Variational Quality Control

Elias Holm

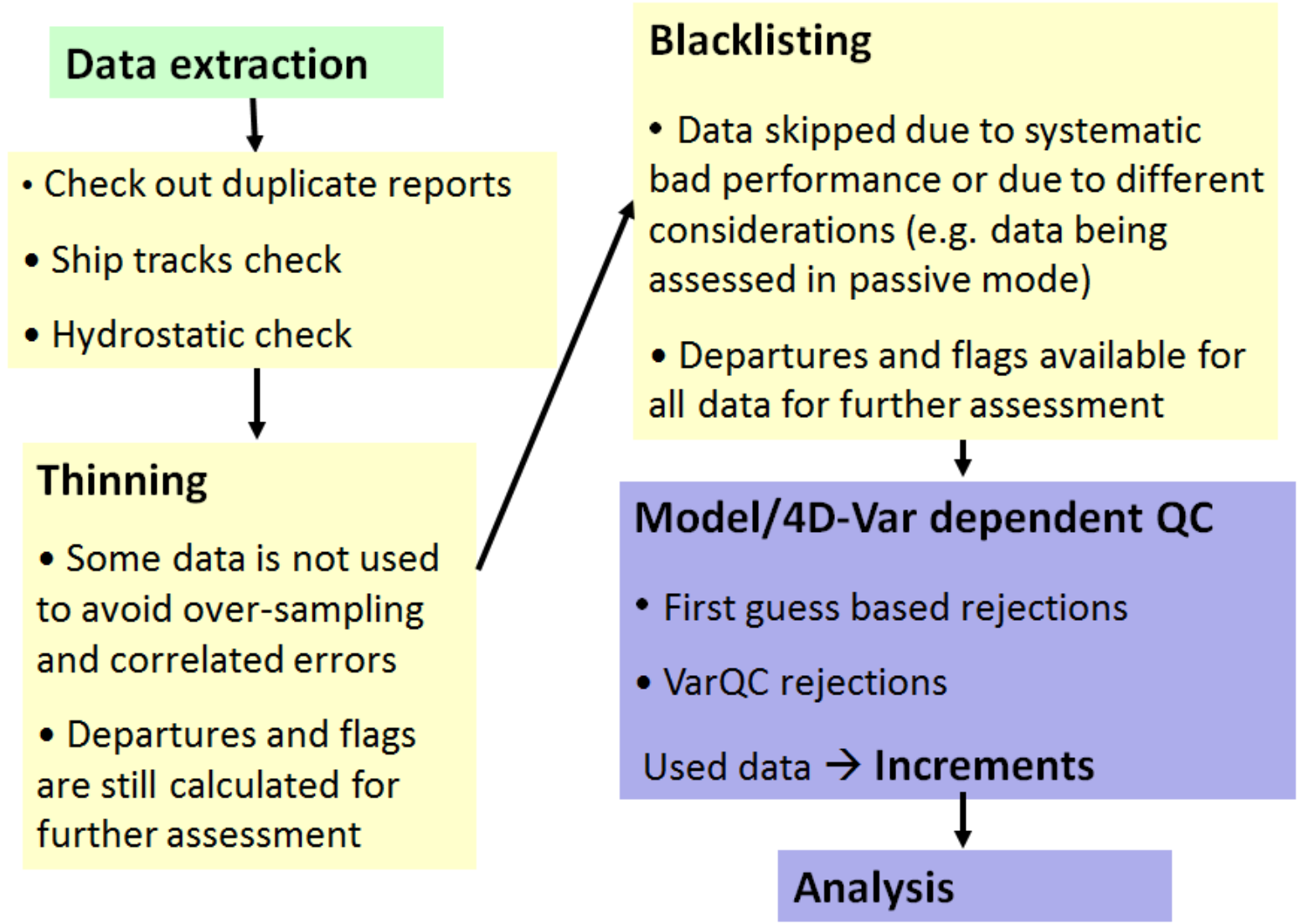
Data Assimilation Section, ECMWF

Contributors: Lars Isaksen, Christina Tavolato,
Erik Andersson and Elias Holm, ECMWF

