



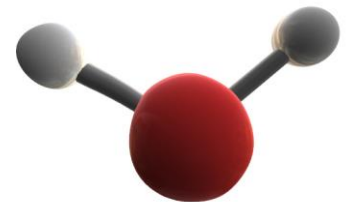
Radiative transfer in numerical models of the atmosphere

Robin Hogan

*Slides contain contributions from
Jean-Jacques Morcrette, Alessio
Bozzo, Tony Slingo and Piers Forster*

Outline

- Lectures 1 & 2
 1. Global context
 2. From Maxwell to the two-stream equations
 3. Gaseous absorption and emission
- Lecture 3 (Alessio Bozzo)
 - The ECMWF radiation scheme
- Lecture 4
 4. Representing cloud structure
 5. Some remaining challenges



Further reading

- Petty, G., 2006: *A first course on atmospheric radiation*
- Liou, K.-N., 1992: *Radiation and Cloud Processes in the Atmosphere*. Oxford University Press, 487 pp.
- Morcrette, J.-J., H. W. Barker, J. N. S. Cole, M. J. Iacono, R. Pincus, 2008: Impact of a New Radiation Package, McRad, in the ECMWF Integrated Forecasting System. *Mon. Wea. Rev.*, **136**, 4773–4798.
- Randall, D. A., 2000: *General circulation model development*
- Manabe, S., and R. F. Strickler, 1964: Thermal equilibrium of the atmosphere with a convective adjustment. *J. Atmos. Sci.*, **21**, 361-385
- Hogan, R. J., and J. K. P. Shonk, 2009: Radiation parametrization and clouds. Proc. ECMWF Seminar 1-4 Sept 2008.
- Scattering animations <http://www.met.rdg.ac.uk/clouds/maxwell>

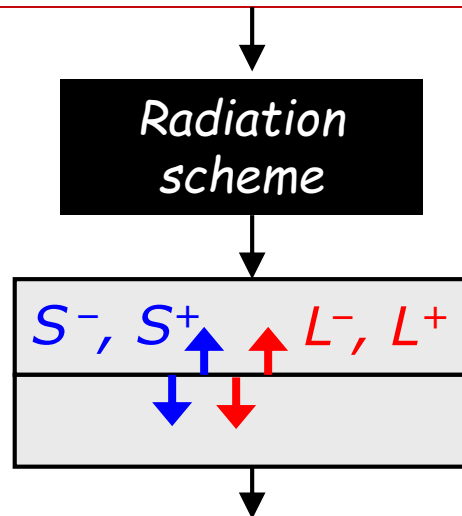
Part 1: Global context



- What does a radiation scheme do?
- How does radiation determine global temperature?
- What is the role of radiation in the global circulation?
- How do we evaluate radiation schemes globally?

What does a radiation scheme do?

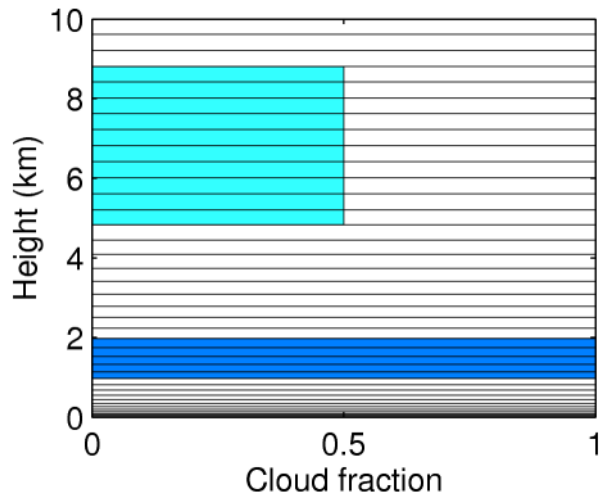
- *Prognostic variables*: temperature, humidity, cloud fraction, liquid and ice mixing ratios, surface temperature
- *Diagnostic variables*: sun angle, surface albedo, pressure, O₃, aerosol; well-mixed gases: CO₂, O₂, CH₄, N₂O, CFC-11 and CFC-12
- *MACC project can provide prognostic aerosols, CO₂ and CH₄*



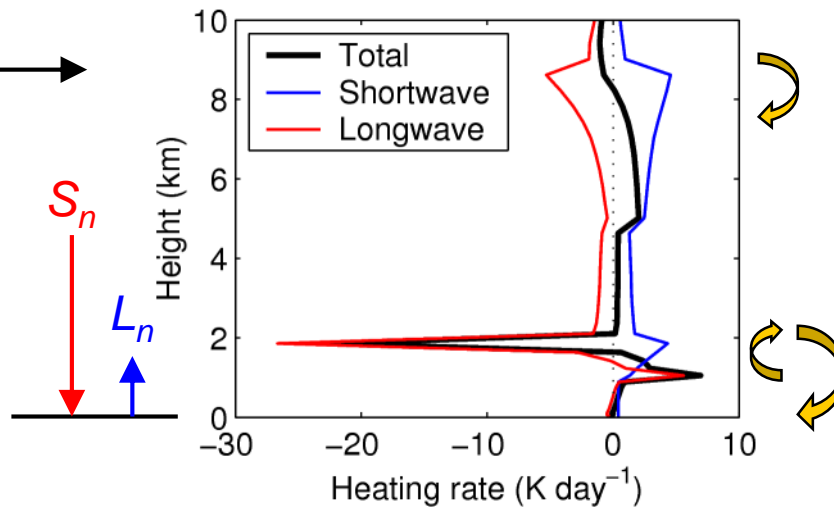
- Fluxes / irradiances between model levels in W m⁻²
- Net flux $R_n = S^- - S^+ + L^- - L^+$

- Thermodynamic equation:
$$\frac{D\bar{\theta}}{Dt} = \frac{1}{\rho C_p} \frac{\partial R_n}{\partial z} + latent + \dots$$
- Radiation terms in surface energy balance: soil & sea temperatures

Heating rate profiles



Radiation scheme



Radiation in the presence of clouds tends to destabilize the atmosphere

Radiation is unlike most other diabatic processes because

- There exists a well known theory:
 - Quantum Mechanics to Spectroscopy
 - Maxwell's equations to Radiative Transfer
- Radiation exchanged with outside space: balance determines the climate
- The sun providing the energy input so seasonal and diurnal forcings
- Radiation at Top Of Atmosphere (TOA) has been globally measured since the 60's (by operational satellites), with real flux measurements from ERB (1978), ERBE (1985), ScaRaB (1993), CERES (1998)
- Surface radiation has been (roughly) measured at points over almost 40 years. Present programs like ARM, BSRN, SURFRAD measure it with high accuracy
- Observations to evaluate the ability to predict spectral fluxes are also coming on line

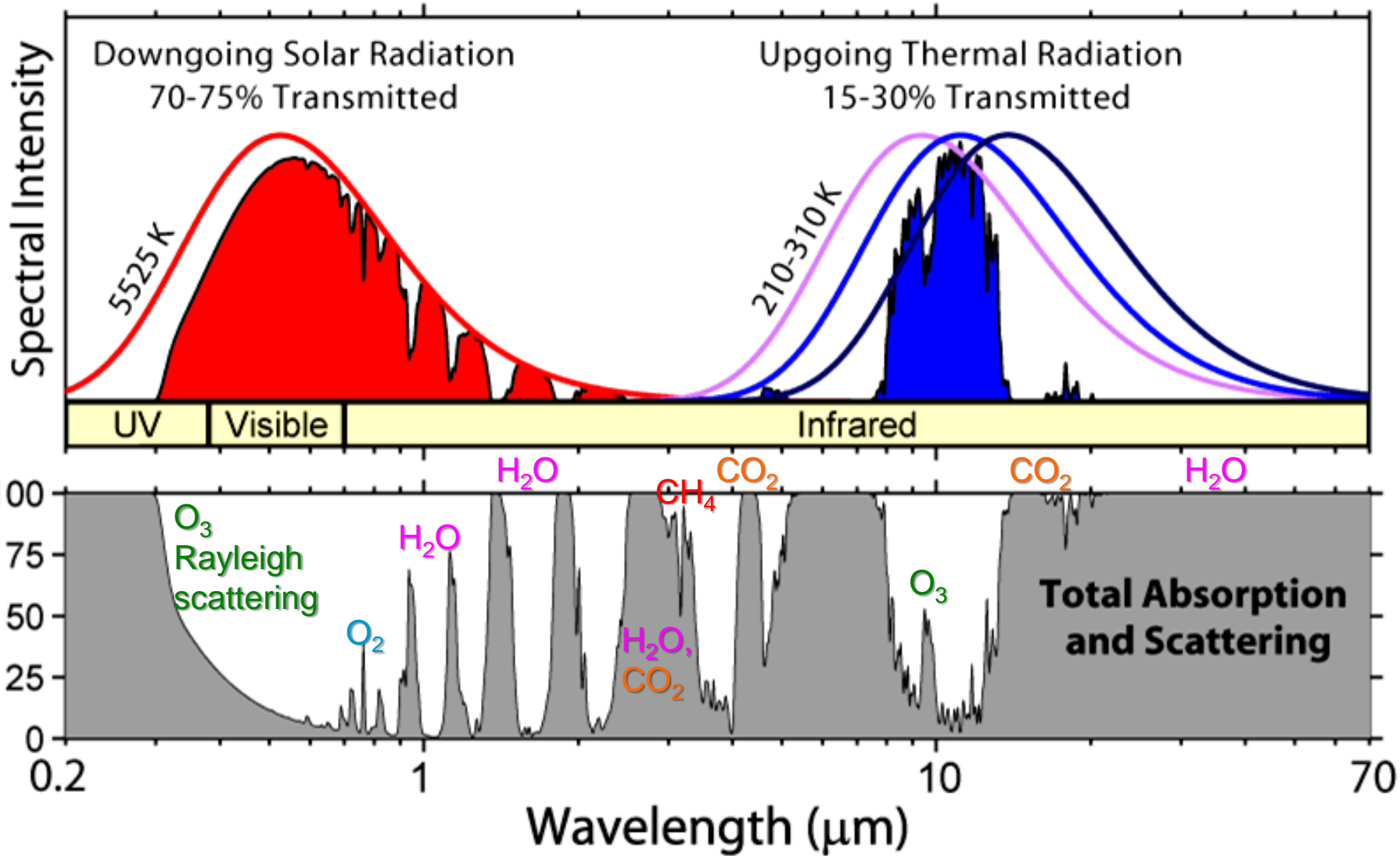
Comparative planetology

<u>Planet</u>	<u>Distance from sun (Earth = 1)</u>	<u>Mean surface temperature (K)</u>	<u>Surface pressure (Earth = 1)</u>	<u>Atmospheric composition</u>	<u>Absorbed solar radiation (Wm^{-2})</u>
Mercury	0.387	103-623	~ 0	-	2000
Venus	0.723	750	92	> 96% CO_2	150
Earth	1	293	1	78% N_2 , etc	235
Mars	1.524	186-268	0.007 - 0.009	96% CO_2	125

Conclusion: the composition of the atmosphere and the magnitude of the greenhouse effect are crucial factors for regulating the surface temperatures of the planets



Spectral distribution of radiation



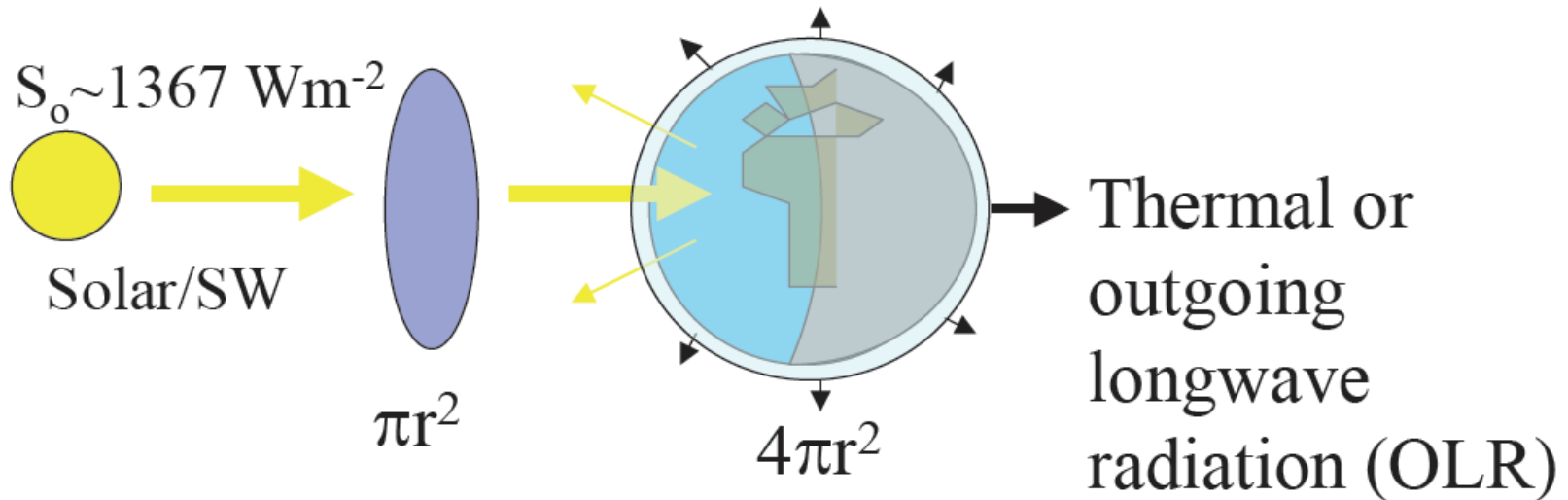
- **Shortwave: atmosphere is mostly transparent**
- **Longwave: atmosphere is mostly opaque**

Composition of the Earth's atmosphere

Gas	Parts by volume	Interaction
<u>Nitrogen</u> (N ₂)	780,840 ppmv (78.084%)	SW (Rayleigh)
<u>Oxygen</u> (O ₂)	209,460 ppmv (20.946%)	SW (Ray+abs)
<u>Water vapour</u> (H ₂ O)	~0.40% full atmosphere, surface ~1%-4%	LW, SW (abs)
<u>Argon</u> (Ar)	9,340 ppmv (0.9340%)	
<u>Carbon dioxide</u> (CO ₂)	390 ppmv (0.039%) <i>rising</i>	LW, SW (abs)
<u>Neon</u> (Ne)	18.18 ppmv (0.001818%)	
<u>Helium</u> (He)	5.24 ppmv (0.000524%)	
<u>Methane</u> (CH ₄)	1.79 ppmv (0.000179%) <i>rising</i>	LW
<u>Krypton</u> (Kr)	1.14 ppmv (0.000114%)	
<u>Hydrogen</u> (H ₂)	0.55 ppmv (0.000055%)	
<u>Nitrous oxide</u> (N ₂ O)	0.319 ppmv (0.00003%) <i>rising</i>	LW
<u>Carbon monoxide</u> (CO)	0.1 ppmv (0.00001%)	
<u>Xenon</u> (Xe)	0.09 ppmv (9×10^{-6} %) (0.000009%)	
<u>Ozone</u> (O ₃)	0.0 to 0.07 ppmv (0 to 7×10^{-6} %)	LW, SW (abs)

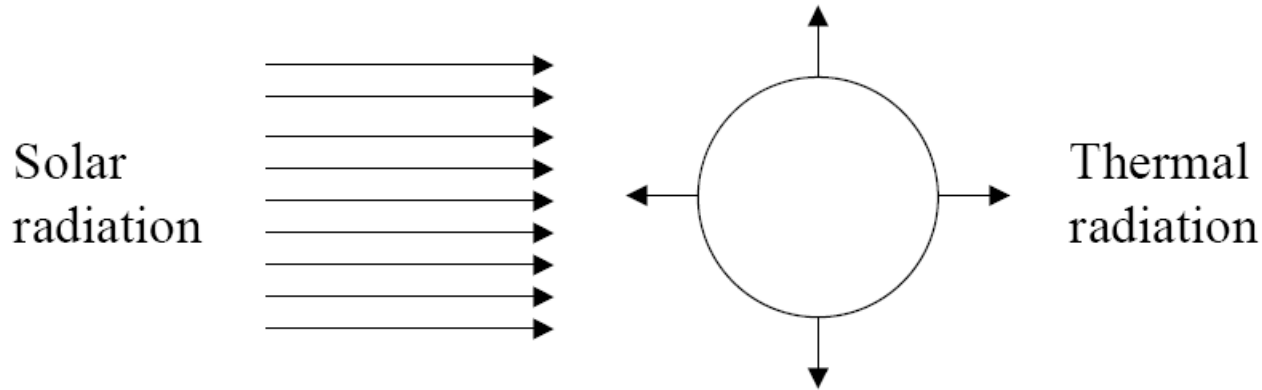
SW “shortwave” solar radiation: Rayleigh scattering (blue sky) or absorption
LW “longwave” terrestrial infrared radiation: absorbing greenhouse gases

Earth's radiation balance



- In equilibrium, the net absorption of solar radiation is balanced by the emission of thermal radiation back to space
- The thermal emission is controlled by the strength of the greenhouse effect
- If there is an increase in the concentrations of greenhouse gases, such as carbon dioxide, then the system warms as it tries to reach a new equilibrium

Overall energy balance of the Earth



$$(1 - \alpha) S_o \pi r^2 = 4 \pi r^2 \sigma T_{\text{eff}}^4$$

Simplifying, we find that;

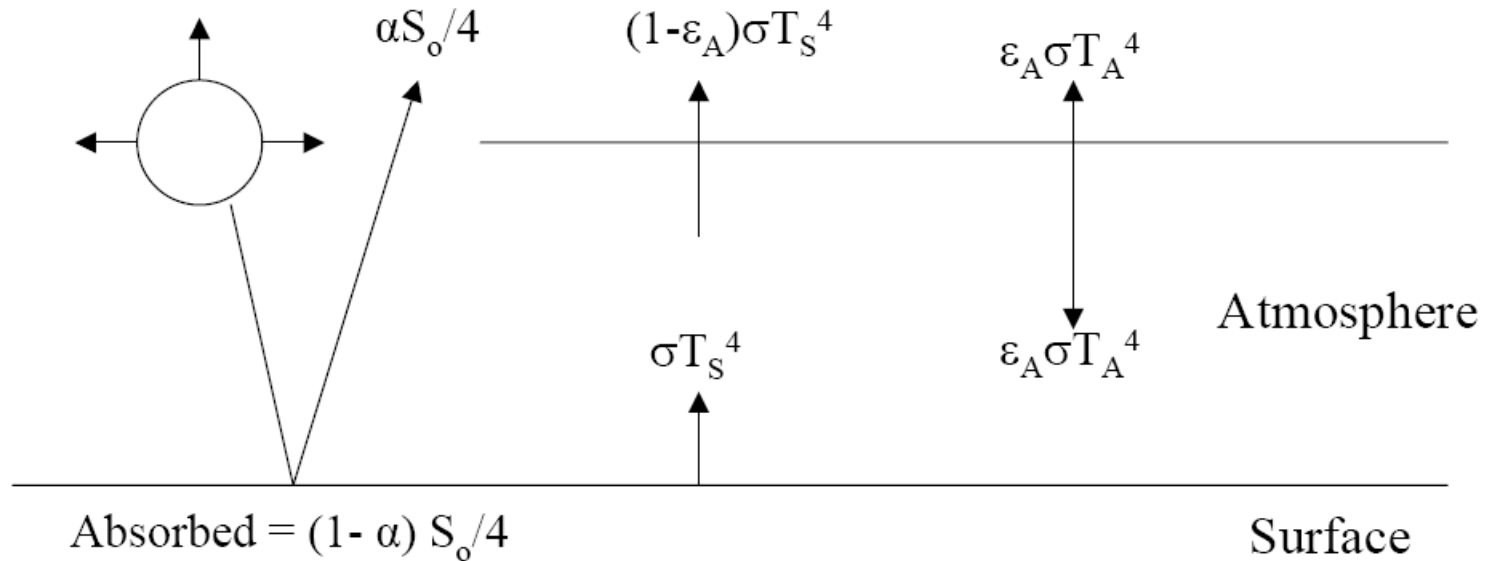
$$\sigma T_{\text{eff}}^4 = (1 - \alpha) S_o / 4$$

and hence

$$T_{\text{eff}} \approx 255 \text{ K}$$

If the Earth was black ($\alpha=0$), $T_{\text{eff}} = 278 \text{ K}$, still lower than observed 288 K

Overall energy balance including the greenhouse effect



Consider the equilibrium of the atmosphere and then of the surface;

$$\epsilon_A \sigma T_s^4 = 2\epsilon_A \sigma T_A^4 \quad (4)$$

$$(1-\alpha)S_o/4 + \epsilon_A \sigma T_A^4 = \sigma T_s^4 \quad (5)$$

Hence

$$\sigma T_S^4 = \{(1 - \alpha)S_o/4\} / (1 - \varepsilon_A/2) \quad (6)$$

and

$$T_A = T_S/2^{1/4} \quad (7)$$

Note that T_S is larger than T_{eff} given by (2), because of the additional downward thermal emission from the atmosphere. So, the greenhouse effect ensures that the surface is warmer with an atmosphere than without. Secondly, the atmosphere is colder than the surface and slightly colder than T_{eff} .

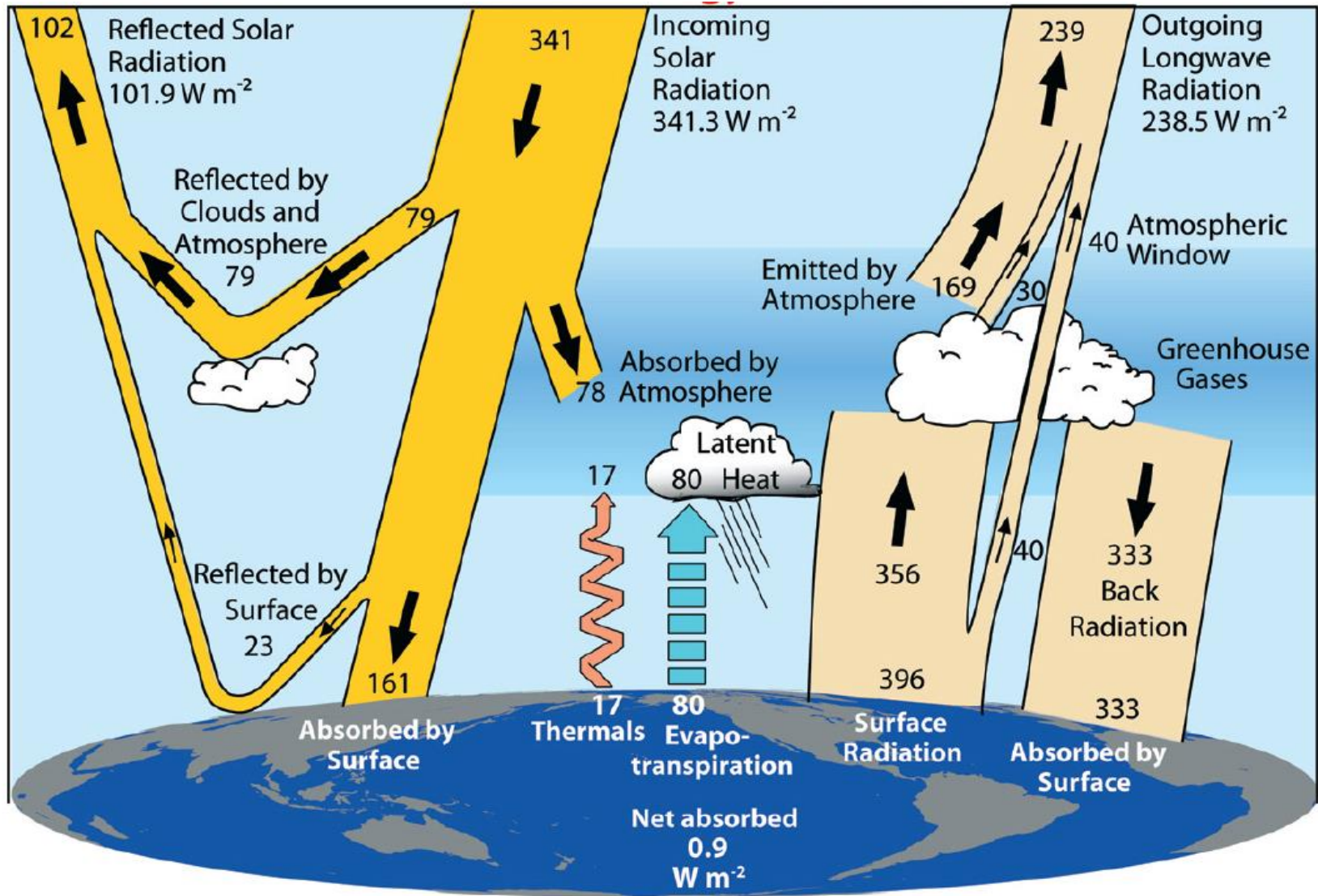
If we assume that $\alpha = 0.3$ and $\varepsilon_A = 0.8$ then we find that;

$$T_S = 289 \text{ K}$$

$$T_A = 243 \text{ K}$$

Which are reasonable values for the global mean surface and atmospheric temperatures.

