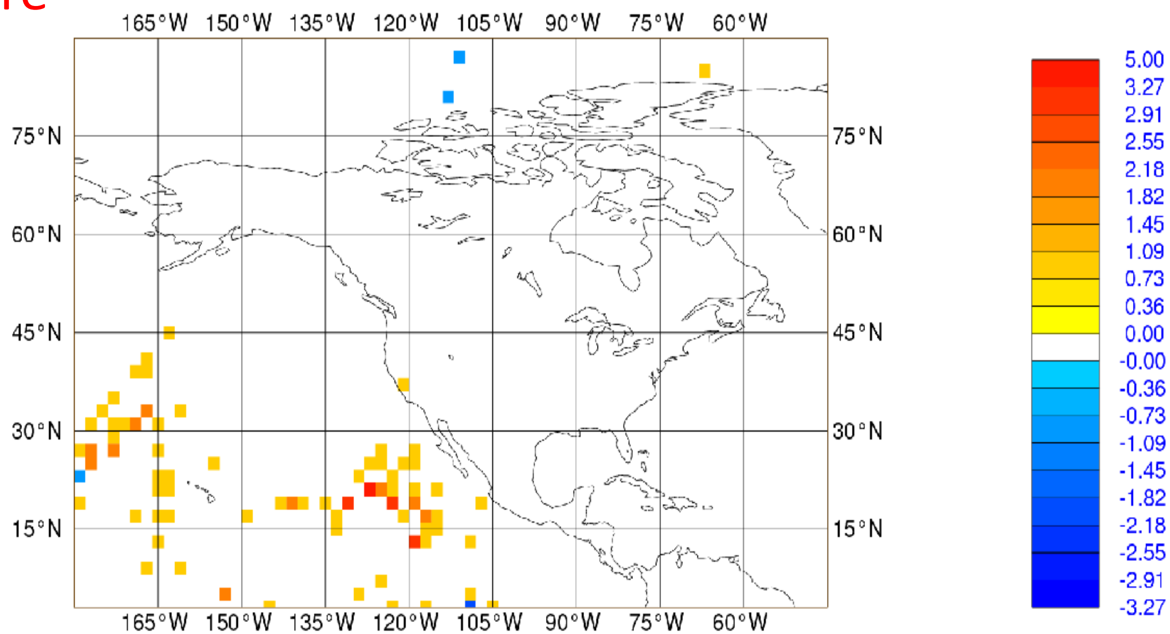


Use of EDA variances in 4DVar

EDA without polar orbiters' data has larger spread than operational EDA

This has two effects: a) Observations are more closely fit and b) More observations pass first guess quality control: $(y-\mathcal{H}(x))^2 \le \alpha(\sigma_b^2+\sigma_o^2)$

In this case more AMVs from geostationary satellites are assimilated



In this case more AMVs from geostationary satellites are assimilated