Outline of Lecture

- Introduction
- VarQC formulation 1: Gaussian+constant
- Rejection limits and tuning
- VarQC formulation 2: Huber norm
- Example
- Summary

Pre-check → Thin → Blacklist → FG-check → VarQC



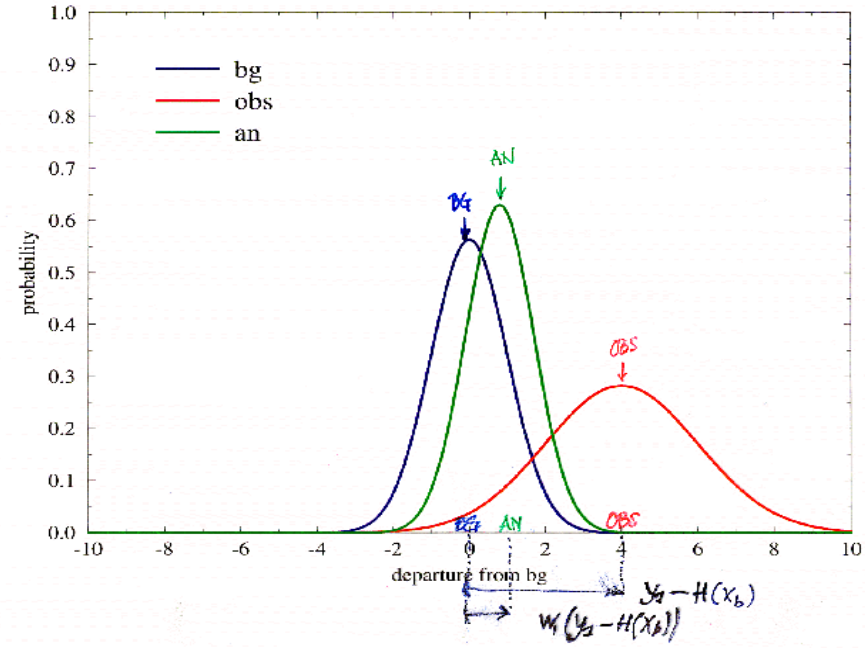
Weight of observations in the analysis

Assuming Gaussian statistics, the maximum likelihood solution to the linear estimation problem results in observation analysis weights (*w*) that are independent of the observed value.