Global energy flows



- Trenberth et al. (2009); modification of Kiehl & Trenberth (1997)

Radiative-convective equilibrium

If we assume that only radiative processes are operating, the equilibrium surface temperature is very high, tropospheric temperatures very low and the profile is strongly superadiabatic.

In reality, convection removes heat from the surface, warms the atmosphere and adjusts the lapse-rate towards that observed.

From the classic paper by Manabe and Wetherald, JAS, 1967

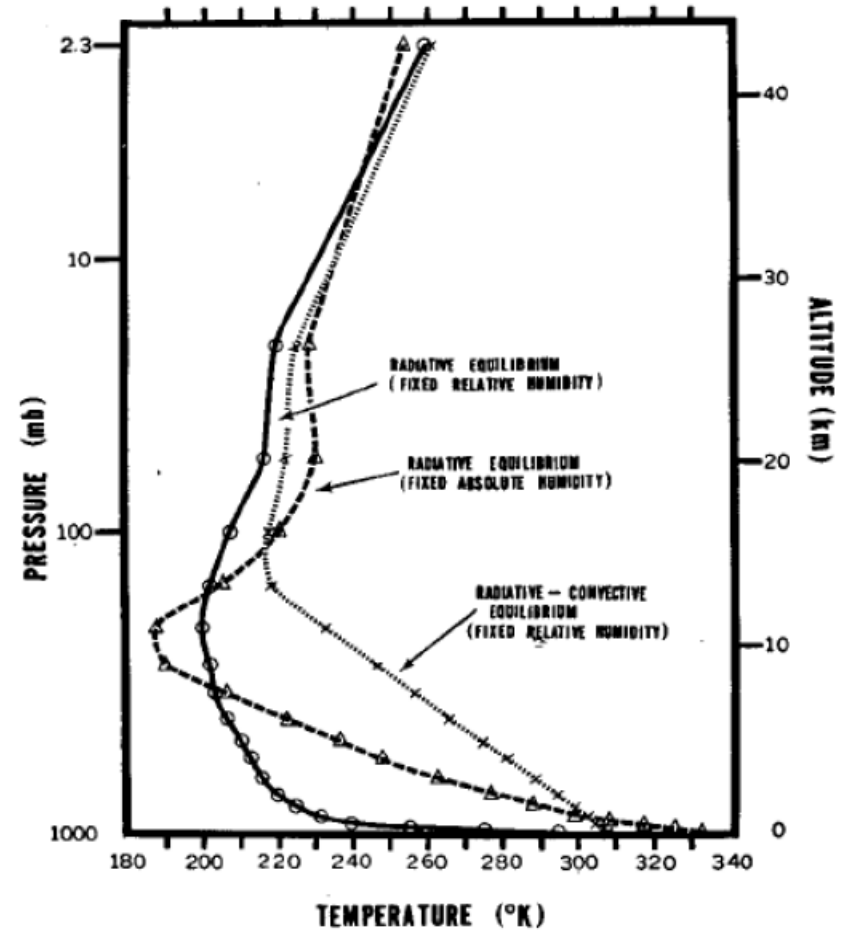
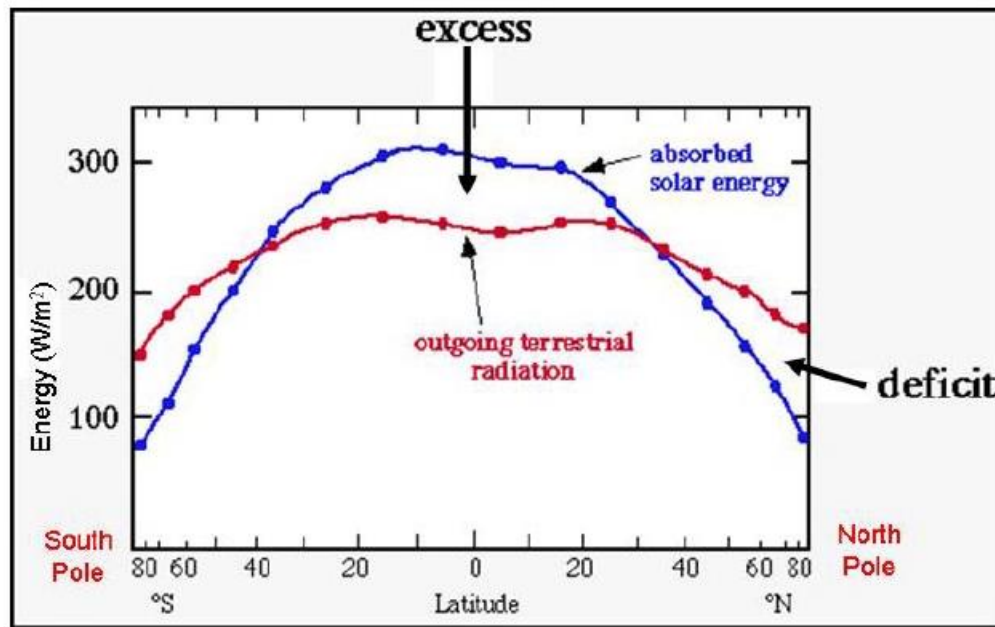
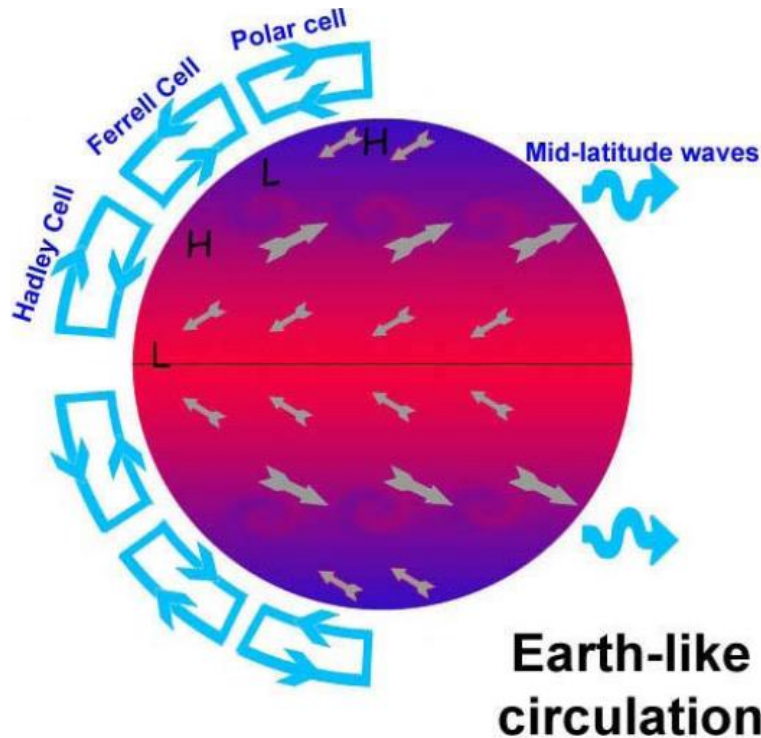


FIG. 5. Solid line, radiative equilibrium of the clear atmosphere with the given distribution of relative humidity; dashed line, radiative equilibrium of the clear atmosphere with the given distribution of absolute humidity; dotted line, radiative convective equilibrium of the atmosphere with the given distribution of relative humidity.

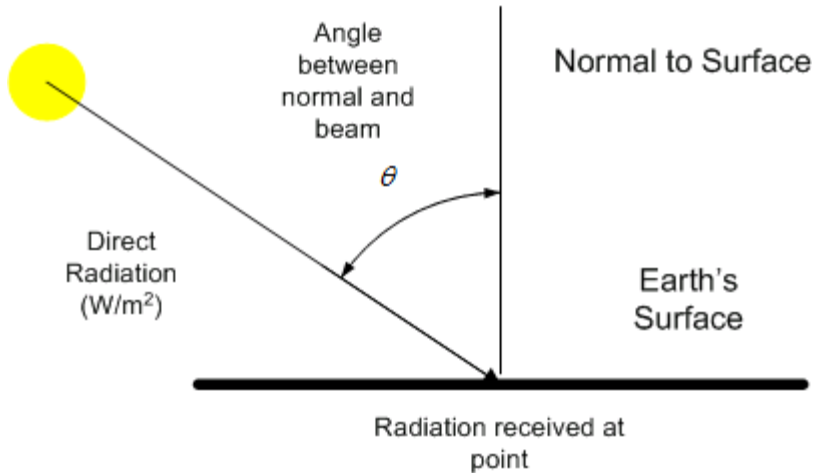
Global circulation



- Warmer tropics means same pressure layers are thicker at equator
- By thermal wind balance there must be westerlies
- Excess heat transported polewards by
 - Disturbances in these westerlies
 - Oceanic transport



Solar zenith angle θ_0



- θ_0 is a function of lat, lon, day of year, time of day
- Often use $\mu_0 = \cos(\theta_0)$

- Irradiance through a plane parallel to the surface:

$$- S_{\text{TOA}} = S_0 (d/d_{\text{mean}})^{-2} \mu_0$$

where S_0 is total solar irradiance, d is earth-sun distance

- Surface flux for absorbing atmosphere:

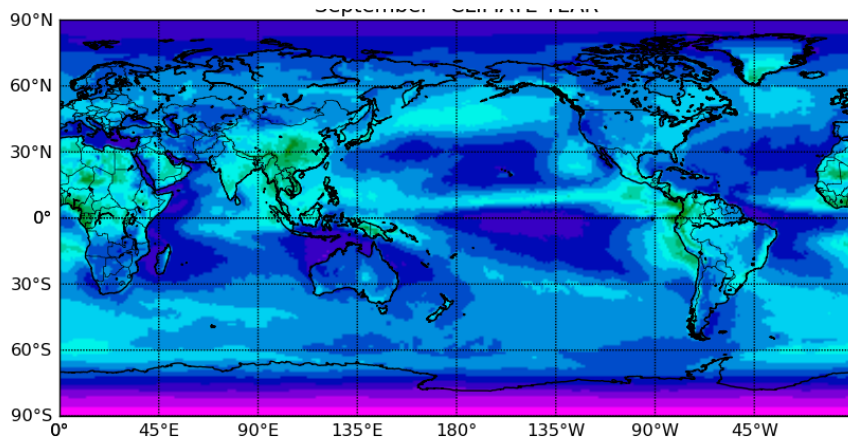
$$- S_{\text{surf}} = S_{\text{TOA}} \exp(-\delta/\mu_0)$$

where δ is the zenith optical depth

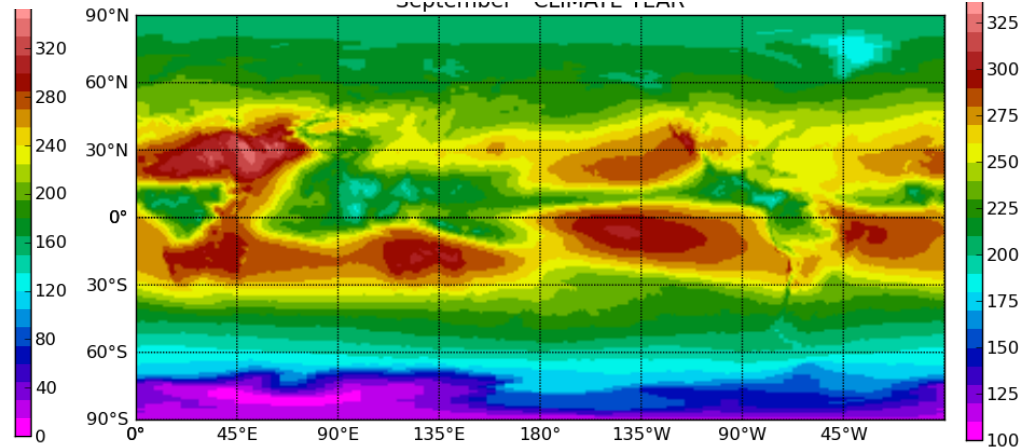
CERES radiometer (Sept)

- TOA total upwelling irradiance

– Shortwave

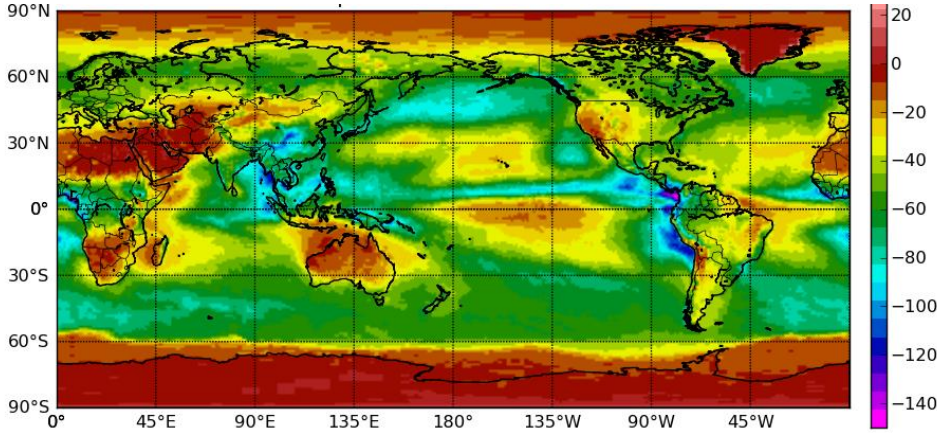


– Longwave

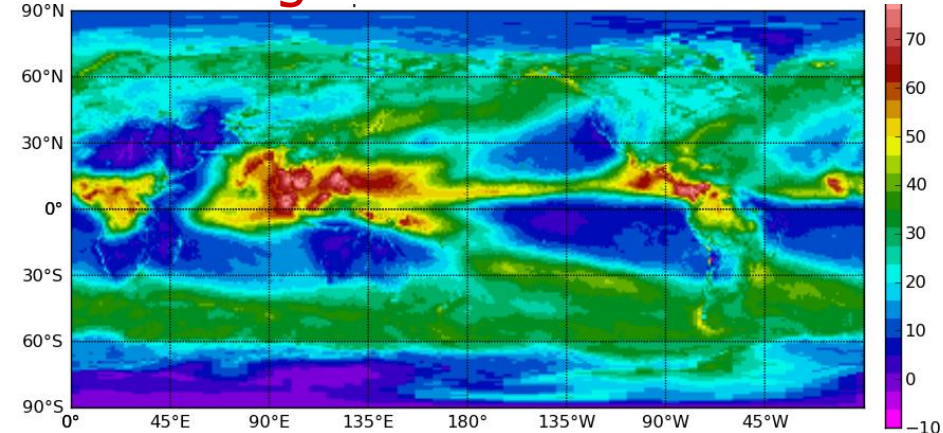


- TOA cloud radiative effect (or "forcing"): $F_n^{\text{cloud}} - F_n^{\text{clear}}$

– Shortwave



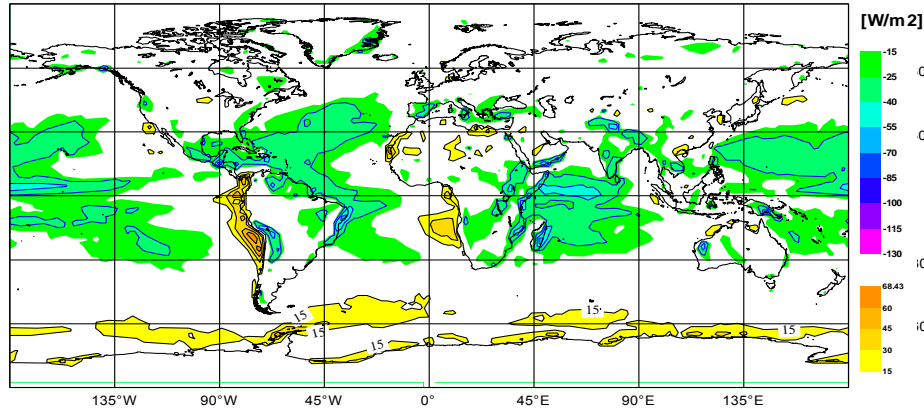
– Longwave



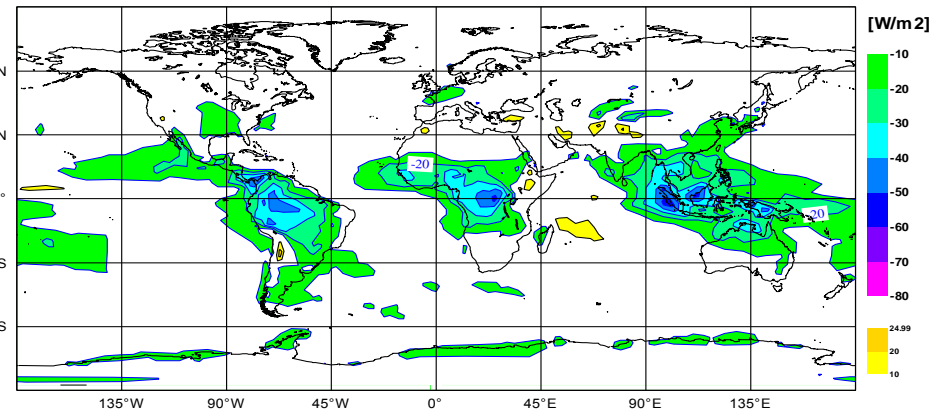
ECMWF model biases versus CERES

- Before McRad (<2007)

– Shortwave

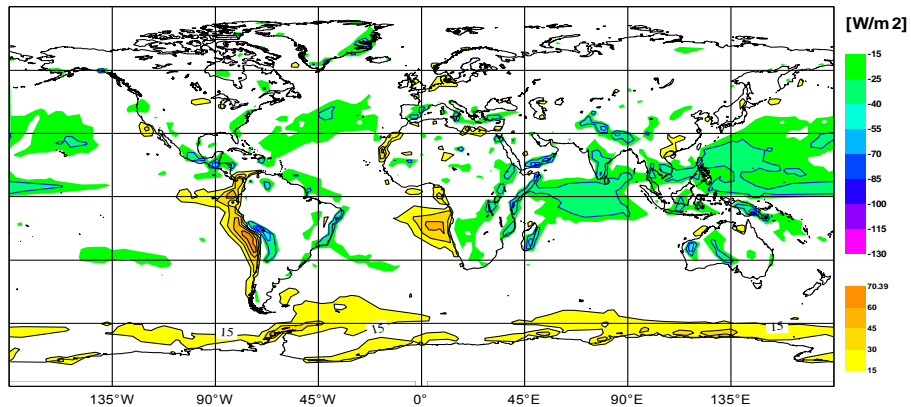


– Longwave

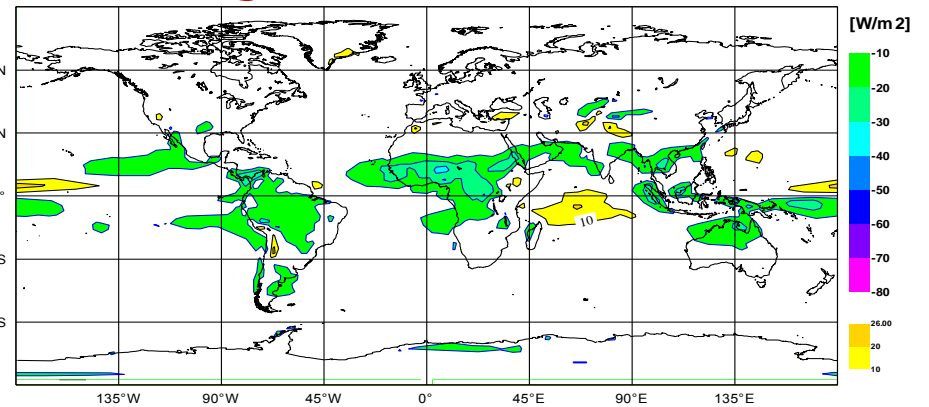


- After McRad (>2007)