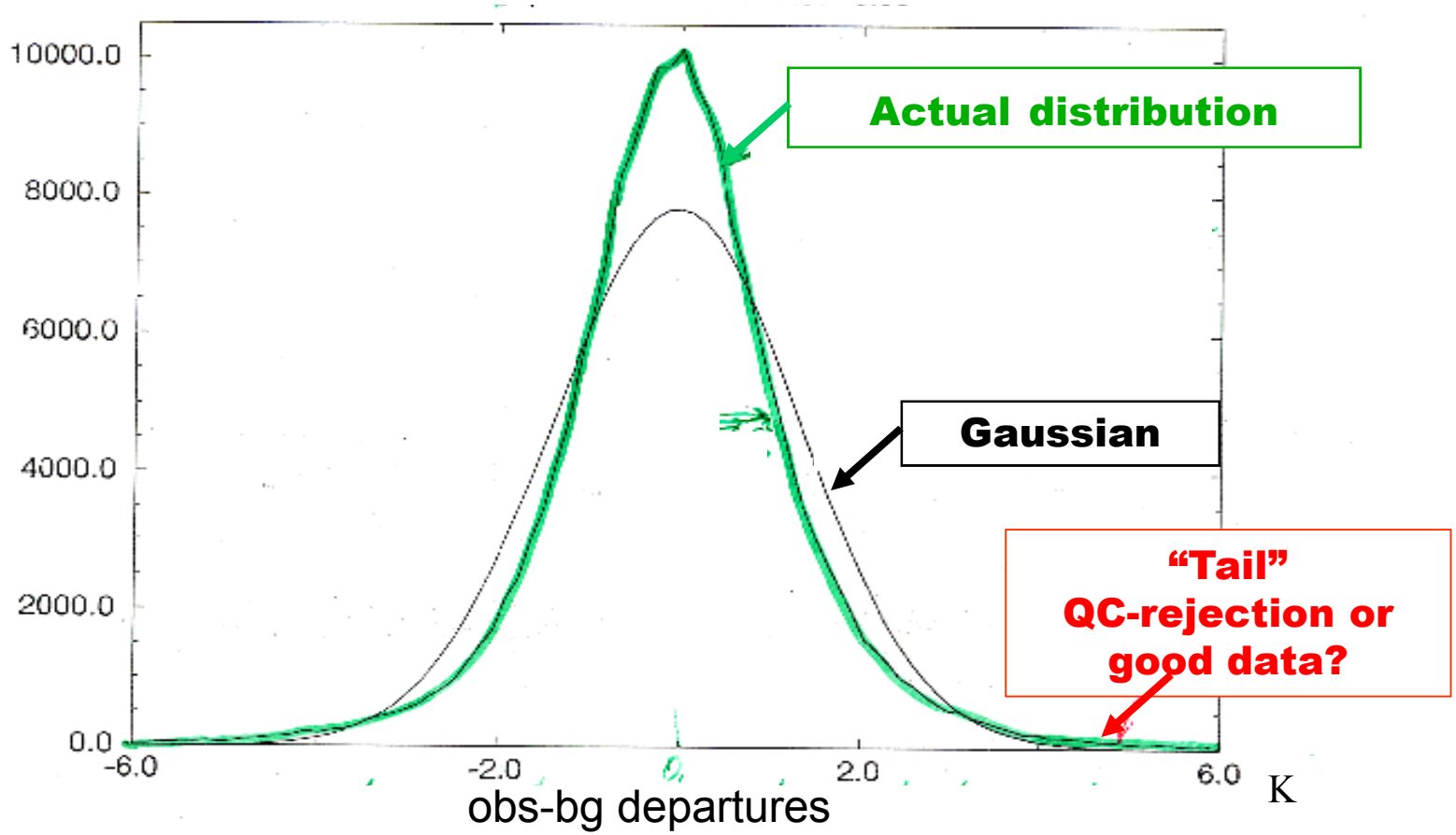
$$x_a - x_b = w(y - Hx_b)$$

$$w = \frac{\sigma_b^2}{\sigma_o^2 + \sigma_b^2}$$

Outliers will be given the same weight as good data, potentially corrupting the analysis



Even good-quality data show significant deviations from the pure Gaussian form



- The real data distribution has fatter tails than the Gaussian
- Aircraft temperature observations shown here

Observation cost function Jo (1)

The general expression for the observation cost function is based on the probability density function (the pdf) of the observation error distribution (see Lorenc 1986):

$$J_o = -\ln p + \text{const}$$

p = probability density function of observation error

Constant chosen such that Jo=0 when y=Hx

Observation cost function Jo (2)

When for *p* we assume the normal (Gaussian) distribution (*N*):

$$N = \frac{1}{\sigma_o \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{y - Hx}{\sigma_o} \right)^2 \right]$$

we obtain the expression

$$J_0^{\mathrm{N}} = -\ln N + \text{const} = \frac{1}{2} \left(\frac{y - Hx}{\sigma_o} \right)^2$$

y: observation
x: represents the model/analysis variables
***H*: observation operators**
σ_o: observation error standard deviation

Normalized departure

In VarQC a non-Gaussian pdf will be used,
resulting in a non-quadratic expression for *Jo*.

Accounting for non-Gaussian effects in Jo

In an attempt to better describe the tails of the observed distributions, Ingleby and Lorenc (1993) suggested a modified pdf (probability density function), written as a sum of two distinct distributions:

$$p^{\mathrm{QC}} = (1 - A)N + Ap^{G}$$

Normal distribution (pdf), as appropriate for 'good' data

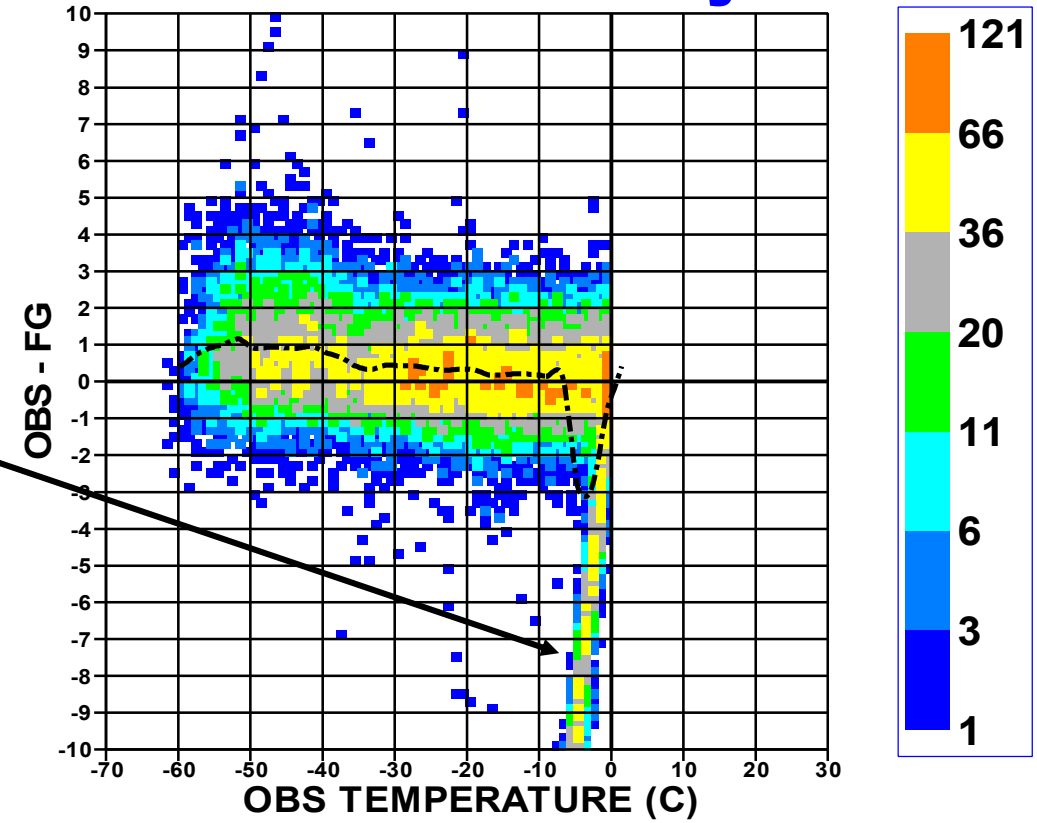
pdf for data affected by gross errors

A is the prior probability of gross error

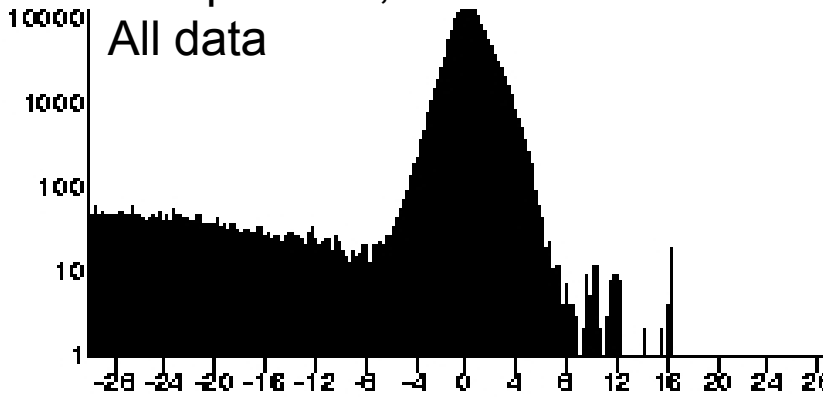
Gross errors of that type occur occasionally...

Positive observed temperatures (˚C) reported with wrong sign.