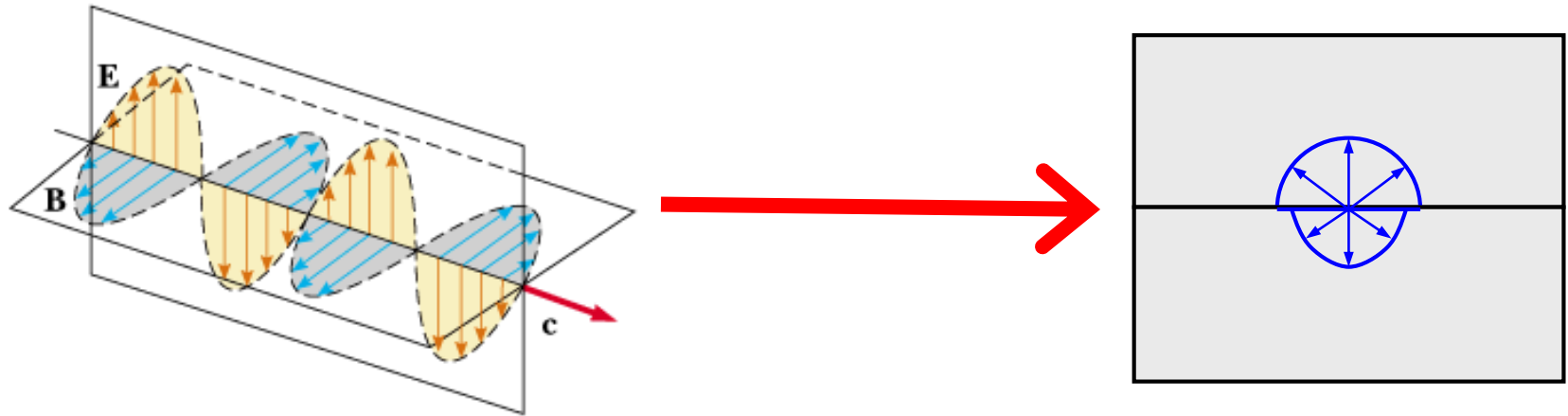
– Shortwave



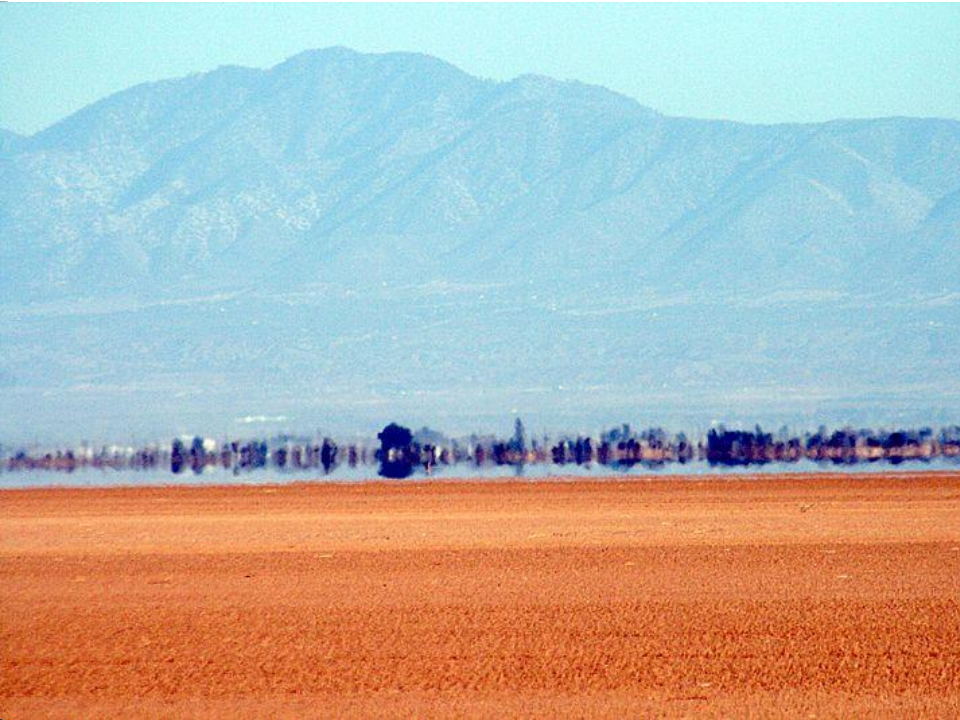
– Longwave



Part 2: Maxwell's equations to the two-stream equations

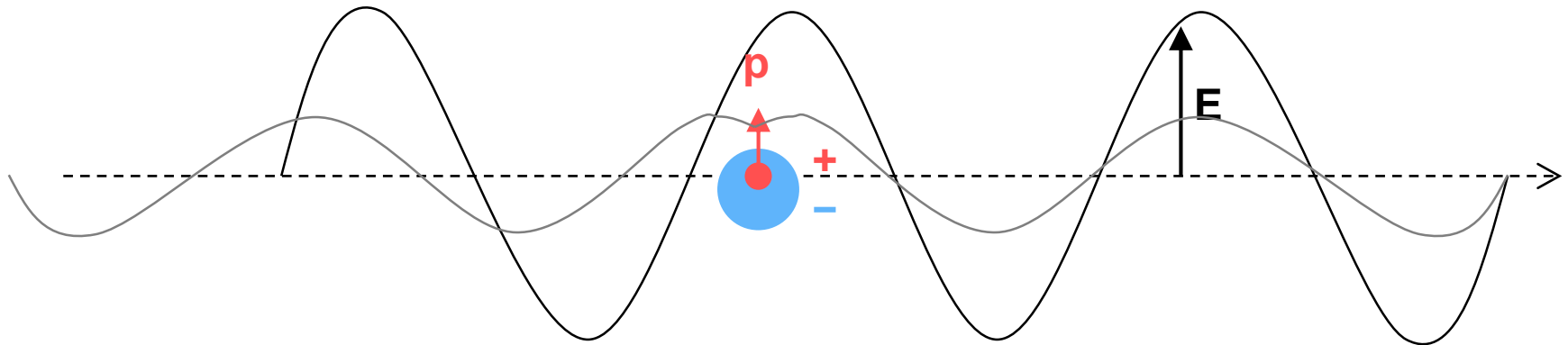


- How do Maxwell's equations explain optical phenomena?
- How do we describe scattering by cloud particles, aerosols and molecules?
- How is radiative transfer implemented in models?



Building blocks of atmospheric radiation

1. Emission and absorption of quanta of radiative energy
 - Governed by quantum mechanics: the Planck function and the internal energy levels of the material
 - Responsible for complex gaseous absorption spectra
2. Electromagnetic waves interacting with a dielectric material
 - An oscillating dipole is excited, which then re-radiates
 - Governed by Maxwell's equations + Newton's 2nd law for bound charges
 - Responsible for *scattering, reflection and refraction*



Oscillating dipole \mathbf{p} is induced, which is typically in phase with the incident electric field \mathbf{E}

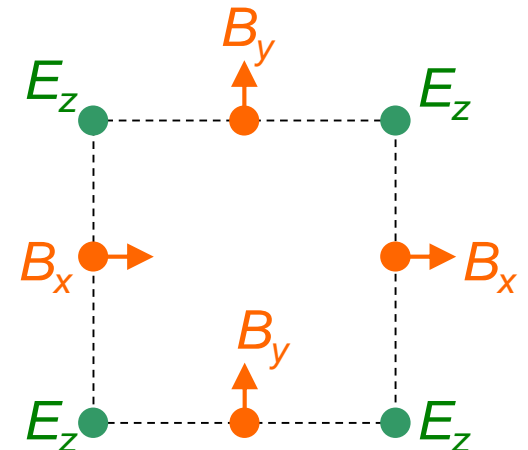
Dipole radiates in all directions (except directly parallel to its axis)

Maxwell's equations

- Almost all atmospheric radiative phenomena are due to this effect, described by the Maxwell curl equations:

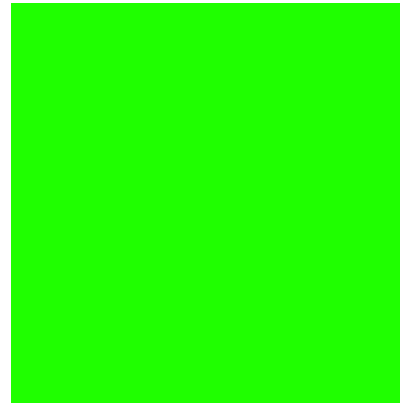
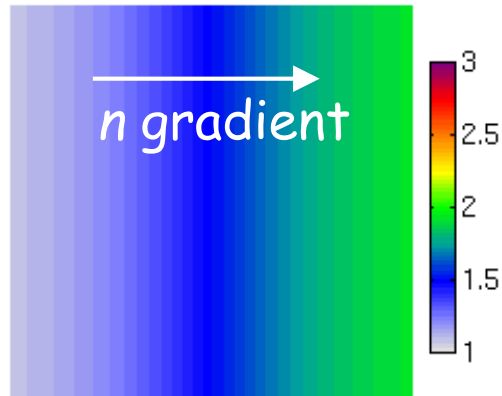
$$\frac{\partial \mathbf{E}}{\partial t} = \frac{c^2}{n^2} \nabla \times \mathbf{B} \quad \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

- where c is the speed of light in vacuum, n is the complex refractive index (which varies with position), and \mathbf{E} and \mathbf{B} are the electric and magnetic fields (both functions of time and position);
- It is illuminating to discretize these equations directly
 - This is known as the Finite-Difference Time-Domain (FDTD) method
 - Use a staggered grid in time and space (Yee 1966)
 - Consider two dimensions only for simplicity
 - Need gridsize of $\sim 0.02 \mu\text{m}$ and timestep of $\sim 50 \text{ ps}$ for atmospheric problems

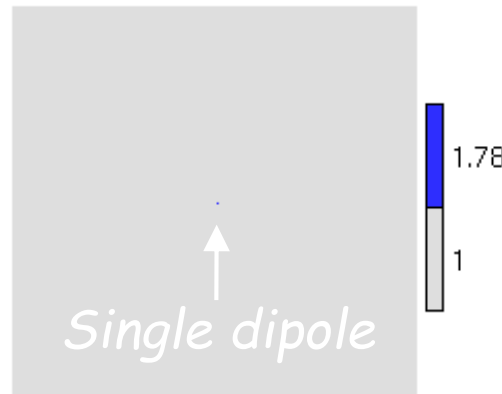


Simple examples

- Refraction (a mirage)



- Rayleigh scattering (blue sky)



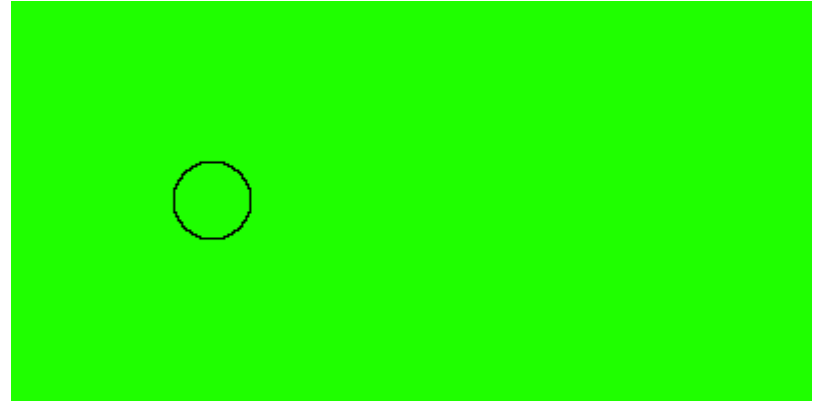
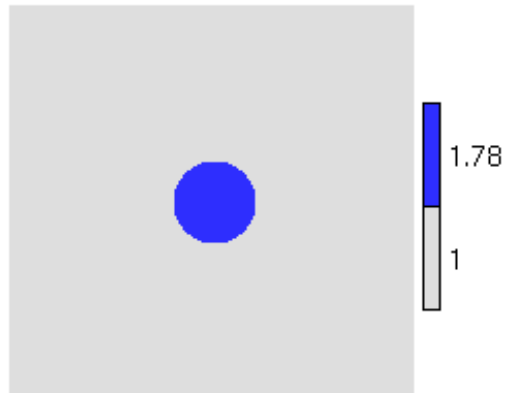
Refractive index

Total E_z field

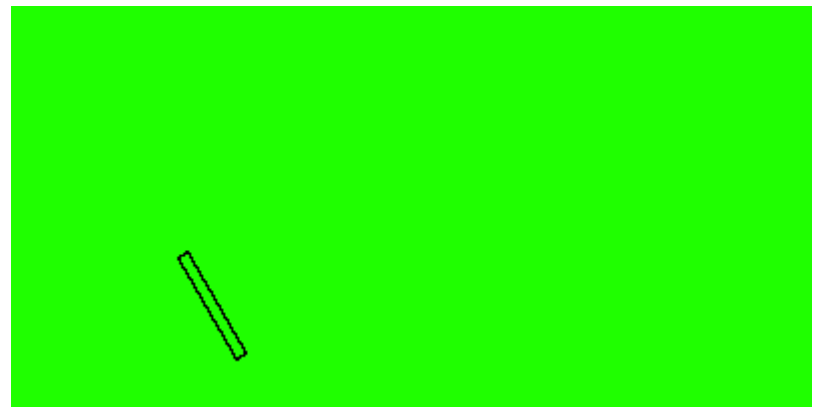
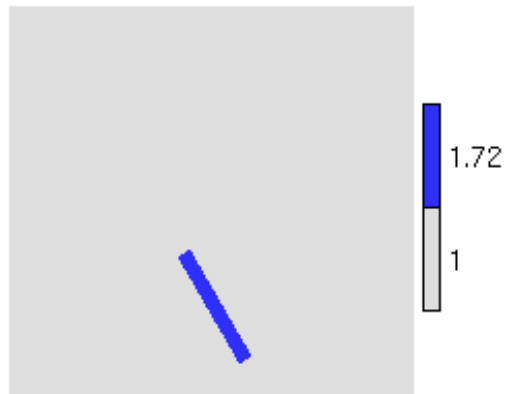
Scattered field
(total - incident)

More complex examples

- A sphere (or circle in 2D)



- An ice column



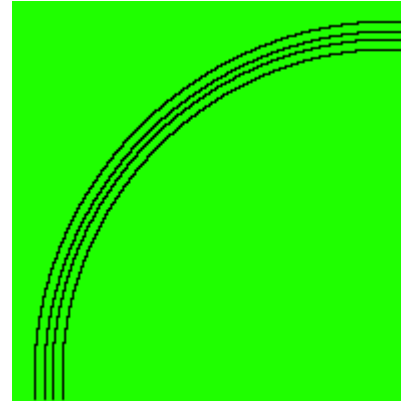
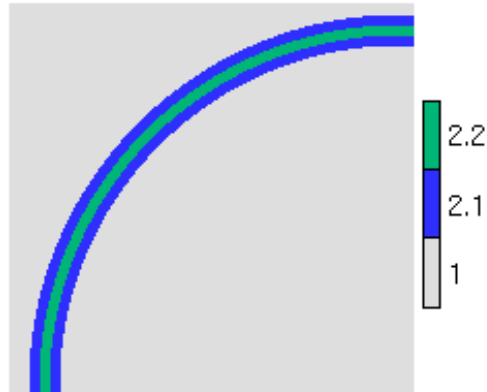
Refractive index

Total E_z field

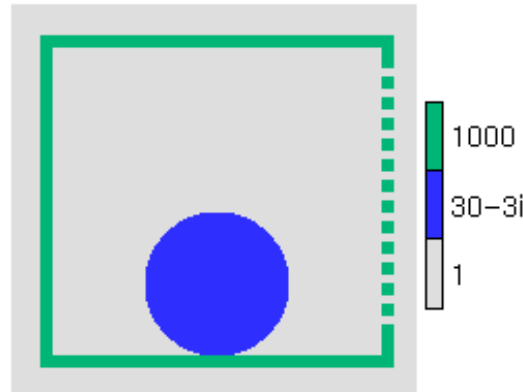
Scattered field
(total - incident)

Non-atmospheric examples

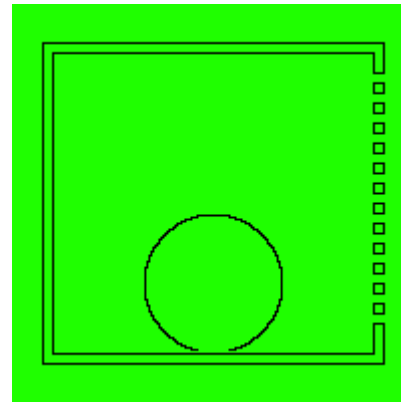
- Single-mode optic fibre



- Potato in a microwave oven



Refractive index



Total E_z field

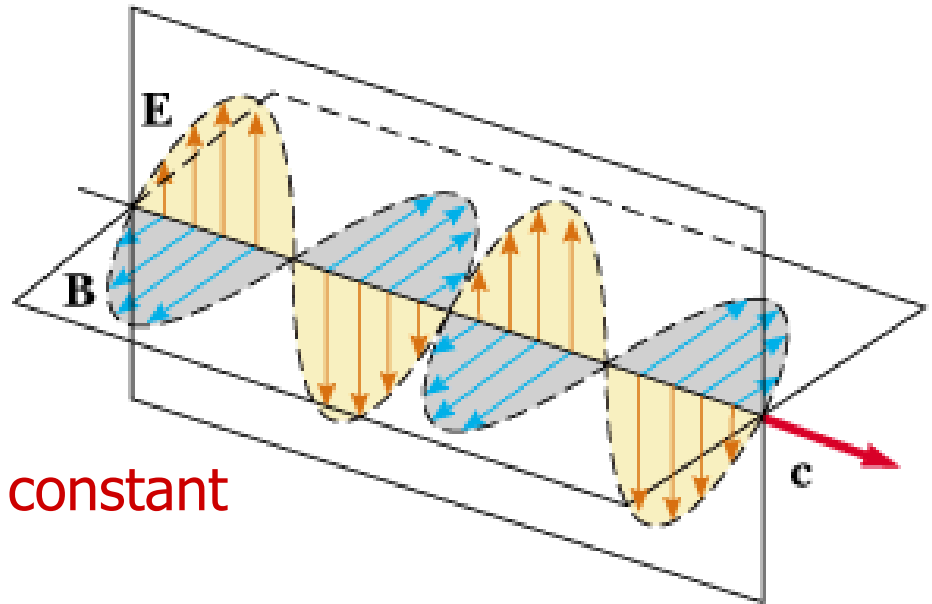
*Many more animations at www.met.rdg.ac.uk/clouds/maxwell
(interferometer, diffraction grating, dish antenna, clear-air radar, laser...)*

How are **E** & **B** fields related to fluxes?

- Poynting vector given by

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

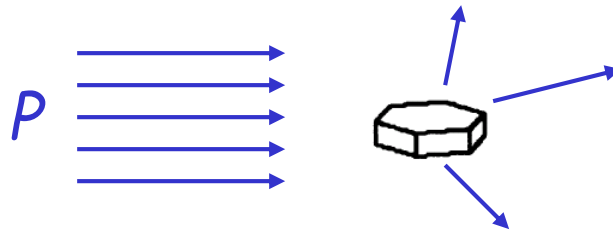
- Here μ_0 is the magnetic constant



- It describes
 - Direction of energy propagation
 - Rate of energy flow in $W\ m^{-2}$ (more useful if time averaged)

Particle scattering

- Maxwell's equations used to obtain scattering properties
- Suppose we illuminate a single particle with monochromatic radiation of flux density P (in W/m^2)



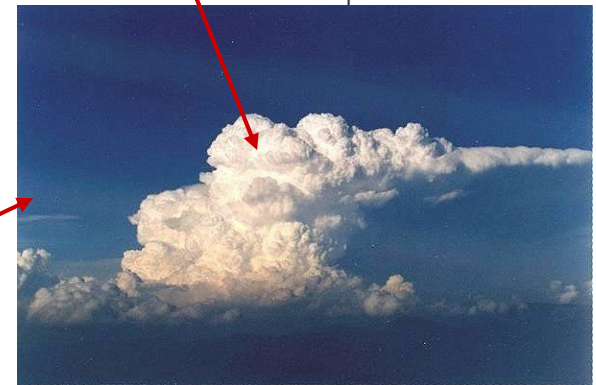
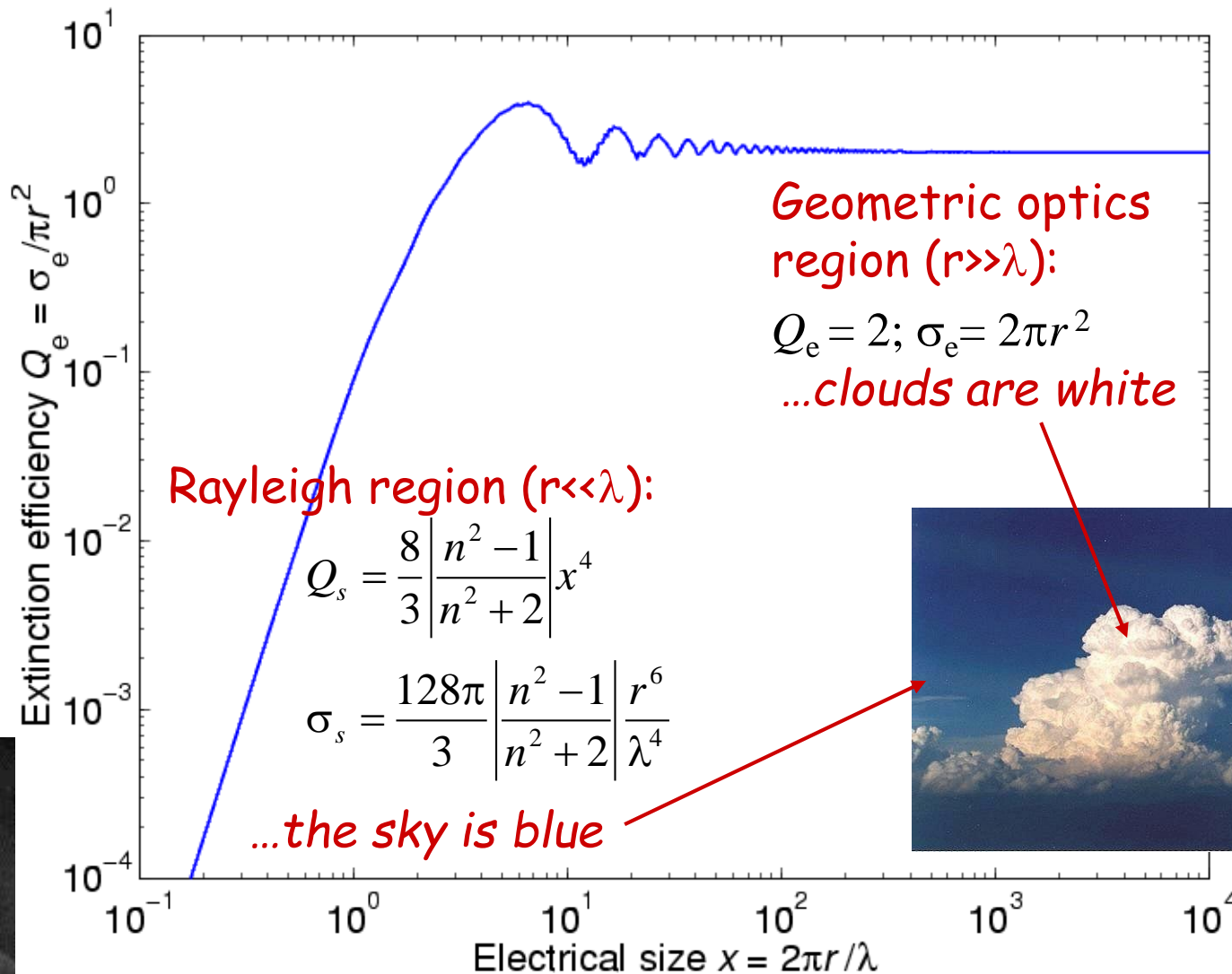
- *Scattering cross-section* σ_s (in m^2) is defined such that the total scattered power (in W) is $P\sigma_s$
 - *Absorption cross-section* σ_a is the same but for absorbed power
 - *Extinction cross-section* $\sigma_e = \sigma_s + \sigma_a$ is the sum of the two
 - *Single scattering albedo* $\omega_0 = \sigma_s / \sigma_e$
- Directional scattering described by the *phase function* $p(\Omega)$
 - Ω is the angle between incident and scattered directions
 - Phase function normalized such that

$$\int_{\Omega} p(\Omega) d\Omega = 4\pi$$

The limits of Mie theory



Gustav Mie



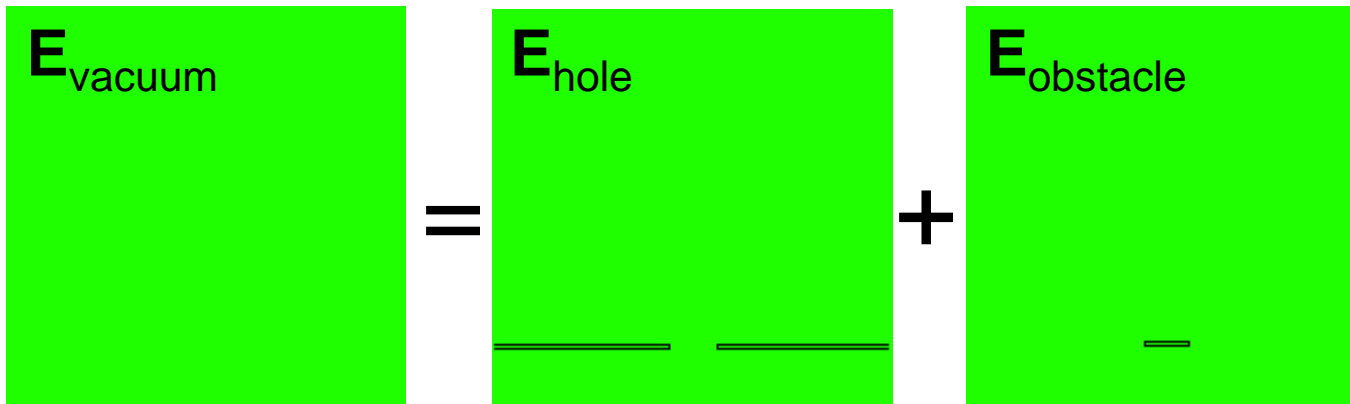
Lord Rayleigh





Why $Q_e \approx 2$ for $r \gg \lambda$?

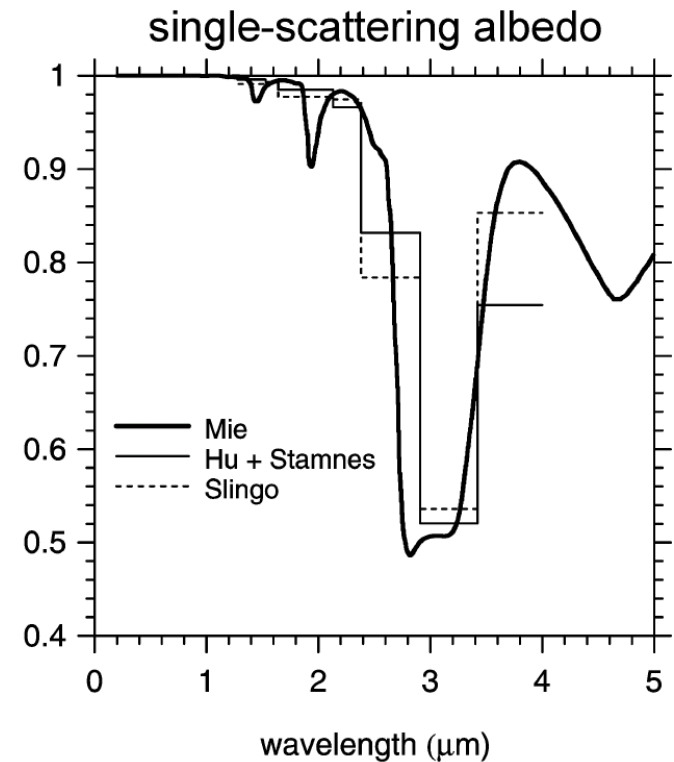
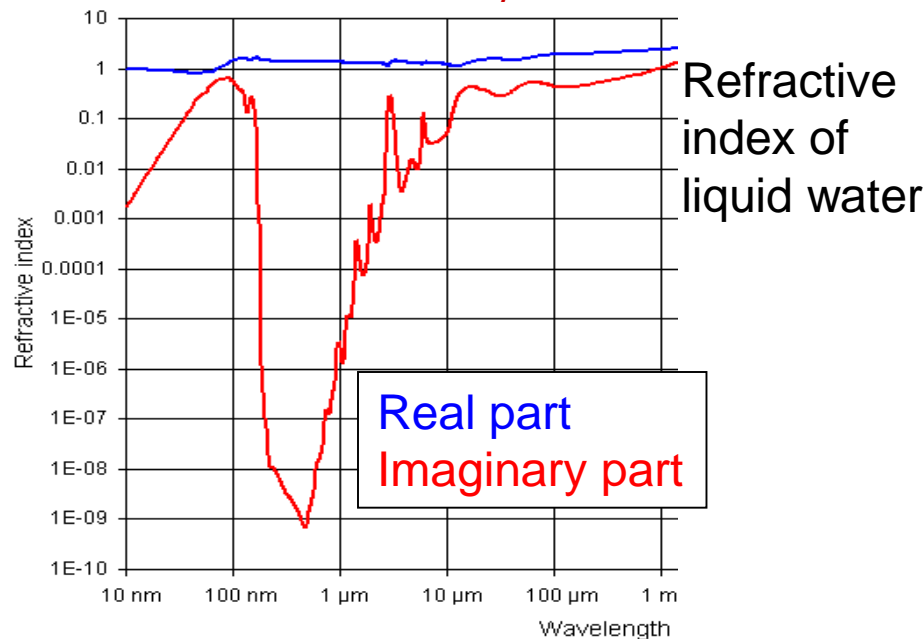
- The extinction paradox: σ_e is *twice* physical cross section
- *Babinet's principle*, valid due to superposition principle:
 - $\mathbf{E}_{\text{vacuum}} = \mathbf{E}_{\text{hole}} + \mathbf{E}_{\text{obstacle}}$
- Diffraction scattering pattern around the obstacle is:
 - $\mathbf{E}_{\text{scat,obstacle}} = \mathbf{E}_{\text{obstacle}} - \mathbf{E}_{\text{vacuum}} = -\mathbf{E}_{\text{hole}}$



- *Energy diffracted around particle equals energy intercepting particle so $Q_e \approx 2$ for any particle shape*

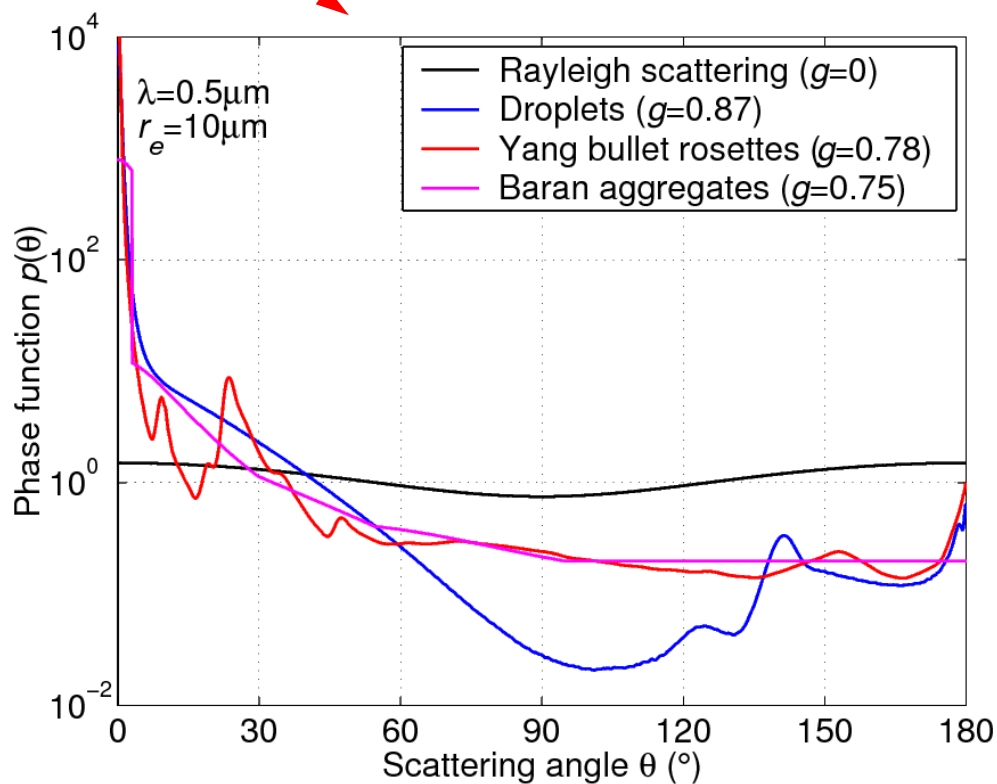
Single scattering albedo $\omega = \sigma_s / \sigma_e$

- Absorption related to imaginary part of refractive index m_i
- For liquid and ice
 - Visible: m_i is very small so ω is close to one (0.999...)
 - Longwave: m_i higher so $\omega \sim 0.5$



- Aerosols in the shortwave
 - Water soluble: 0.9-0.95; Black carbon ~ 0.3

The scattering phase function



- The distribution of scattered energy is known as the “scattering phase function”
- Different methods are suitable for different types of scatterer

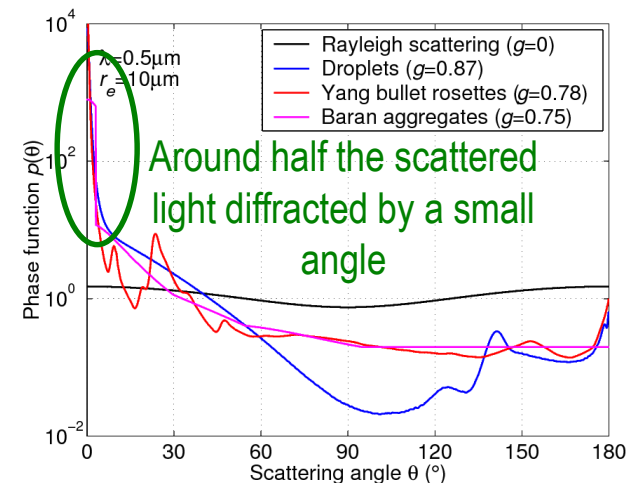
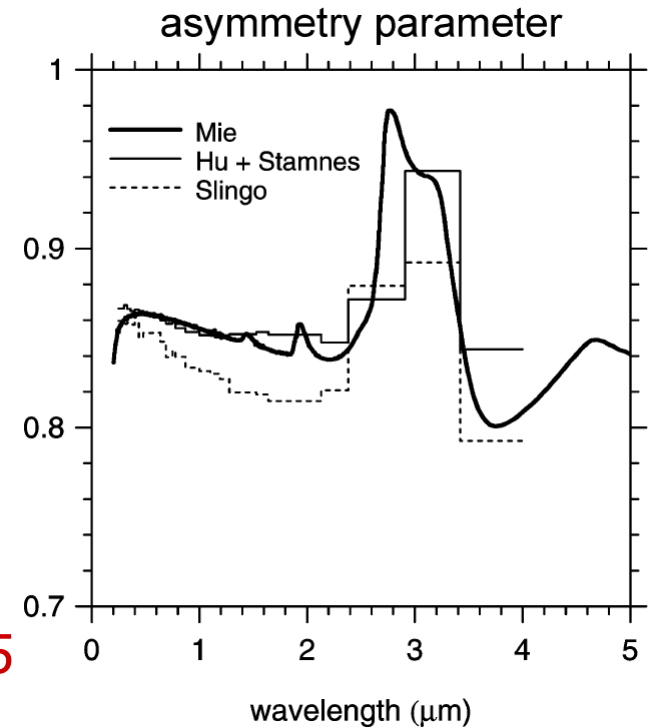
- *Spheres*: Mie theory (Mie 1908) provides a solution to Maxwell's equations as a series expansion
- *Arbitrary ice particle shapes*: depending on D/λ , use the Discrete Dipole Approximation, FDFT or ray tracing (Yang et al. 2000)
- But observations (Baran) suggest smoother phase functions implying that the surface of ice particles is “rough”

Asymmetry factor

- Radiation schemes can't use full phase function: approximate by *asymmetry factor*:

$$g = \overline{\cos \theta}$$

- Isotropic and Rayleigh scattering: $g = 0$
- Droplets larger than wavelength $g \approx 0.85$
- Ice larger than wavelength $g \approx 0.7-0.8$
- Delta-Eddington scaling (Joseph et al. 1976)
 - Treat some of forward scattered radiation as if it had not been scattered at all
 - $\delta' = \delta(1 - \omega g^2)$
 - $\omega' = \omega(1 - g^2)/(1 - \omega g^2)$
 - $g' = g/(1 + g)$



Size distributions

- Describe size distribution by $n(r)$ [m^{-4}], where $n(r)dr$ is number concentration of particles with radius between r and $r + dr$
 - Extinction coefficient [m^{-1}]: $\beta_e = \int n(r)\sigma_e(r)dr$
 - Scattering coefficient [m^{-1}]: $\beta_s = \int n(r)\sigma_s(r)dr$
 - Single scattering albedo: $\omega = \beta_s/\beta_e$
 - Bulk asymmetry factor: $g = \frac{1}{\beta_s} \int n(r)\sigma_s(r)g(r)dr$
- In geometric optics region ($r \gg \lambda$):
 - Geometric optics extinction coefficient $\beta_{e,go} = 2\pi \int r^2 n(r)dr$
- But model holds liquid water mixing ratio or
 - Liquid water content [kg m^{-3}]: $\text{LWC} = \frac{4}{3}\pi\rho_l \int r^3 n(r)dr$
- *What's best way to convert LWC to radiation coefficients?*

Effective radius

- Can convert liquid water mixing ratio to extinction with

- Liquid effective radius: $r_e = \frac{\int r^3 n(r) dr}{\int r^2 n(r) dr} = \frac{3LWC}{2\rho_l \beta_{e,go}}$

- For non-spherical ice, equivalent expression is:

- Ice effective radius: $r_e = \frac{3IWC}{2\rho_i \beta_{e,go}}$

- In each part of the spectrum, a radiation scheme parameterizes β_e/LWC , ω and g as a function of r_e (e.g. Slingo and Schrecker 1982)

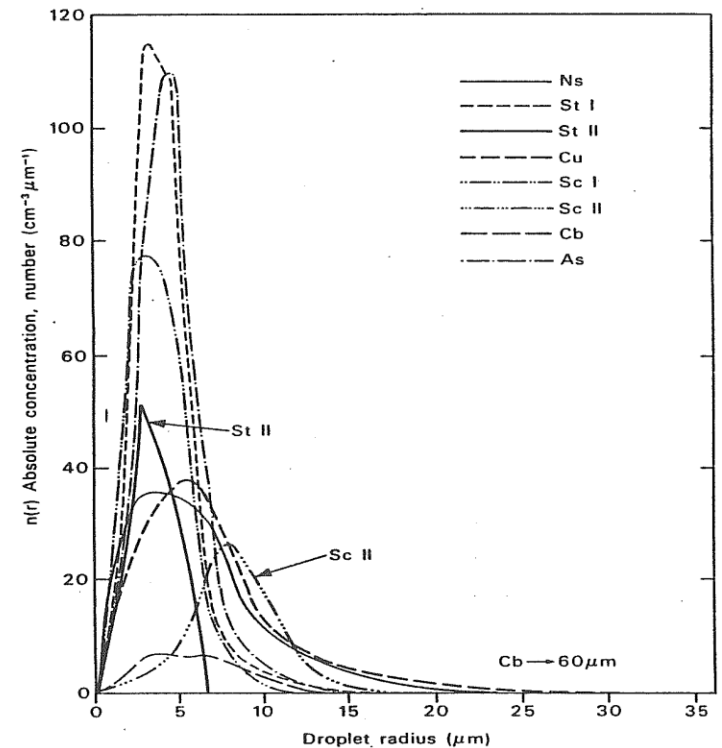


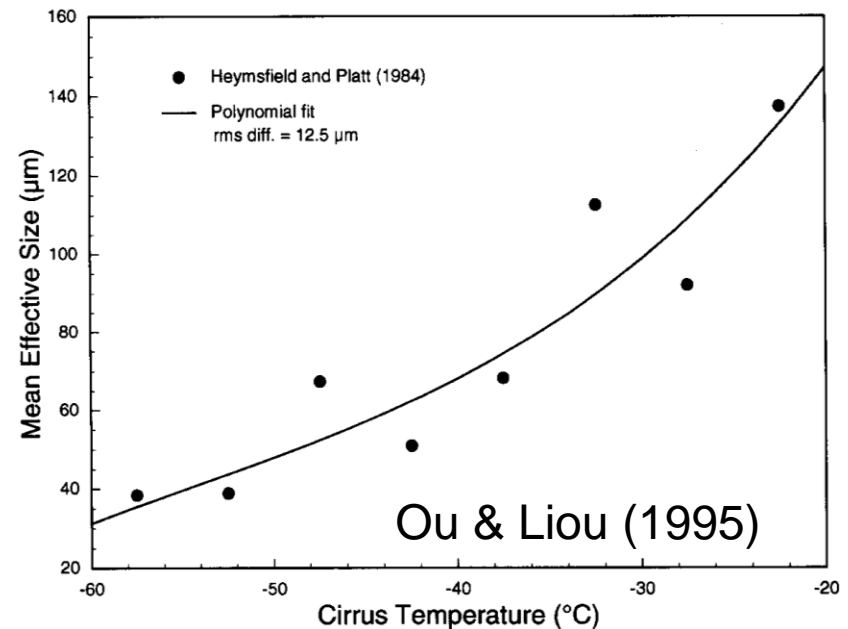
Fig. 3.6 The droplet distribution of eight cloud models. After Stephens (1979).

Parametrizing effective radius

- Liquid water clouds
 - ECMWF: varies with height from 10 to 40 microns
 - Some models have different values over land and sea
 - Met Office UK forecast model has prognostic aerosol which is used for droplet number conc.
- *Radiation schemes need to use more information from cloud schemes!*

- Ice clouds

- ECMWF and most models parametrize vs temperature:



- Field et al. (2005) parametrized full ice size distribution, enabling $r_e(T, IWC)$

From Maxwell to radiative transfer

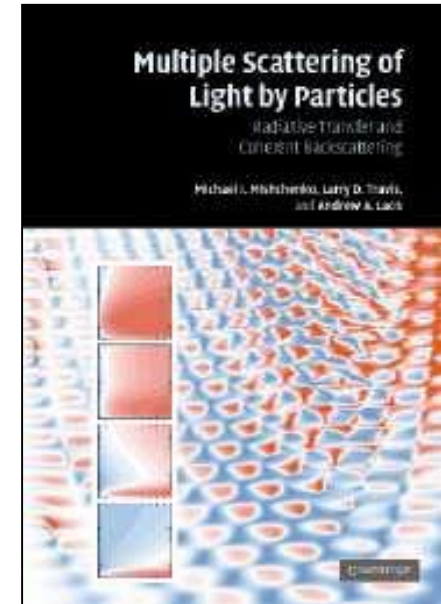
Maxwell's equations in terms of fields $\mathbf{E}(\mathbf{x}, t)$, $\mathbf{B}(\mathbf{x}, t)$



Reasonable assumptions:

- Ignore polarization
- Ignore time-dependence (sun is a continuous source)
- Particles are randomly separated so intensities add incoherently and phase is ignored
- Random orientation of particles so phase function doesn't depend on absolute orientation
- No diffraction around features larger than individual particles

Mishchenko et al. (2007)



3D radiative transfer in terms of monochromatic radiances $I(\mathbf{x}, \Omega, \nu)$ in $\text{W m}^{-2} \text{sr}^{-1} \text{Hz}^{-1}$



The 3D radiative transfer equation

- Also known as the “Boltzmann transport equation”, this describes the radiance I in direction Ω (where the \mathbf{x} and ν dependence of all variables is implicit):

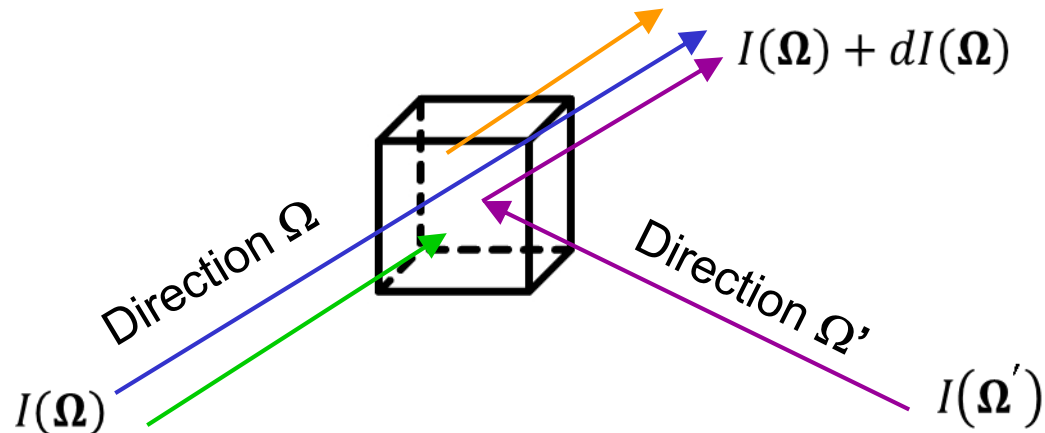
$$\Omega \cdot \nabla I(\Omega) = -\beta_e I(\Omega) + \beta_s \int_{4\pi} p(\Omega, \Omega') I(\Omega') d\Omega' + S(\Omega)$$