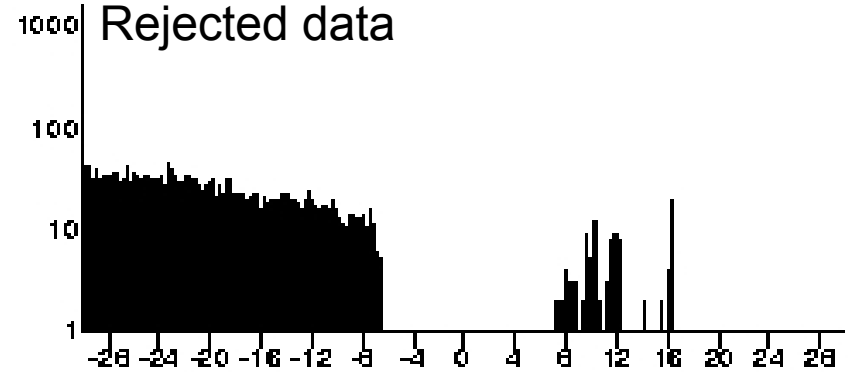
(Chinese aircraft data 1-21 May 2007)



Innovation Statistics
Sample=429,000
All data



Rejected data



Gross error pdf as flat distribution

Thus, a pdf for the data affected by gross errors (p^G) needs to be specified. Several different forms could be considered.

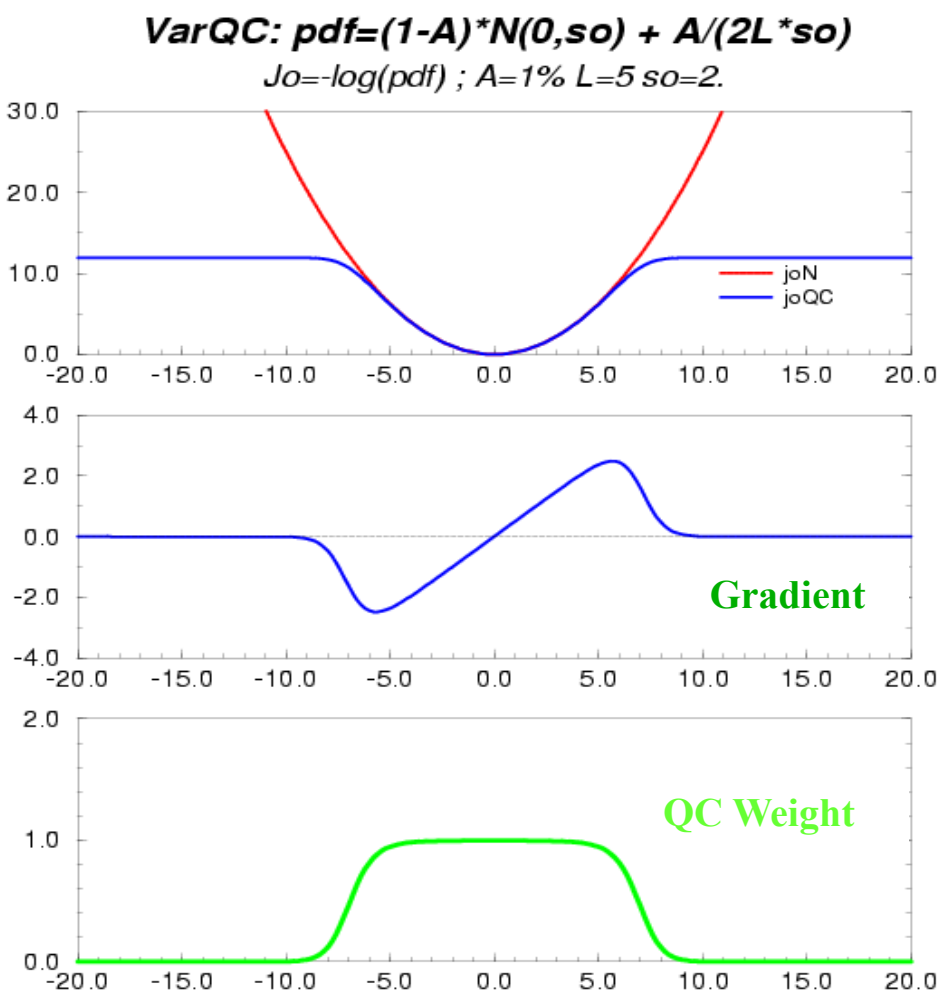
In the ECMWF 1998-2009 implementation (Andersson and Järvinen 1999, QJRMS) a flat distribution was chosen.