Spatial derivative
representing how much radiation is upstream

Loss by absorption or scattering

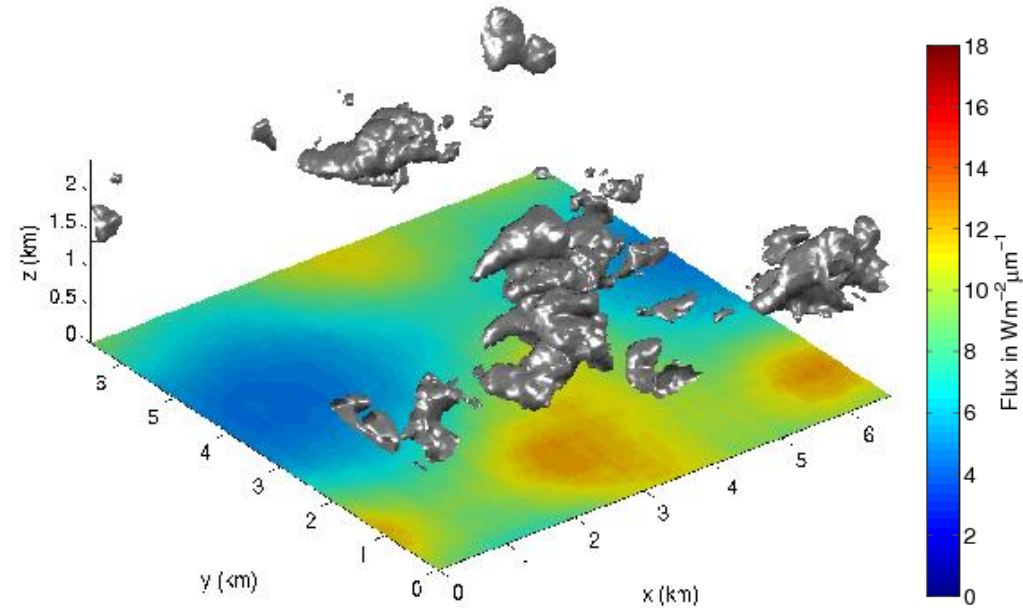
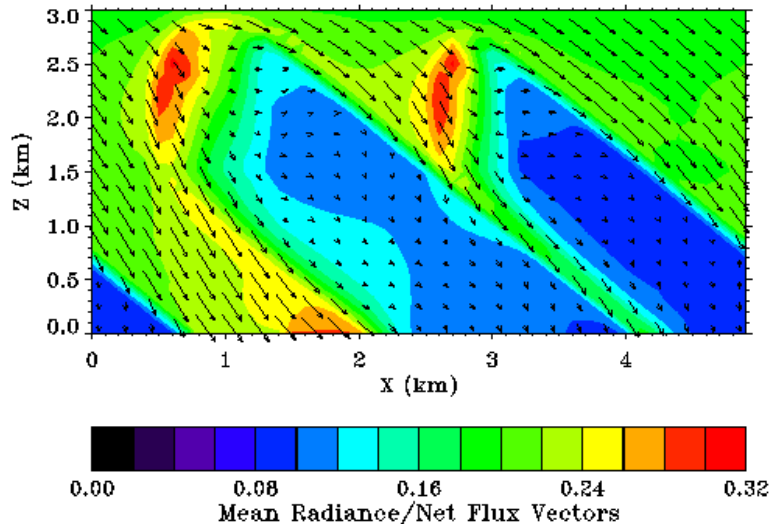
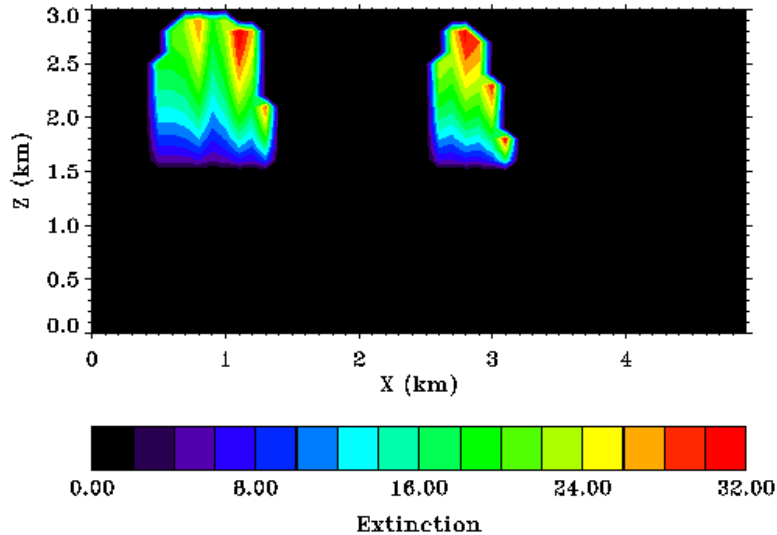
Gain by scattering
Radiation scattered from all other directions

Source
Such as thermal emission



Explicit 3D radiation calculations

- Freely available Monte Carlo and SHDOM codes can compute radiance fields everywhere
- Very slow: 5D problem
- Need to approximate for GCMs



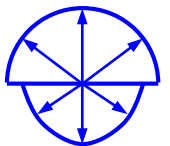
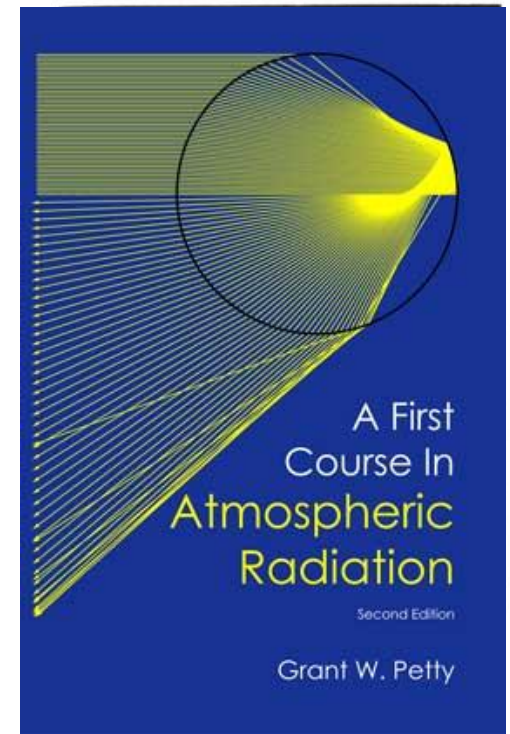
Two-stream approximation

3D radiative transfer in terms of monochromatic radiances $I(\mathbf{x}, \Omega, \nu)$

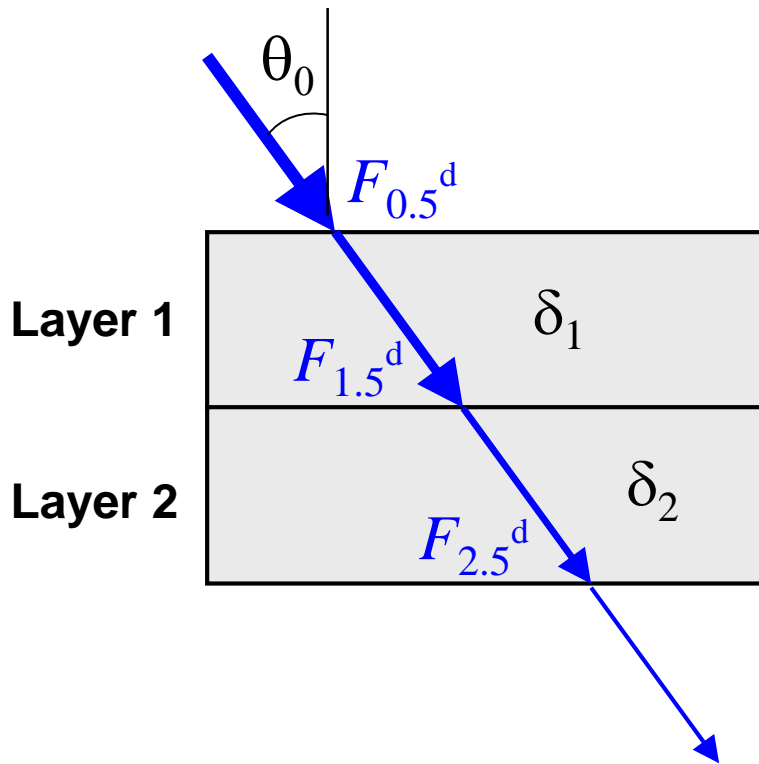
Unreasonable assumptions:

- Radiances in all directions represented by only 2 (or sometimes 4) discrete directions
- Atmosphere within a model gridbox is horizontally infinite and homogeneous
- Details of the phase functions represented by one number, the asymmetry factor $g = \overline{\cos \theta}$

1D radiative transfer in terms of two monochromatic fluxes $F^\pm(z, \nu)$ in $\text{W m}^{-2} \text{ Hz}^{-1}$



Direct solar flux



- TOA flux:
 - $F_{0.5}^d = S_0 \cos(\theta_0)$
- Zenith optical depth in layer i is δ_i , calculated simply as the vertical integral of extinction coefficient β_e across the layer
- Fluxes at layer interfaces are
 - $F_{i+0.5}^d = F_{i-0.5}^d \exp(-\delta_i / \cos \theta_0)$
- For the moment we assume the model layers to be horizontally homogeneous and infinite: no representation of radiation transport between adjacent model columns

Two-stream equations

Gradient of flux
with height

Loss of flux by
scattering or
absorption

Gain in flux by
scattering from
other direction

- Upwelling flux:

$$\frac{\partial F^+}{\partial z} = -\beta_e (\gamma_1 F^+ - \gamma_2 F^-) + S^+$$

Source from
scattering of the
direct solar beam
(shortwave) or
emission
(longwave)

- Downwelling flux: $-\frac{\partial F^-}{\partial z} = -\beta_e (\gamma_1 F^- - \gamma_2 F^+) + S^-$

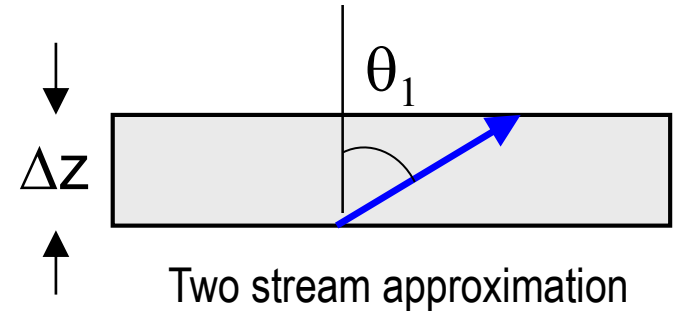
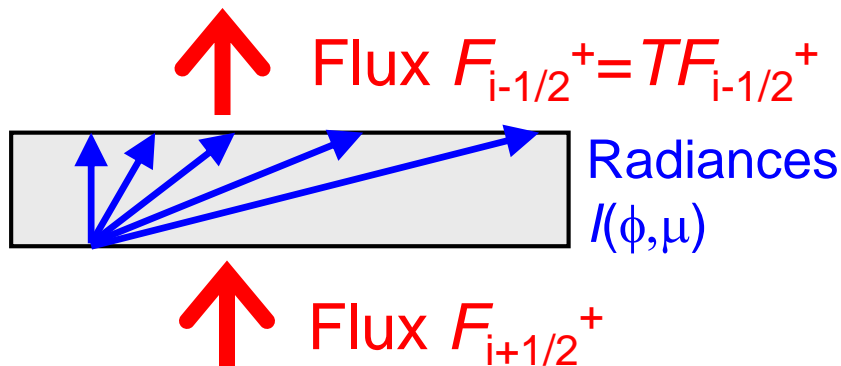
- Where coefficients given by (use delta-Eddington scaling):

$$\gamma_1 = \frac{1}{\mu_1} \left[1 - \frac{\omega(1+g)}{2} \right] \quad \text{and}$$

$$\gamma_2 = \frac{1}{\mu_1} \left[\frac{\omega(1-g)}{2} \right],$$

Two-stream angle / diffusivity factor

- $\mu_1 = \cos(\theta_1)$ is the *effective* zenith angle that diffuse radiation travels at to get the right *transmittance* T

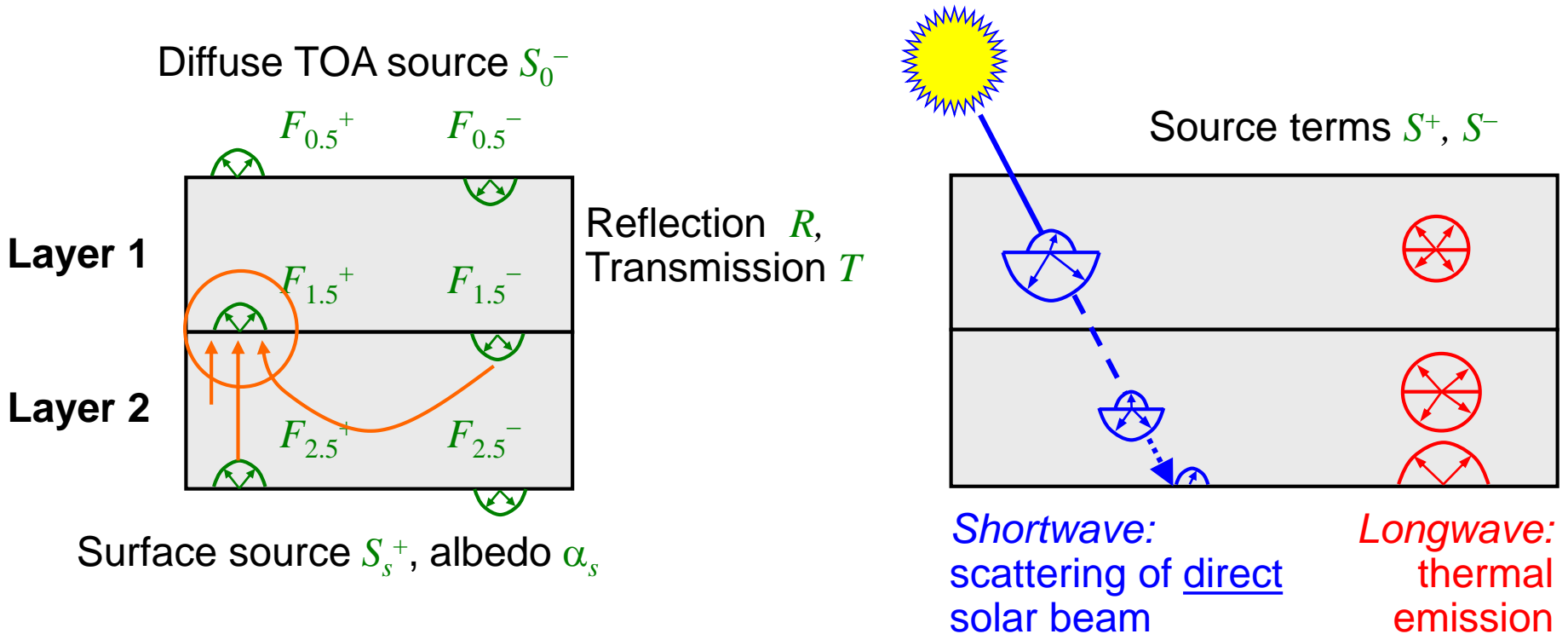


$$T = \frac{\int_0^{2\pi} \int_0^1 I(\phi, \mu) \exp(-\Delta z / \mu) \mu d\mu d\phi}{\int_0^{2\pi} \int_0^1 I(\phi, \mu) \mu d\mu d\phi}$$

$$T = \exp\left(-\frac{\Delta z}{\mu_1}\right)$$

- Most schemes use Elsasser (1942) diffusivity factor of $1/\mu_1 = 1.66$, equivalent to $\theta_1 = 53^\circ$

Discretized two-stream scheme



- Equations relating diffuse fluxes between levels take the form:

$$F_{i-0.5}^+ = T_i F_{i+0.5}^+ + R_i F_{i-0.5}^- + S_i^+$$

- Terms T , R and S found by solving two-stream equations for single homogeneous layers: solutions given by Meador and Weaver (1980)

Solution for two-level atmosphere

- Solve the following tri-diagonal system of equations

$$\begin{pmatrix} 1 & & & & & & \\ 1 & -R_1 & -T_1 & & & & \\ & -T_1 & -R_1 & 1 & & & \\ & & 1 & -R_2 & -T_2 & & \\ & & & -T_2 & -R_2 & 1 & \\ & & & & 1 & -\alpha_s & \end{pmatrix} \begin{pmatrix} F_{0.5}^+ \\ F_{0.5}^- \\ F_{1.5}^+ \\ F_{1.5}^- \\ F_{2.5}^+ \\ F_{2.5}^- \end{pmatrix} = \begin{pmatrix} S_0^- \\ S_1^+ \\ S_1^- \\ S_2^+ \\ S_2^- \\ S_s^+ \end{pmatrix}$$

- Efficient to solve and simple to extend to more layers
- Typical schemes also include separate regions at each height for cloud and clear-sky

Physical interpretation of tri-diagonal solution

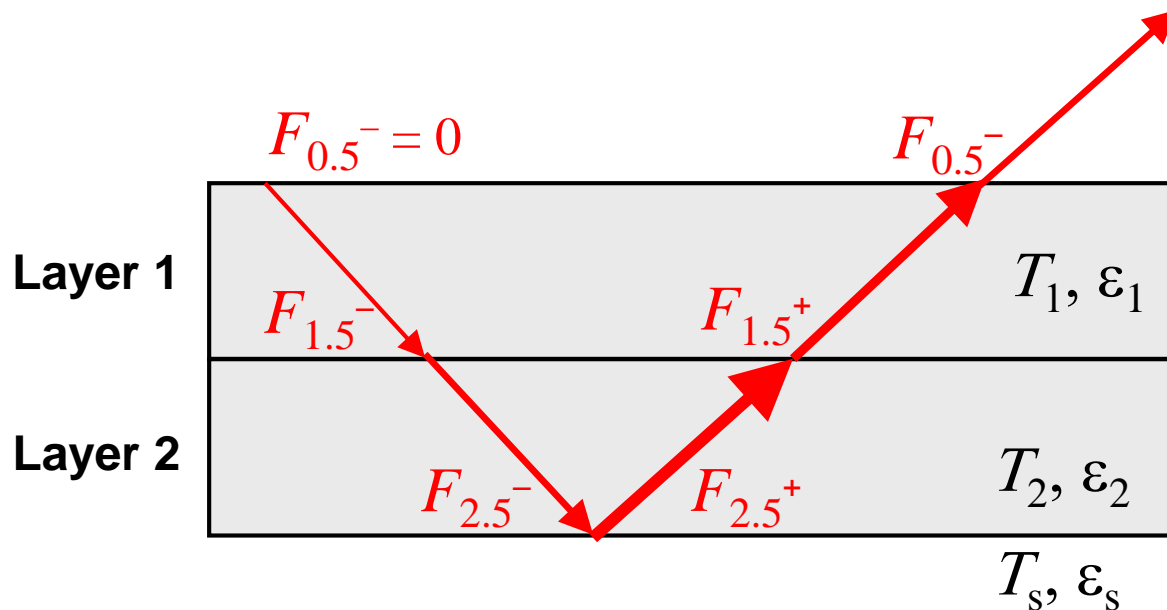
- It is conceptually convenient to solve the system by
 - Working up from the surface calculating the albedo α_i and upward emission G_i of the whole atmosphere below half-level i .

$$\begin{pmatrix} 1 & -\alpha_{0.5} & & & & & \\ & 1 & & & & & \\ & \beta_1 \alpha_1 T_1 & 1 & & & & \\ & -T_1 & -R_1 & 1 & & & \\ & & & \beta_2 \alpha_2 T_2 & 1 & & \\ & & & -T_2 & -R_2 & 1 & \\ & & & & & & 1 \end{pmatrix} \begin{pmatrix} F_{0.5}^+ \\ F_{0.5}^- \\ F_{1.5}^+ \\ F_{1.5}^- \\ F_{2.5}^+ \\ F_{2.5}^- \end{pmatrix} = \begin{pmatrix} G_{0.5} \\ S_{TOA}^- \\ \beta_1 (G_{1.5} + S_1^- \alpha_{1.5}) \\ S_2^- \\ \beta_2 (S_s^+ + S_2^- \alpha_s) \\ S_2^- \end{pmatrix}$$

- Then working down from TOA, calculating the upwelling and downwelling fluxes from α_i and G_i .

Neglecting longwave scattering

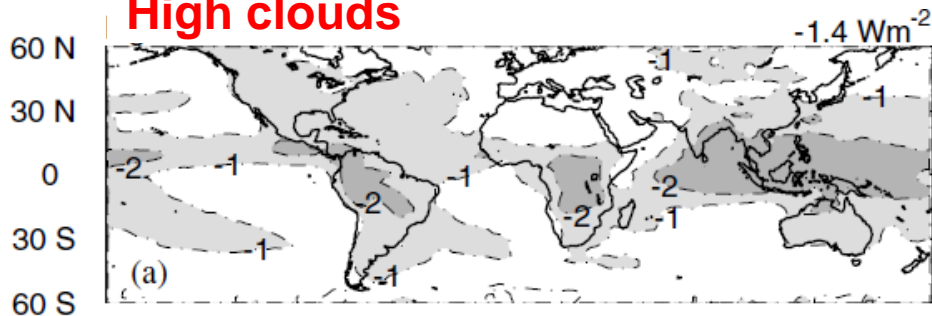
- Many (most?) GCMs neglect longwave scattering by clouds and aerosols, simplifying the algorithm
- Layer monochromatic emissivity $\varepsilon = 1 - \exp(-\delta_a/\cos\theta_l)$



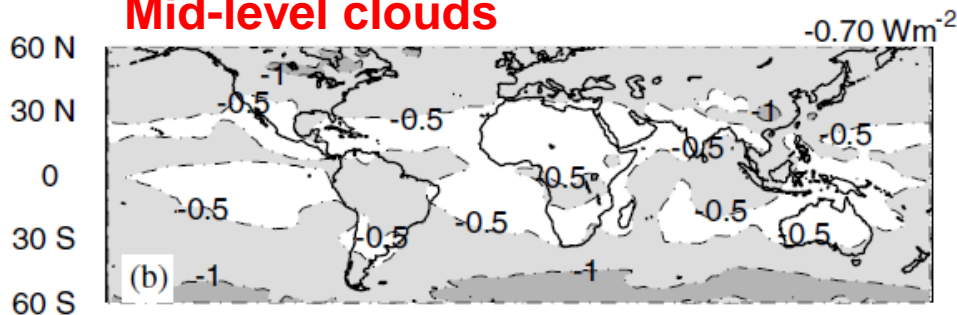
$$\begin{aligned}
 F_{0.5}^- &= 0 \\
 &\vdots \\
 F_{i+1/2}^- &= (1 - \varepsilon_i) F_{i-1/2}^- + \varepsilon_i B(T_i) \\
 &\vdots \\
 F_s^+ &= (1 - \varepsilon_s) F_s^- + \varepsilon_s B(T_s) \\
 &\vdots \\
 F_{i-1/2}^+ &= (1 - \varepsilon_i) F_{i+1/2}^+ + \varepsilon_i B(T_i) \\
 &\vdots
 \end{aligned}$$

- What is the error incurred by neglecting longwave scattering?

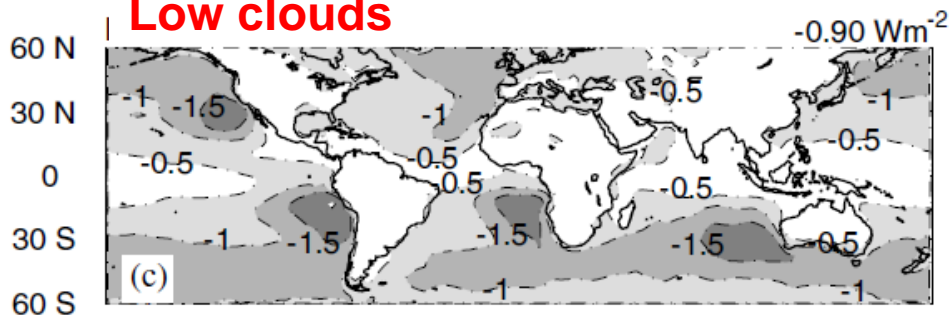
High clouds



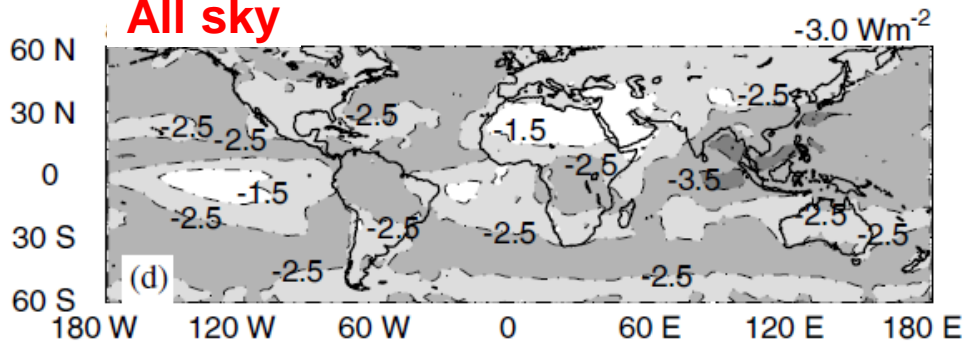
Mid-level clouds



Low clouds



All sky

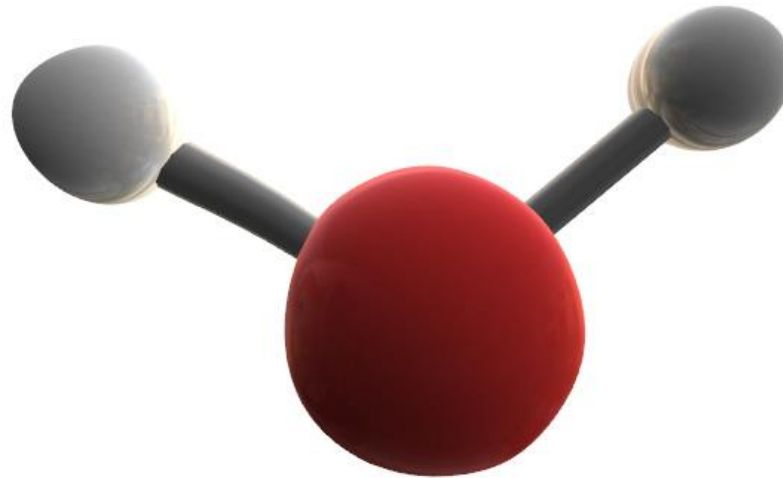


Effect of longwave scattering on OLR

- Outgoing longwave radiation reduced by 3 W m^{-2} , changing cloud longwave radiative effect by 10% and net radiative effect by 20%
- This is due to Kirchhoff's law: scattering means cloud can reflect, so emissivity ($= 1 - \text{albedo}$) is reduced below black body value of 1
 - If cloud albedo is 0.1, it will emit 10% less than a black body

Part 3: Gaseous absorption and emission

- Part 2 considered monochromatic radiative transfer only



- What causes complex emission/absorption spectra of gases?
- How do we represent this efficiently in models?