$$p^G = \frac{1}{2d}$$

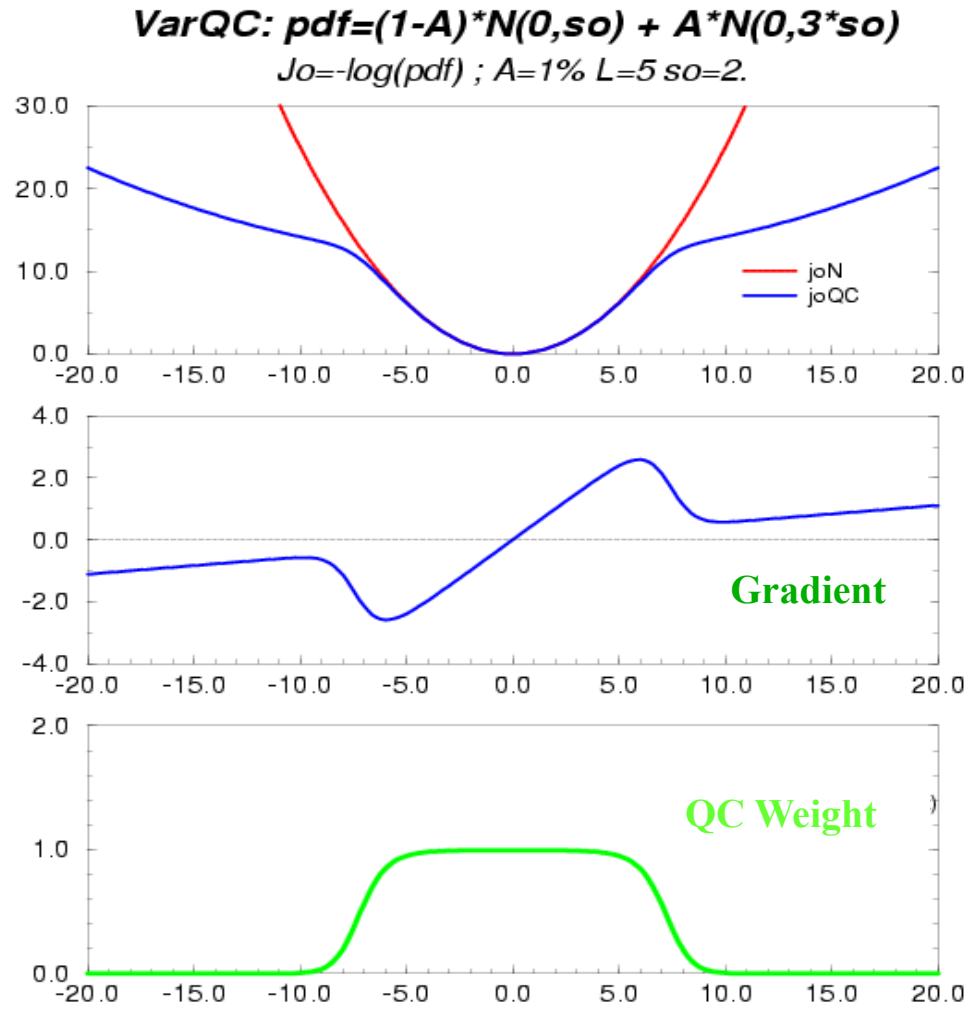
2d is the width of the distribution

The consequence of this choice will become clear in the following

Gaussian + flat PDF



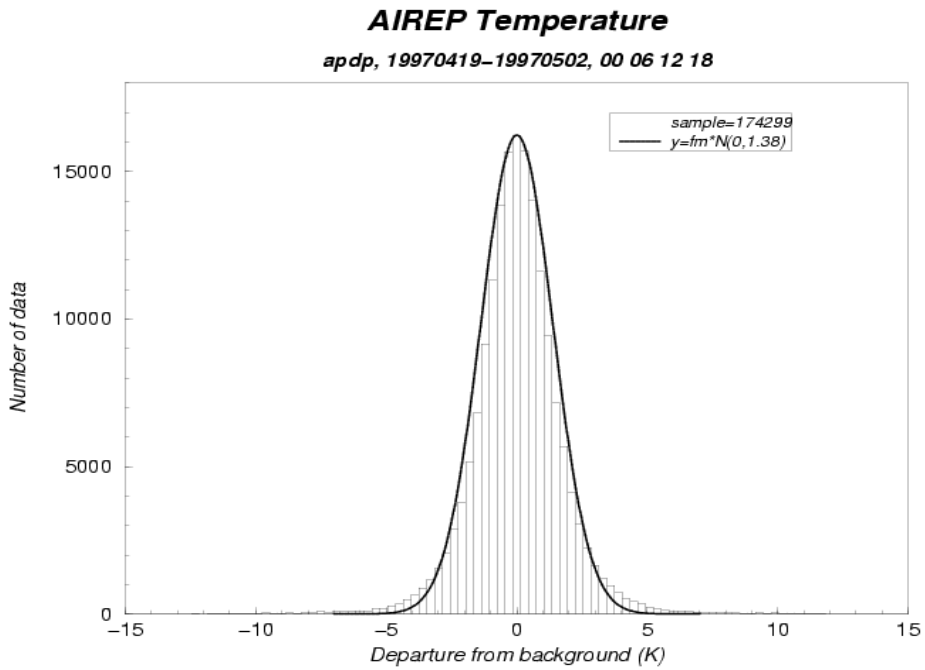
Sum of 2 Gaussians

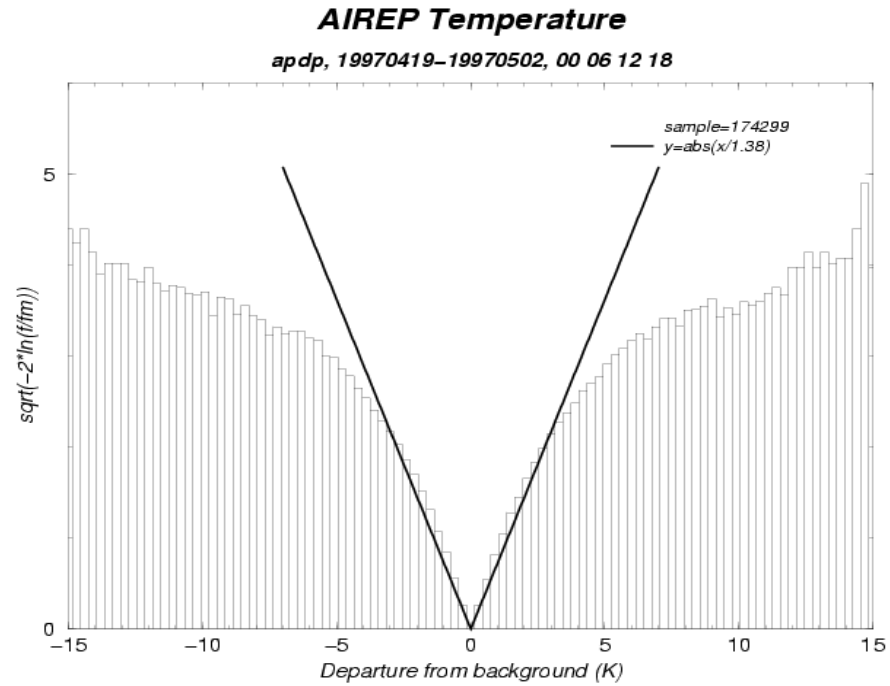