Planck's law



Max Planck

- Spectral radiance [$\text{W m}^{-2} \text{sr}^{-1} \text{Hz}^{-1}$] emitted by a black body at temperature T is

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \left\{ \exp\left(\frac{h\nu}{kT}\right) - 1 \right\}^{-1}$$

h = Planck's constant $6.626 \times 10^{-34} \text{ J s}$

k = Boltzmann's const $1.381 \times 10^{-23} \text{ J K}^{-1}$

c = speed of light in vacuum $299792458 \text{ m s}^{-1}$

- Can change to per-unit-wavelength via $B_\nu d\nu = B_\lambda d\lambda$:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \left\{ \exp\left(\frac{hc}{\lambda kT}\right) - 1 \right\}^{-1}$$

Related laws

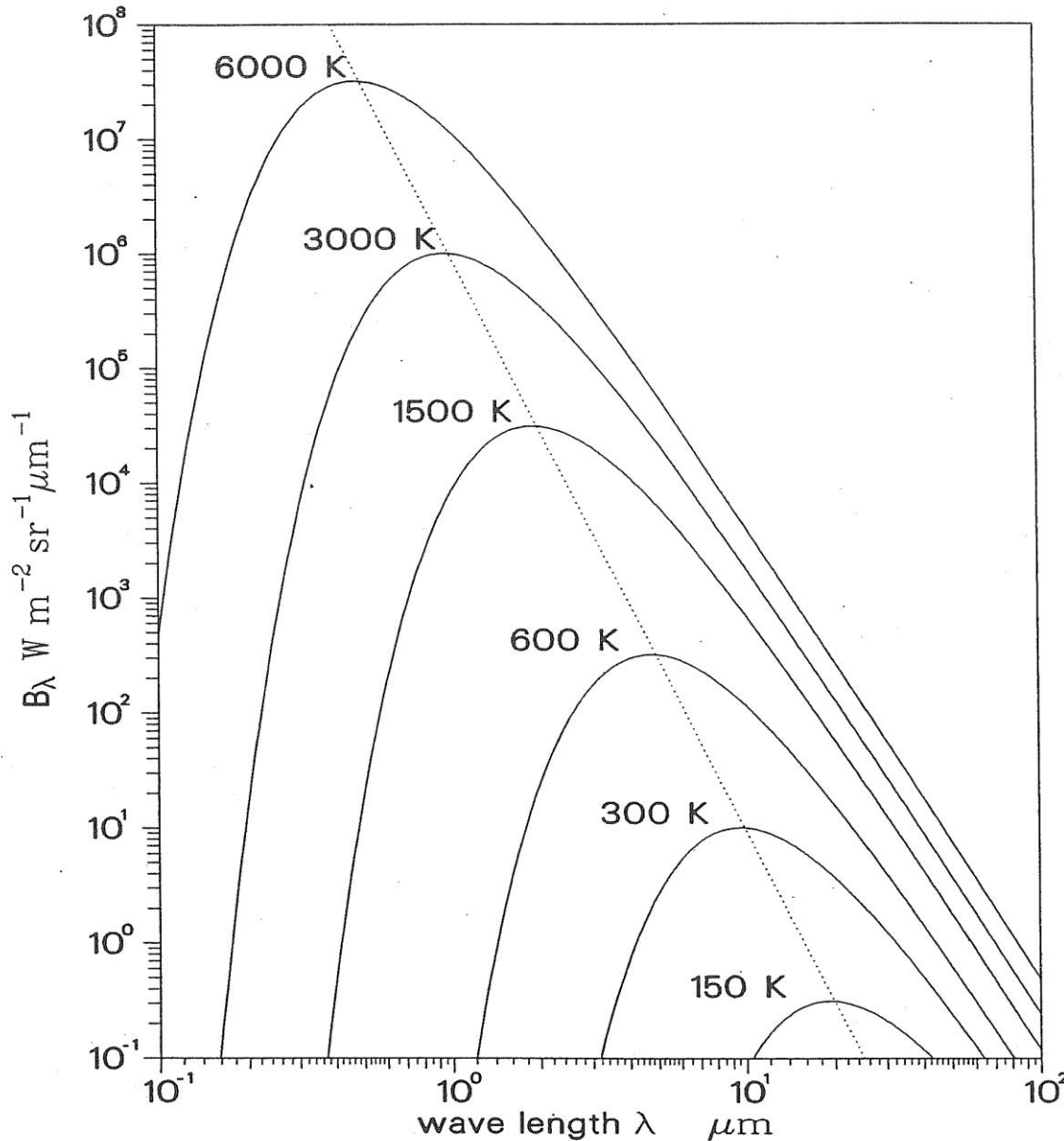


Fig. 2.3 The dependence of the Planck function on wavelength and temperature. Wavelengths of maximum emission are connected by a dotted line.

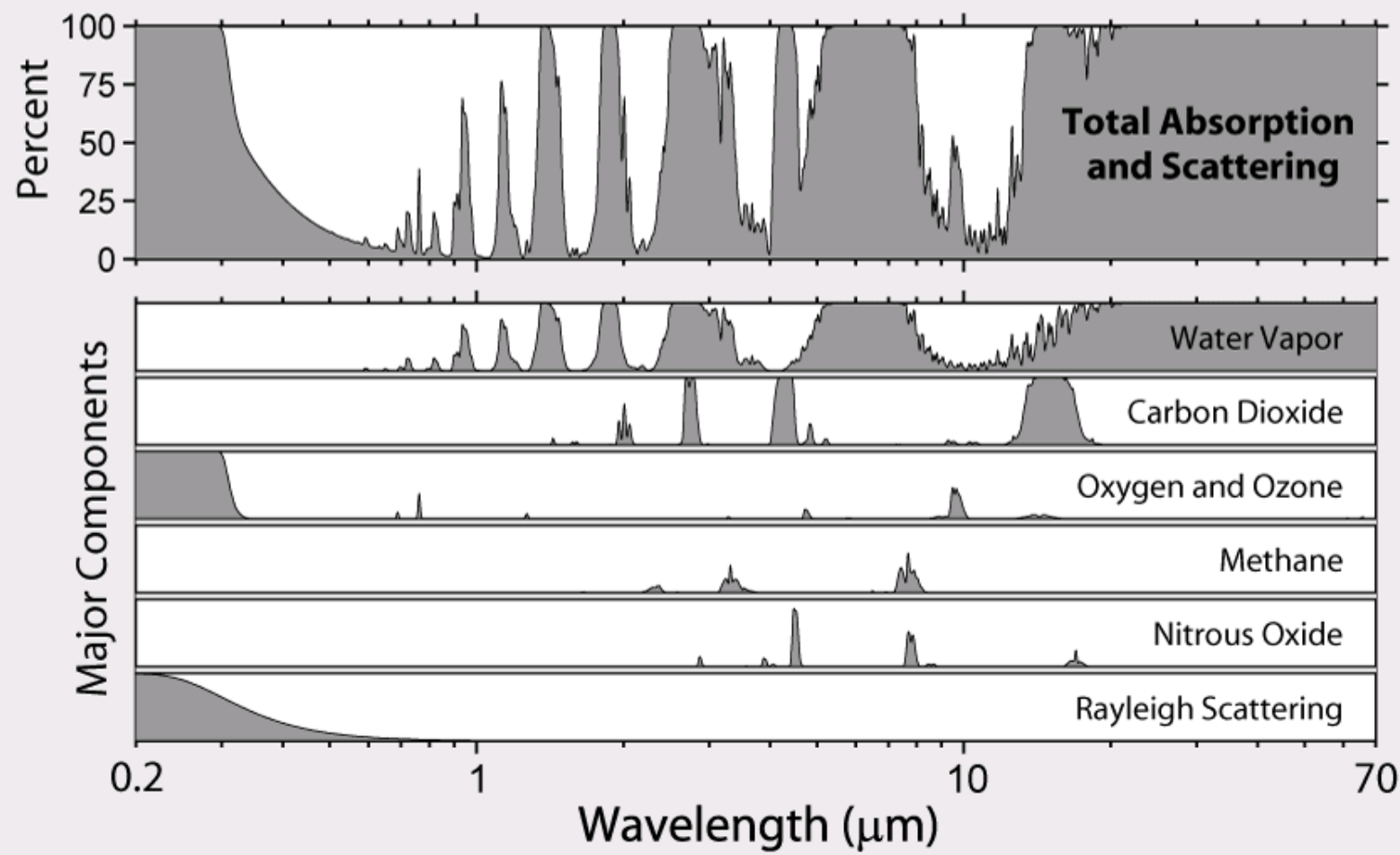
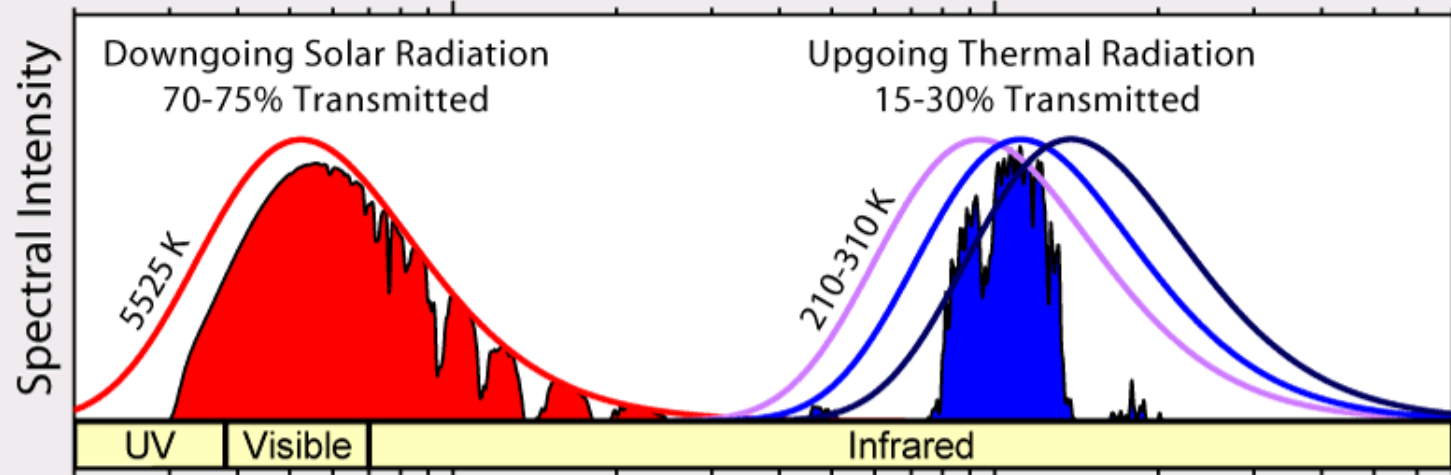
- Stefan-Boltzmann law
 - Integrated flux from black body:

$$F = \pi \int B_\lambda d\lambda = \sigma T^4$$

- Kirchoff's law
 - In thermodynamic equilibrium (up to 50-70 km), at a given wavelength:
 - emissivity ε_λ
 - = absorptivity
 - = $1 - \text{albedo}$

Emission by gases

- Planck function has a continuous spectrum at all temperatures: maximum possible emission by medium in thermal equilibrium
- Absorption by gases is an interaction between molecules and photons and obeys quantum mechanics
 - Not quantized: kinetic energy $\sim kT/2$
 - Quantized: changes in levels of energy occur by $\Delta E = h \Delta \nu$ steps
 - rotational energy: lines in the far infrared $\lambda > 20\mu\text{m}$
 - vibrational energy (+rotational): lines in the 1 - 20 μm
 - electronic energy (+vibr.+rot.): lines in the visible and UV
- Radiation schemes are benchmarked to spectroscopic databases from laboratory measurements
 - For example, HITRAN database (Rothman et al. JQSRT 2009)



Spectral lines

- Spectral lines are of frequency $\nu = \Delta E/h$
- Absorption cross-section per molecule: $\sigma_\nu = S f(\nu - \nu_0)$
 - S = line strength
 - ν_0 = centre frequency
 - $f(\nu - \nu_0)$ = line shape (normalized to unit area)
- Natural broadening
 - Due to Heisenberg's principle (negligible)
- Pressure broadening
 - Molecular collisions disrupt energy levels (troposphere and stratosphere)
- Doppler broadening
 - Due to random motion of molecules, absorption/emission is Doppler-shifted from natural line position (mesosphere)

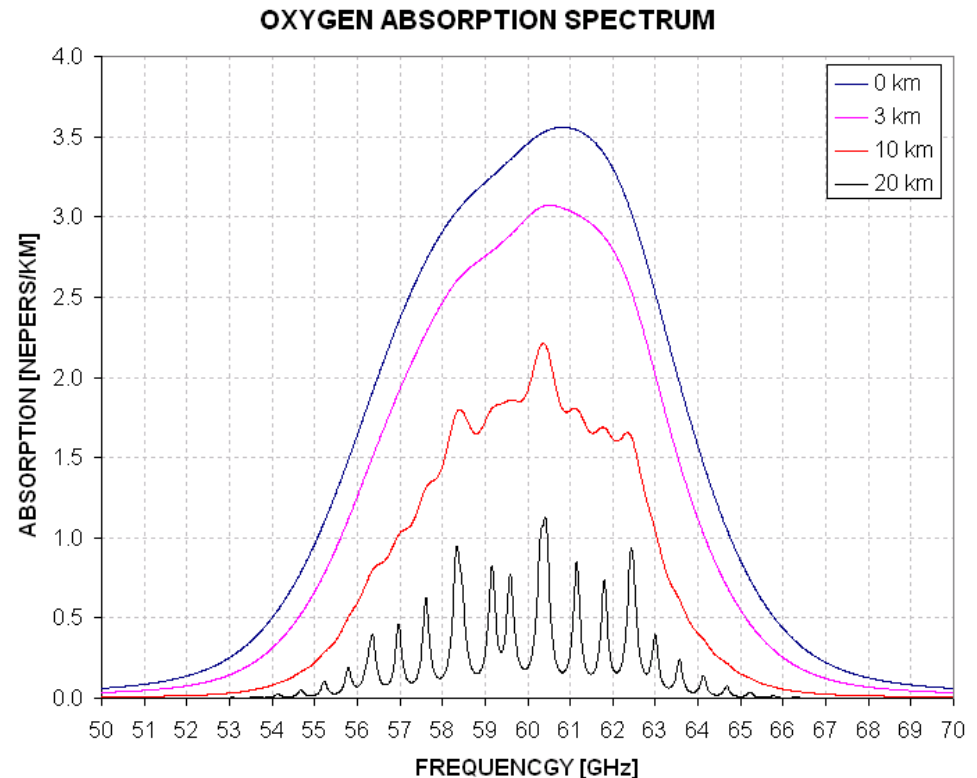
Pressure broadening

- Theory is rather heuristic; usually described adequately but not perfectly by the *Lorenz* line shape:

$$f_L(\nu) = \frac{\alpha_L}{\pi \left[(\nu - \nu_0)^2 + \alpha_L^2 \right]}$$

- With the half-width at half the maximum roughly proportional to the frequency of collisions, modelled by:

$$\alpha_L = \alpha_{L0} \frac{P}{P_0} \left(\frac{T_0}{T} \right)^{0.5}$$



Doppler broadening

- Molecular velocity distribution is Gaussian:

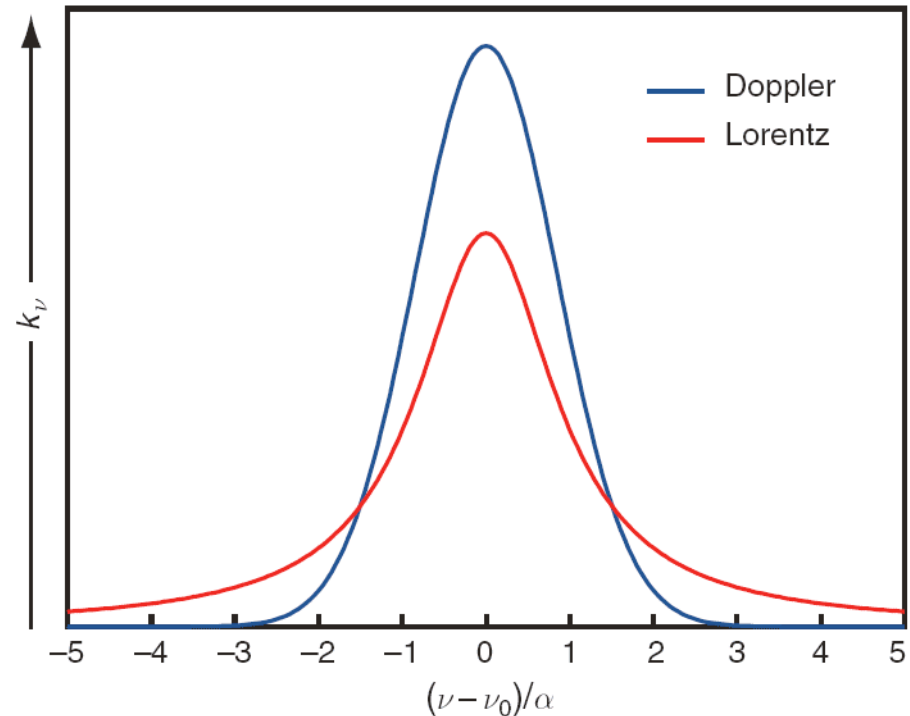
$$P(v) = \left(\frac{m}{2\pi kT} \right)^{0.5} \exp\left(-\frac{mv^2}{2kT} \right)$$

- Doppler shift $\nu' = \nu (1 - v/c)$ so line shape is Gaussian

$$f_D(\nu) = \frac{1}{\alpha_D(\pi)^{0.5}} \exp\left\{ -\left[\frac{\nu - \nu_0}{\alpha_D} \right]^2 \right\}$$

- where

$$\alpha_D = \frac{v_0}{c} \left(\frac{2kT}{m} \right)^{0.5}$$

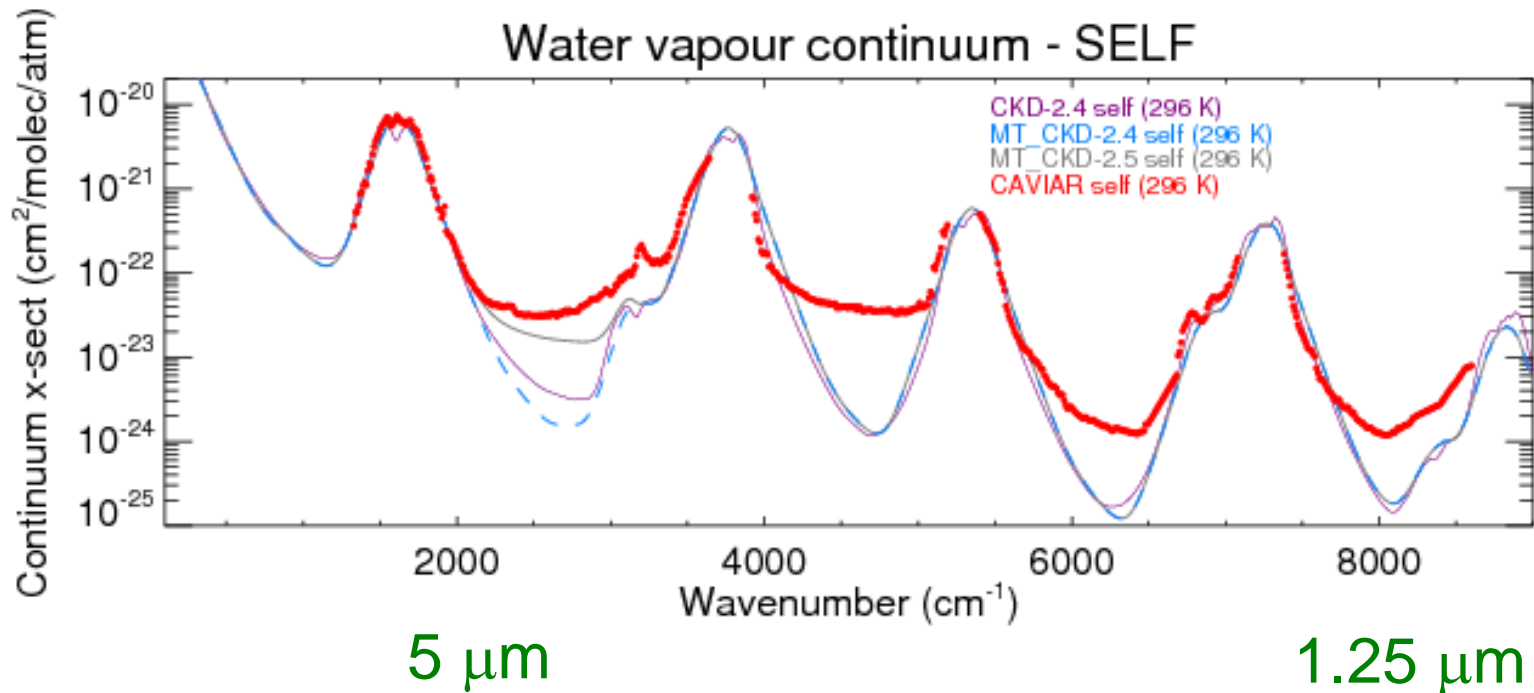


Continuum absorption

- In addition to spectral lines, some absorption does not exhibit line structure – this is due to:
- Photoionization
 - High energy photons (X/ γ -rays) strip electrons from atoms
 - Kinetic energy of resulting ion and electron not quantized, so will be continuum absorption above ionization energy
- Photodissociation
 - Ultraviolet light can break molecules (e.g. O_2 , O_3) into constituent atoms: protects us from hard UV at surface
- Water vapour continuum uncertain: mechanism is either
 - Far wings of lines (due to underestimate by Lorentz shape)
 - Temporary water vapour clusters (dimers, trimers etc.)

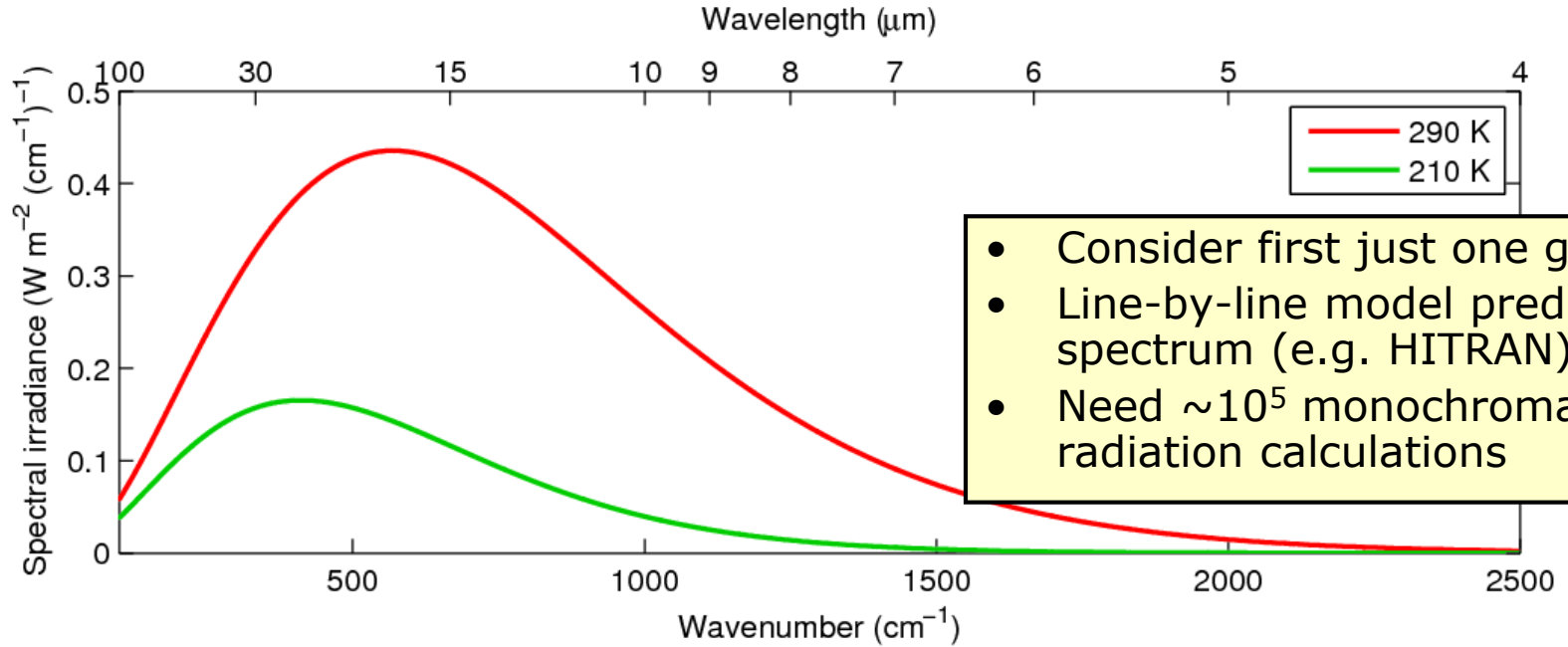
Water vapour continuum

- Shine et al. in CAVIAR project have found that current water vapour continuum models can significantly underestimate absorption in windows between bands, particularly in the near infrared



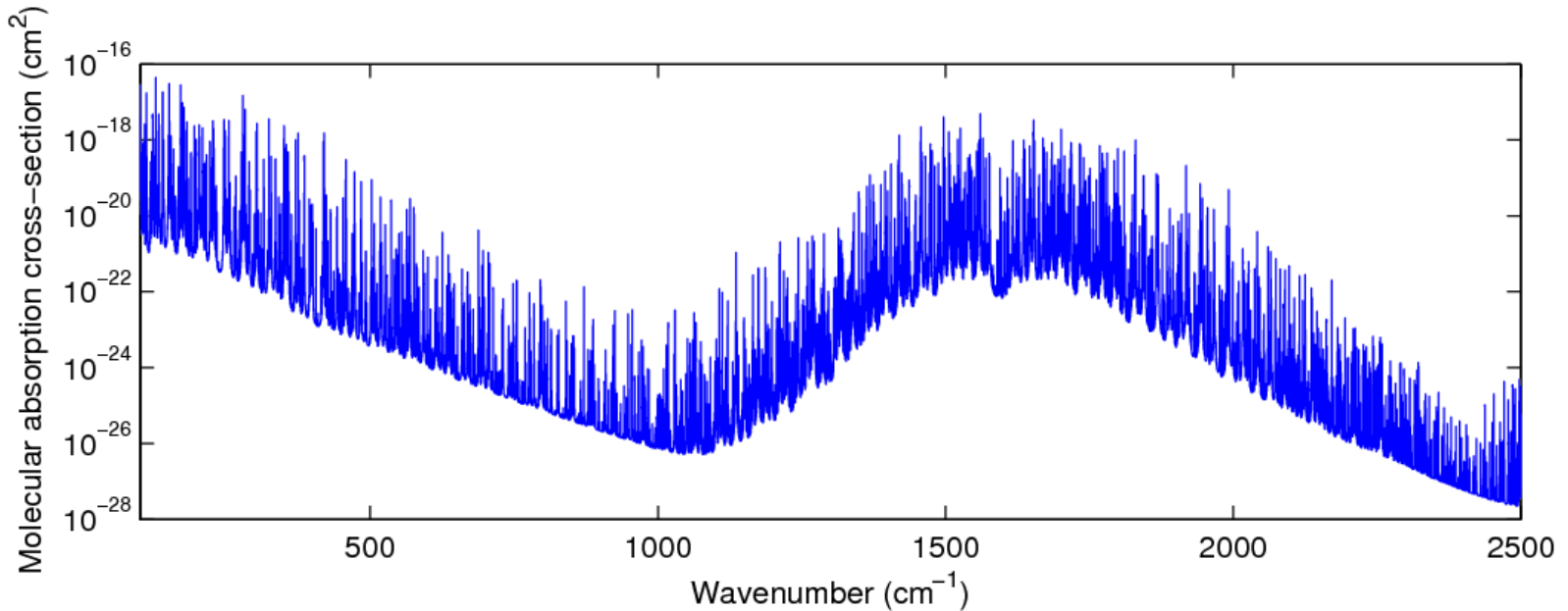
How do we integrate across the spectrum?

Planck function



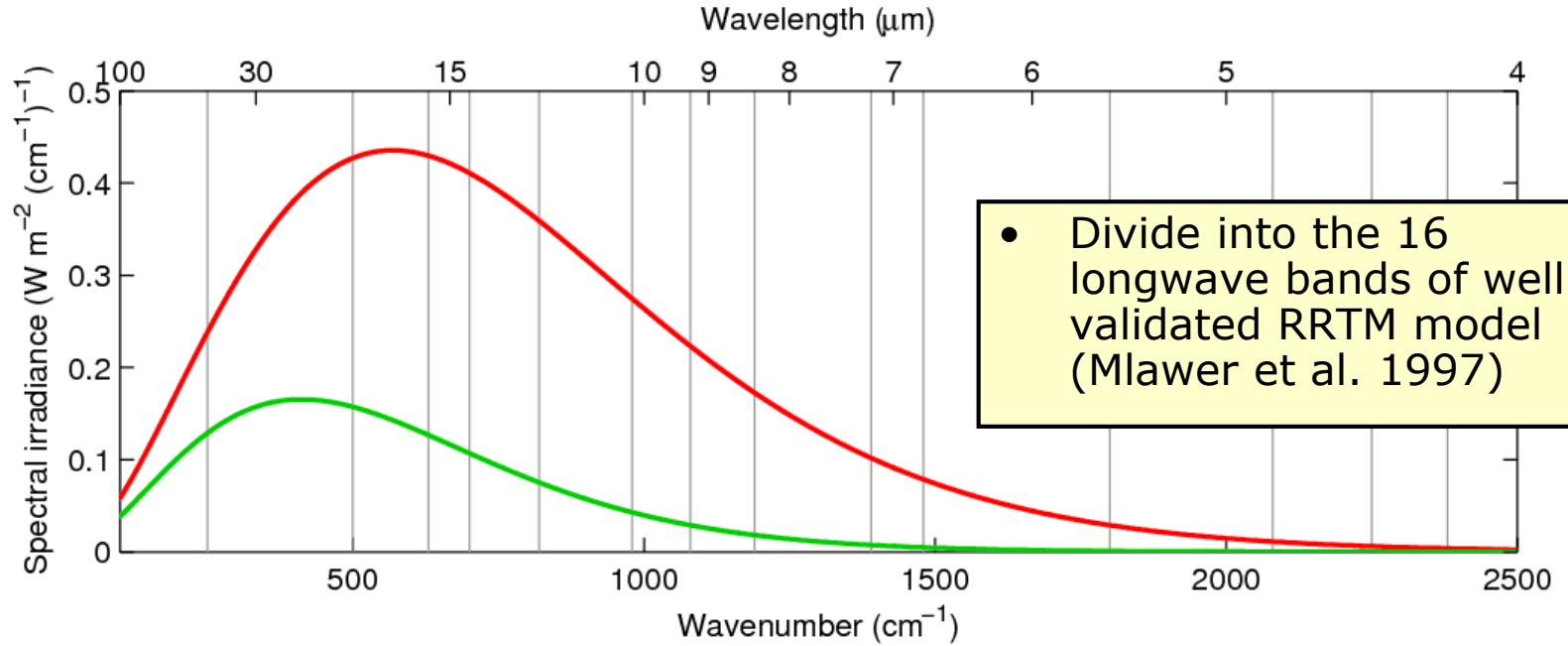
- Consider first just one gas
- Line-by-line model predicts spectrum (e.g. HITRAN)
- Need $\sim 10^5$ monochromatic radiation calculations