Tuning the rejection limit

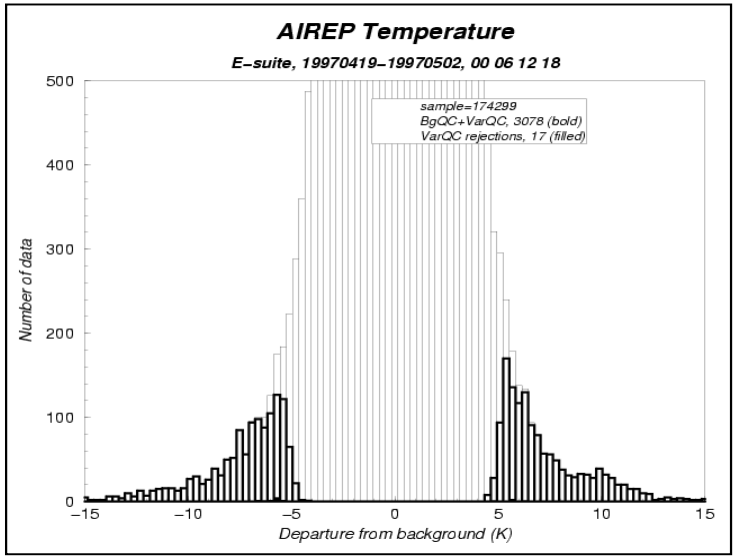
The left histogram on the left has been transformed into the right histogram such that the Gaussian part appears as a pair of straight lines forming a ‘V’ at zero.
The slope of the lines gives the standard deviation of the Gaussian.