Water vapour spectrum

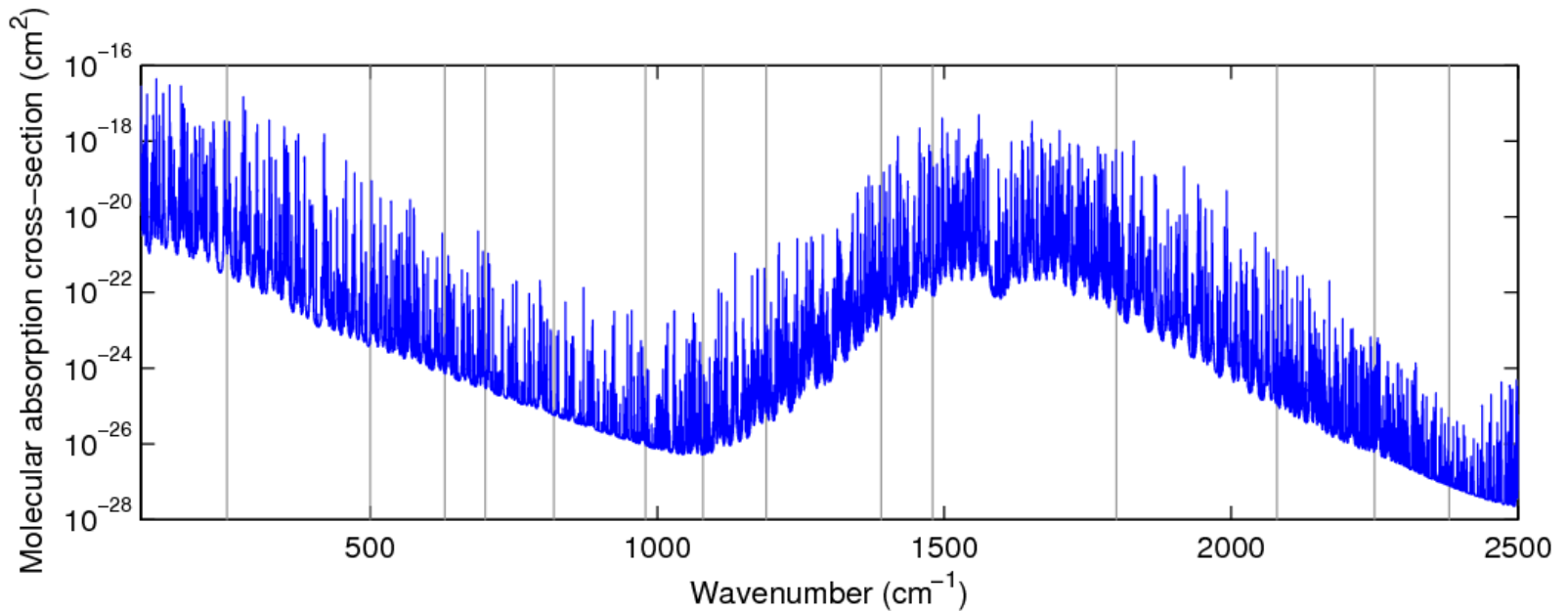


Divide into bands

Planck function

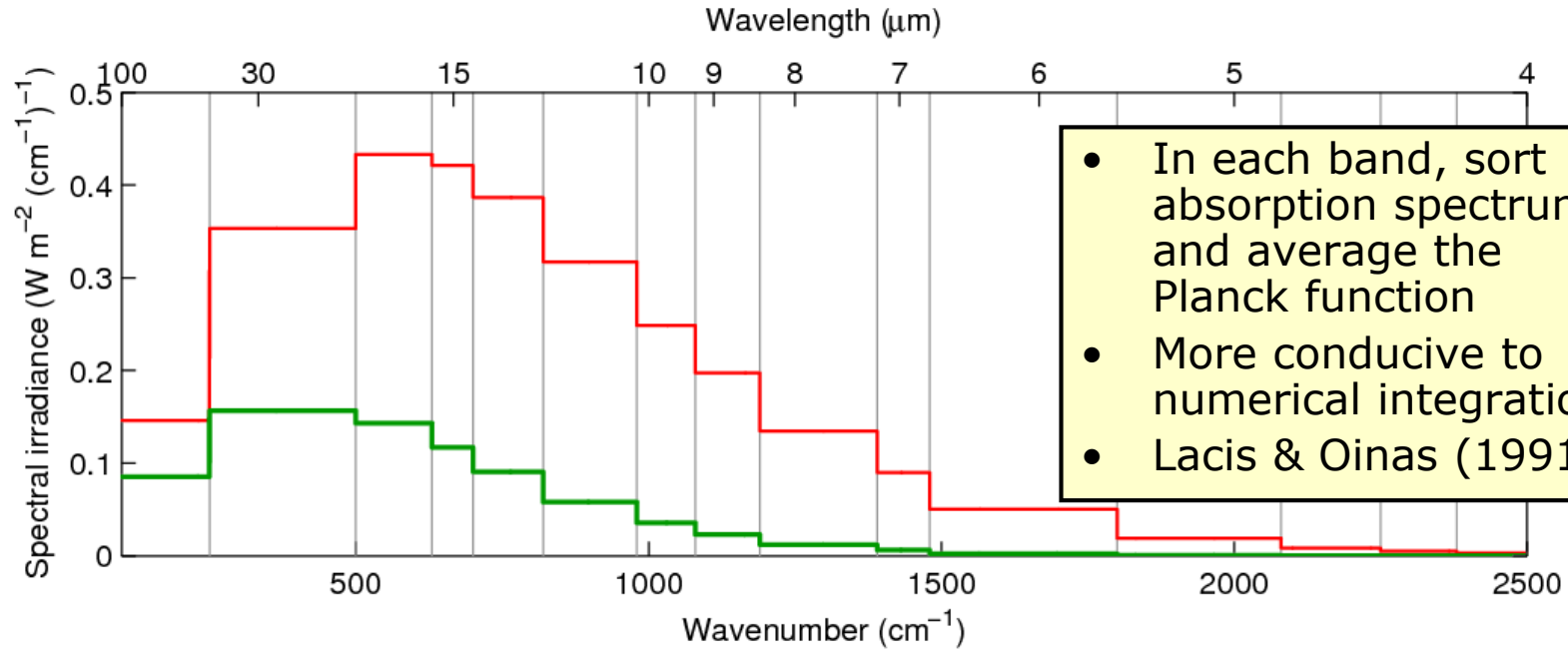


Water vapour spectrum

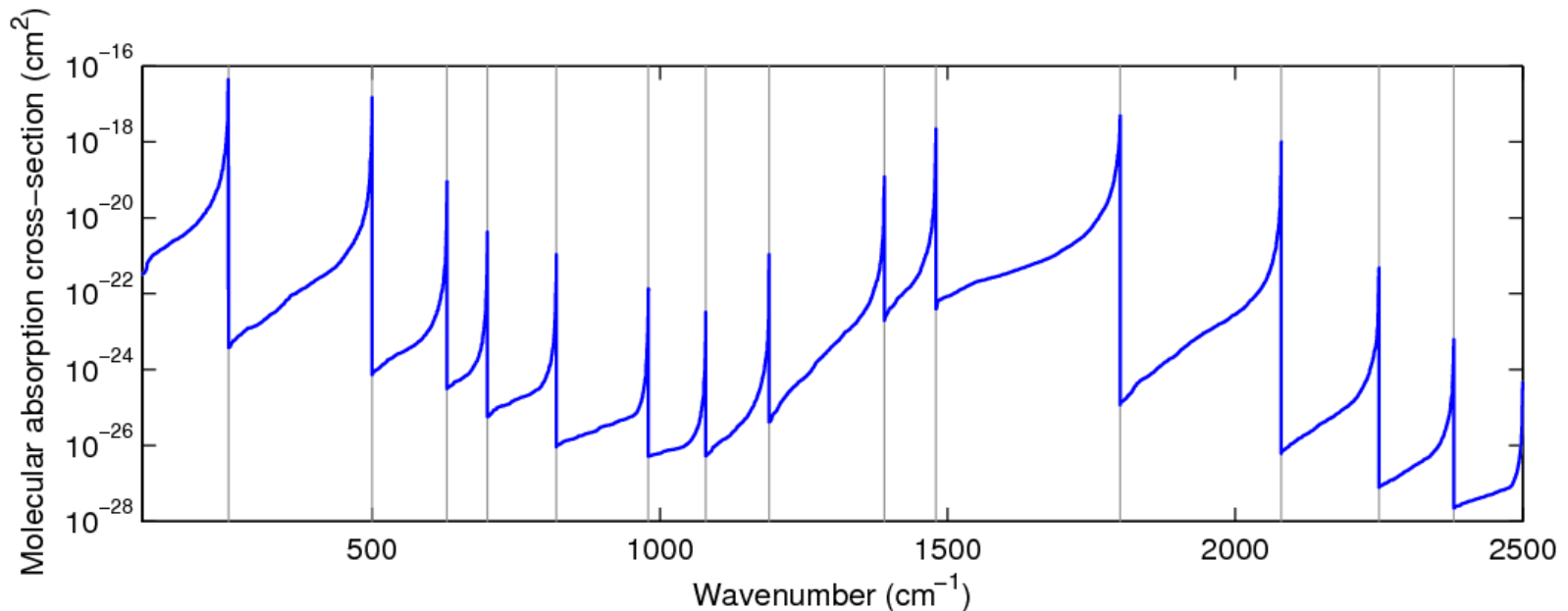


The correlated k-distribution (CKD) method

Planck function

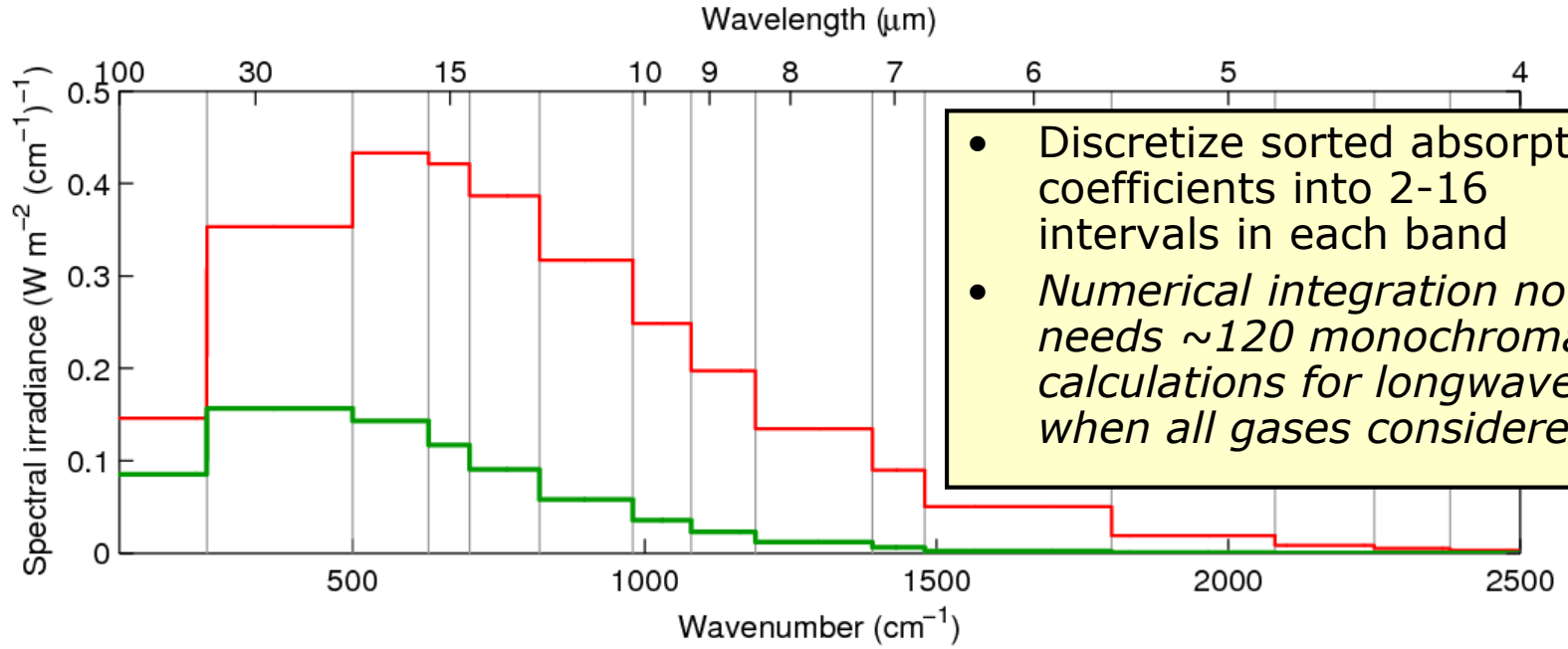


Water vapour spectrum

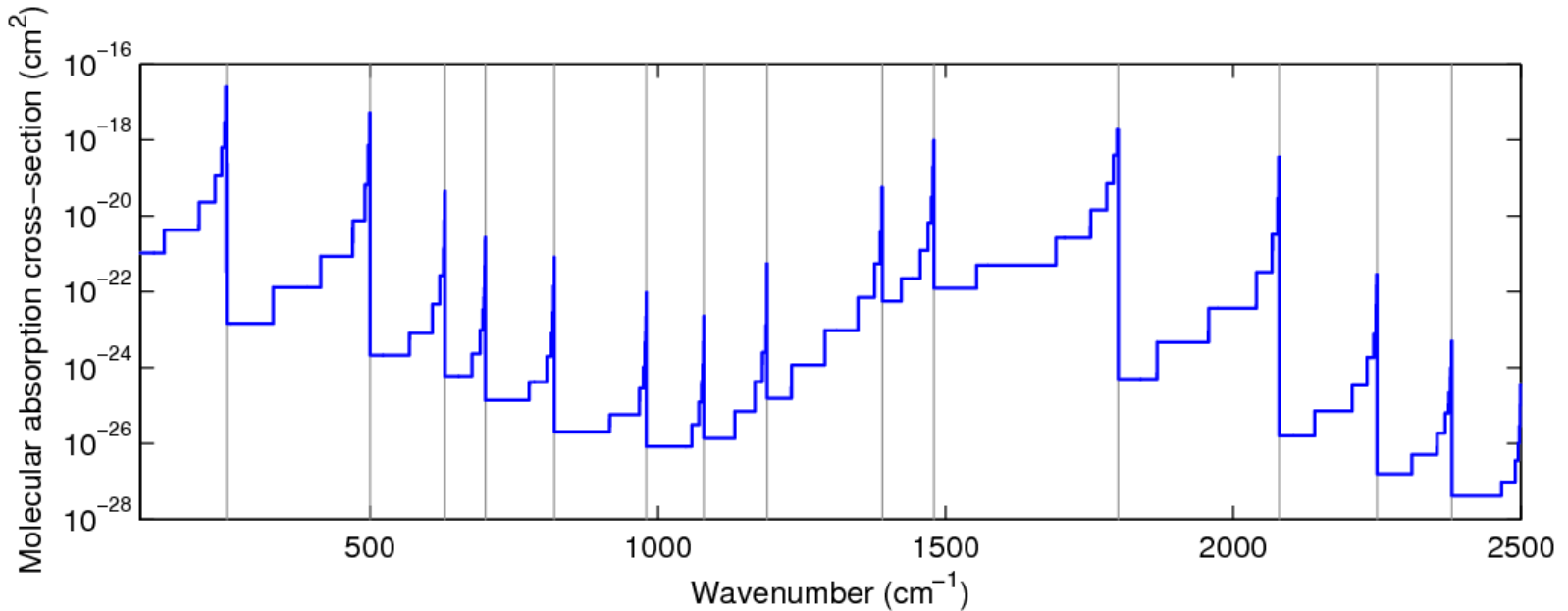


The correlated k-distribution (CKD) method

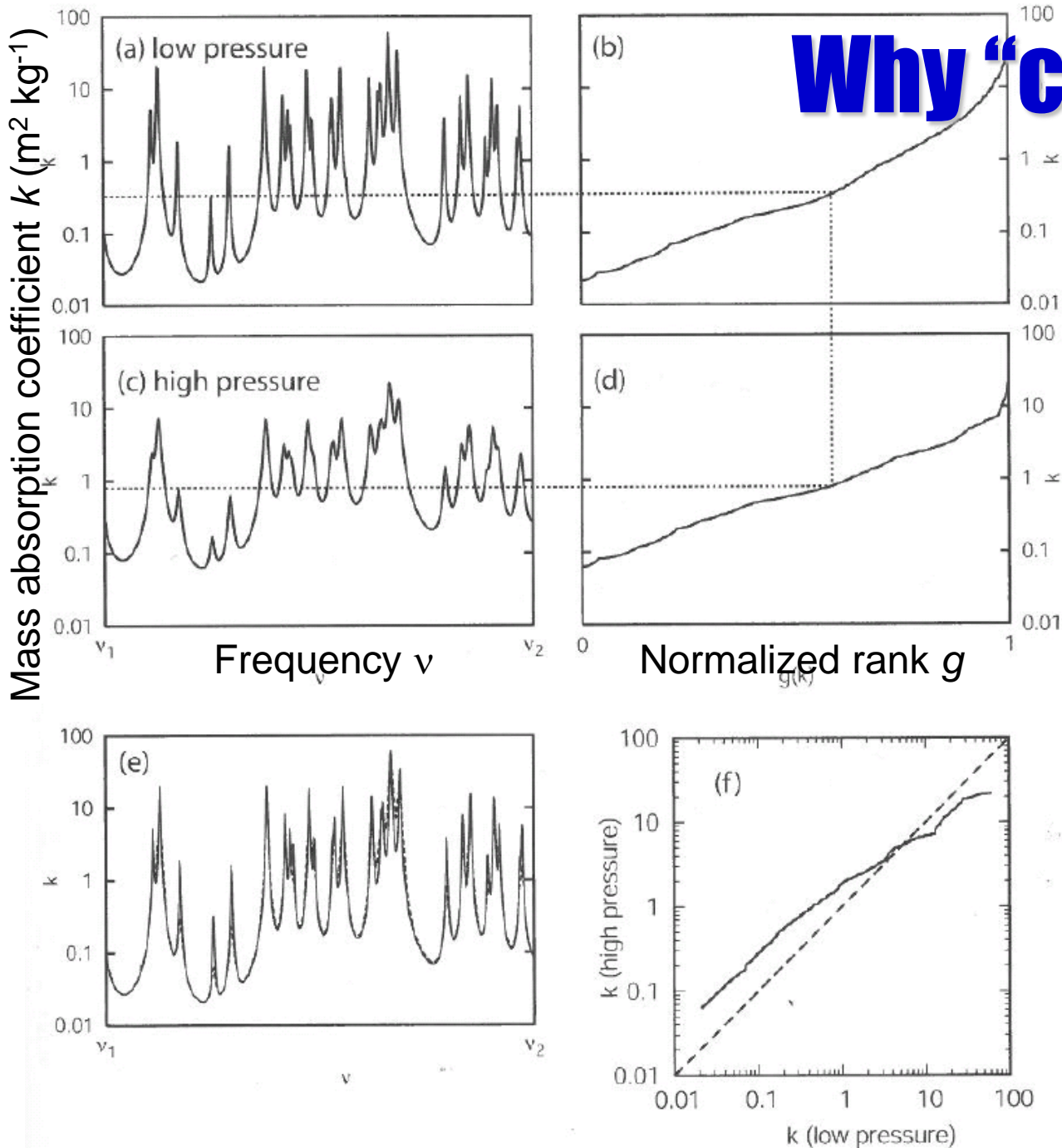
Planck function



Water vapour spectrum

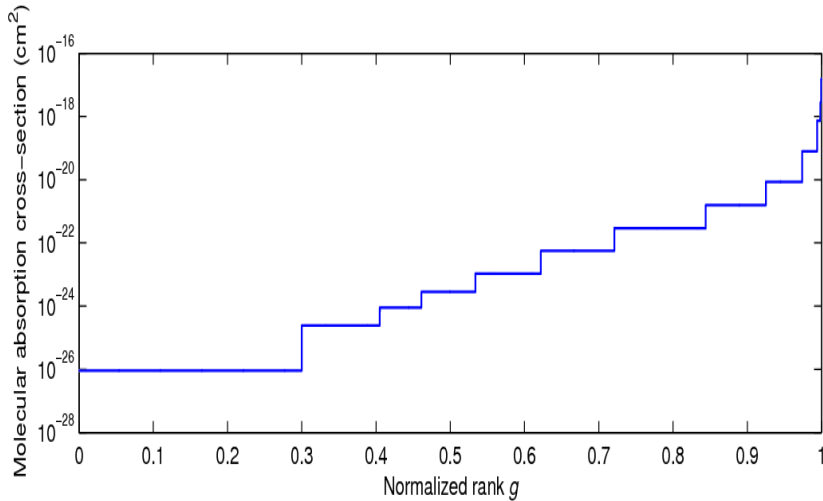


Why “correlated”?

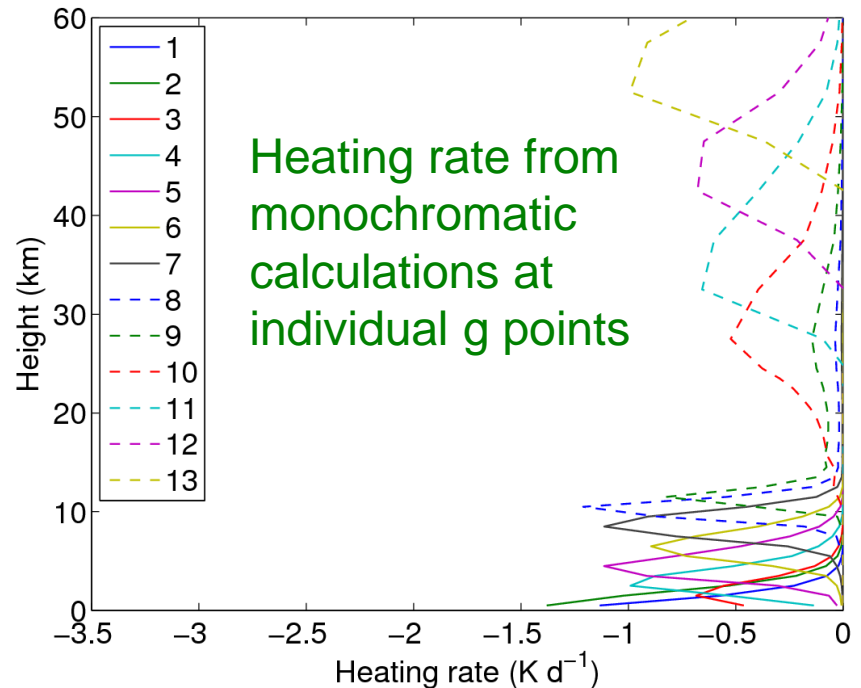
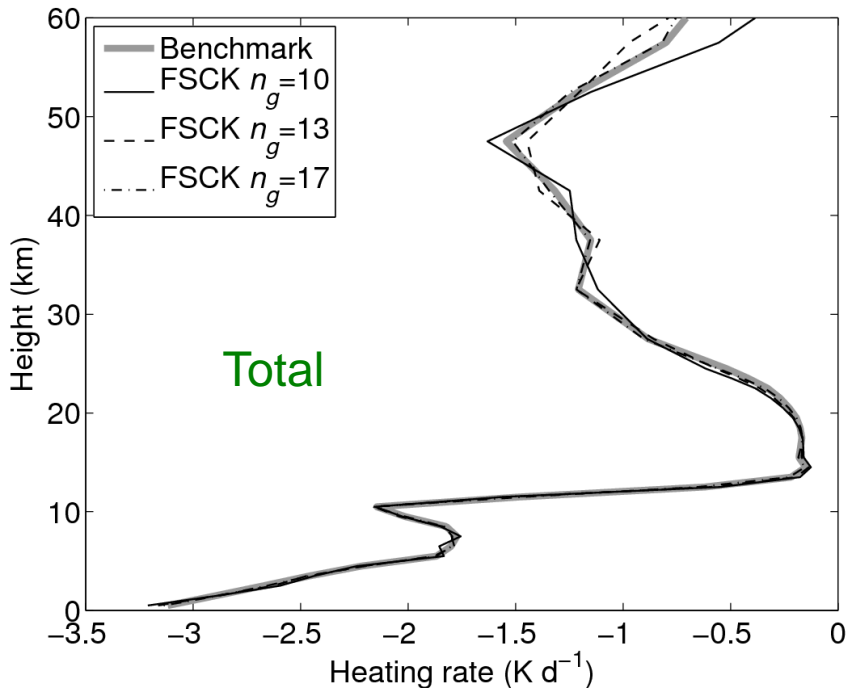


- Despite broadening, absorption spectra at different heights are very well correlated (in terms of *rank correlation*)
- Method assumes perfect rank correlation, so imperfect correlation will cause small errors even for infinite resolution in g space

How best to choose intervals in g space?



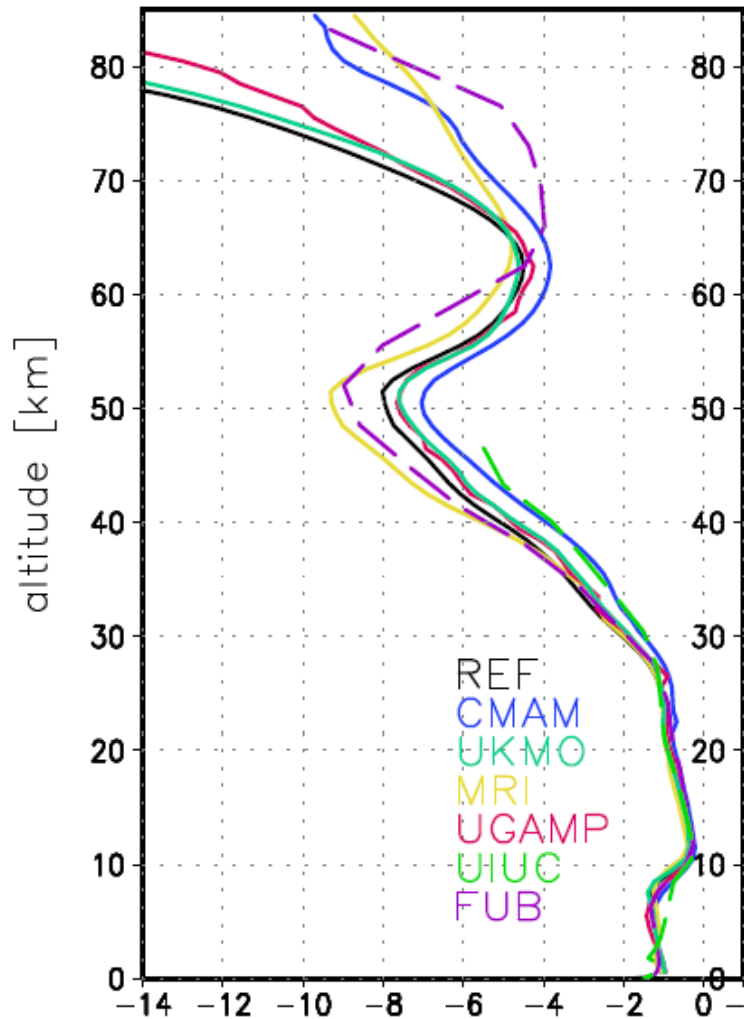
- Minimize errors in heating rate profile
- Tiny parts of the spectrum where the absorption is strongest are responsible for upper-stratospheric heating rates



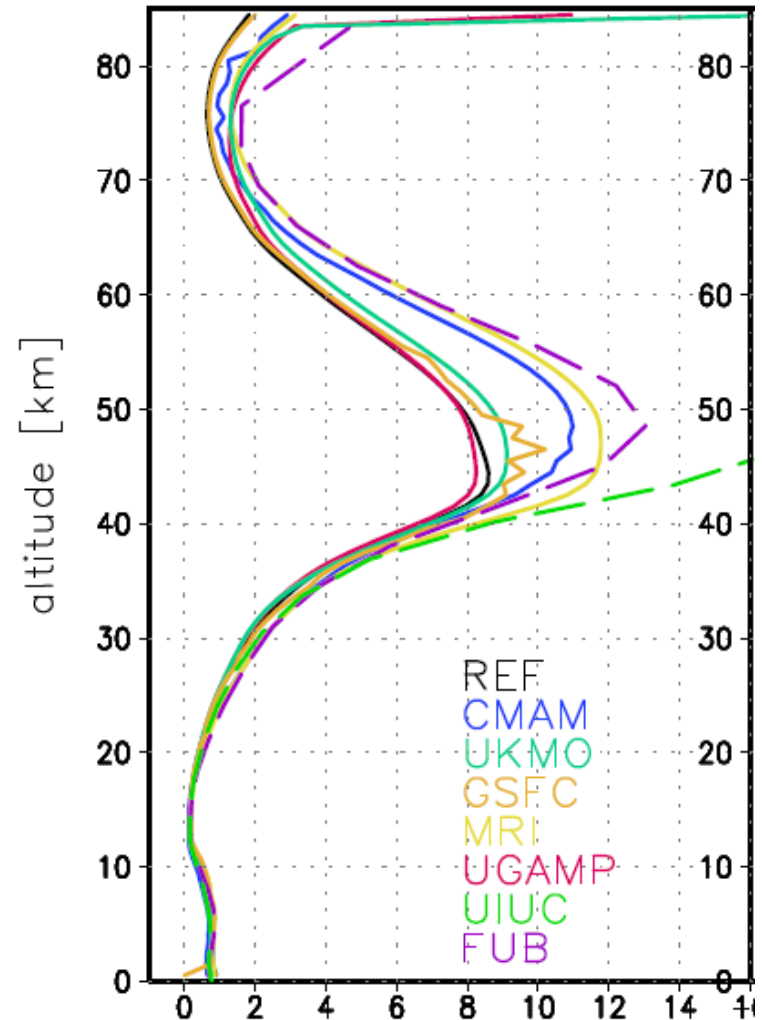
Large discrepancies
in ozone layer and
mesosphere

GRIPS intercomparison

Longwave Heating rates
[K/day]

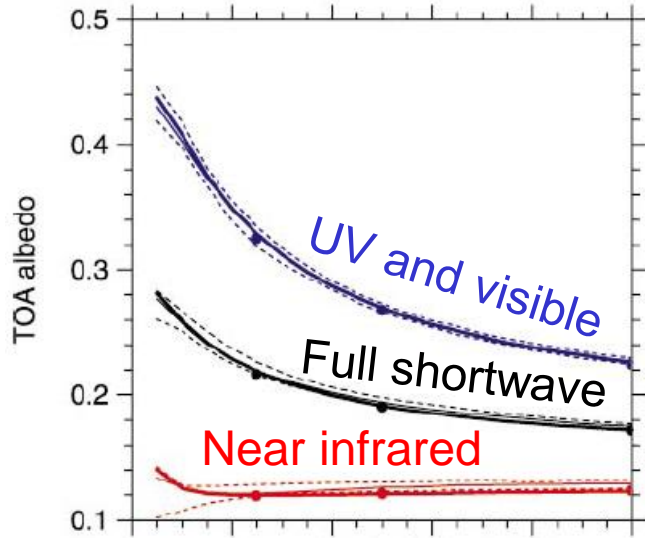


Shortwave Heating rates
[K/day]

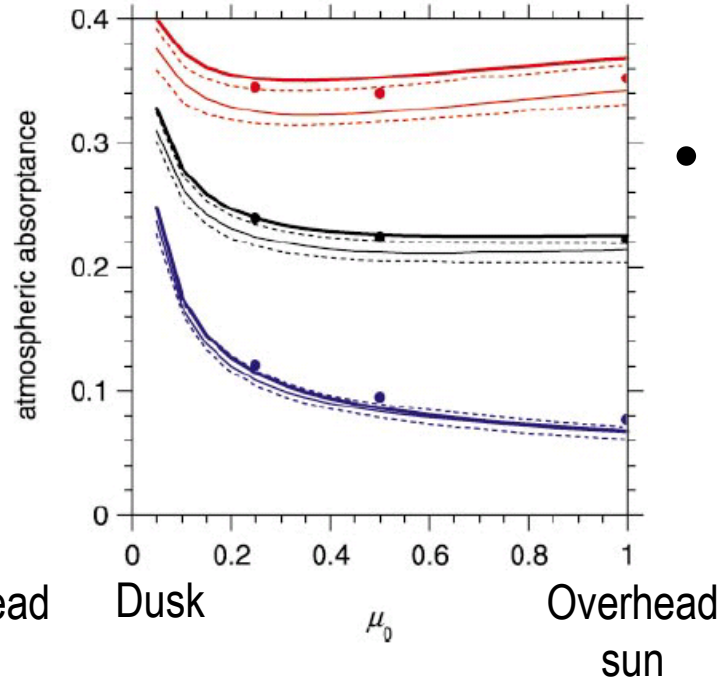
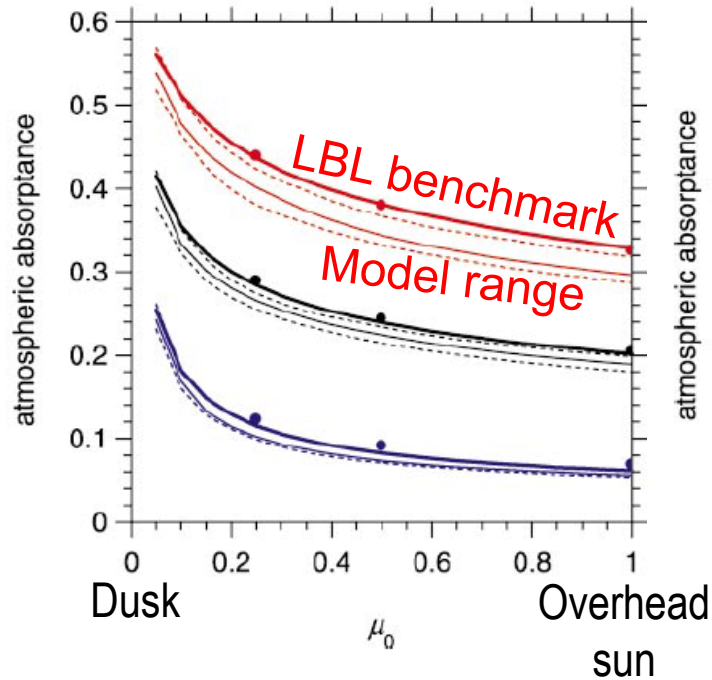
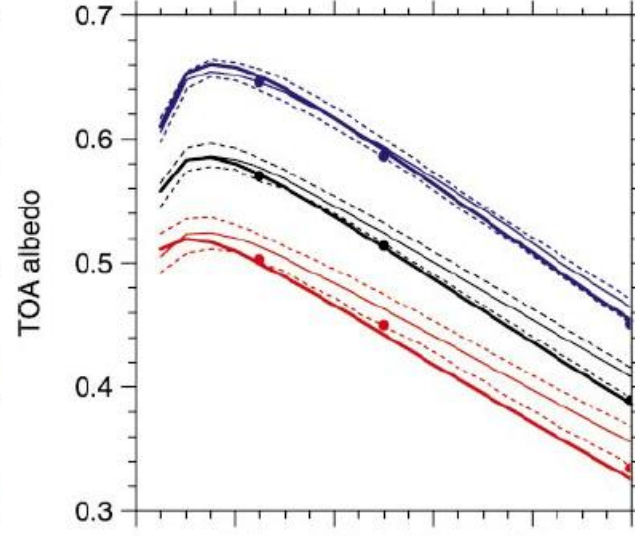


Shortwave comparison

Clear tropical profile



Liquid cloud (plane parallel)



- Barker et al. (JCLim 2003)
- Most models underestimate clear-sky near-IR absorption
 - Poor continuum
- Most models underestimate liquid cloud near-IR absorption

Summary so far

- Radiation is the fundamental driver of the climate system
- The radiative transfer aspects of a radiation scheme can be traced back to Maxwell's equations, including
 - Particle scattering
 - The two-stream equations
- Complex absorption spectra arise due to quantum mechanics
 - The correlated-k-distribution is the state-of-the-art for representing gaseous absorption spectra in models
 - Discrepancies remain between models, especially in representing the water vapour continuum and stratosphere/mesosphere infrared cooling rates