The rejection limit can be chosen to be where the actual distribution is some distance away from the ‘V’ - around 6 to 7 K in this case, would be appropriate.





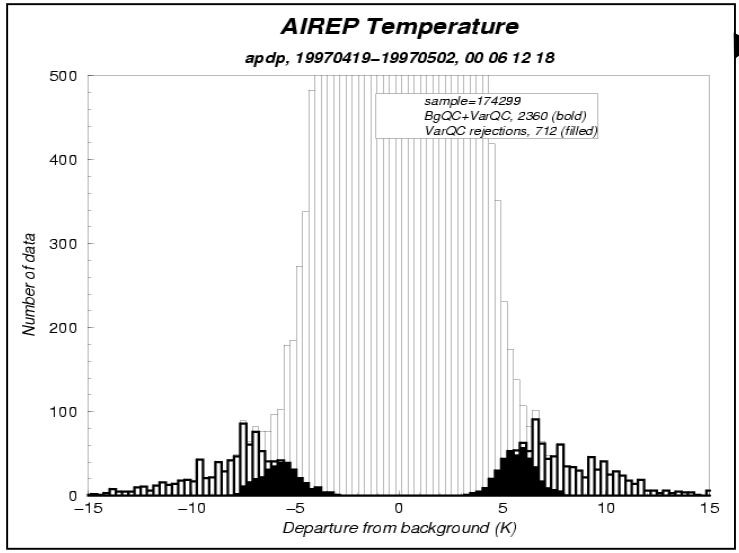
Tuning example

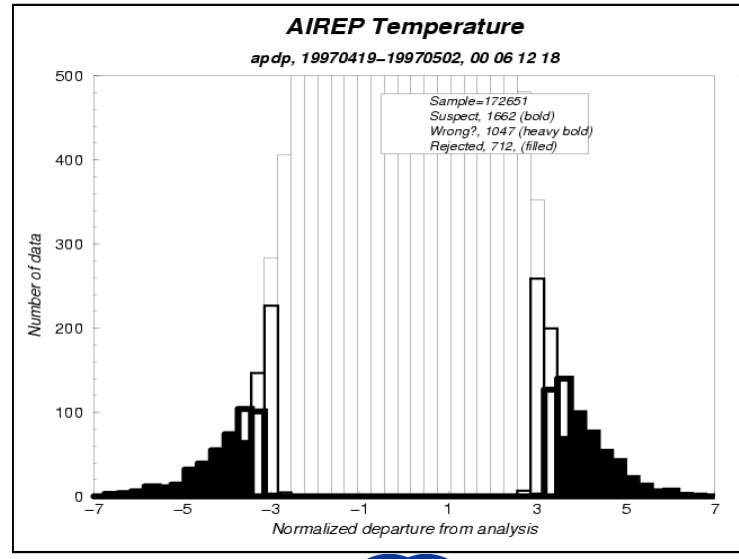


BgQC too tough

BgQC and VarQC correctly tuned

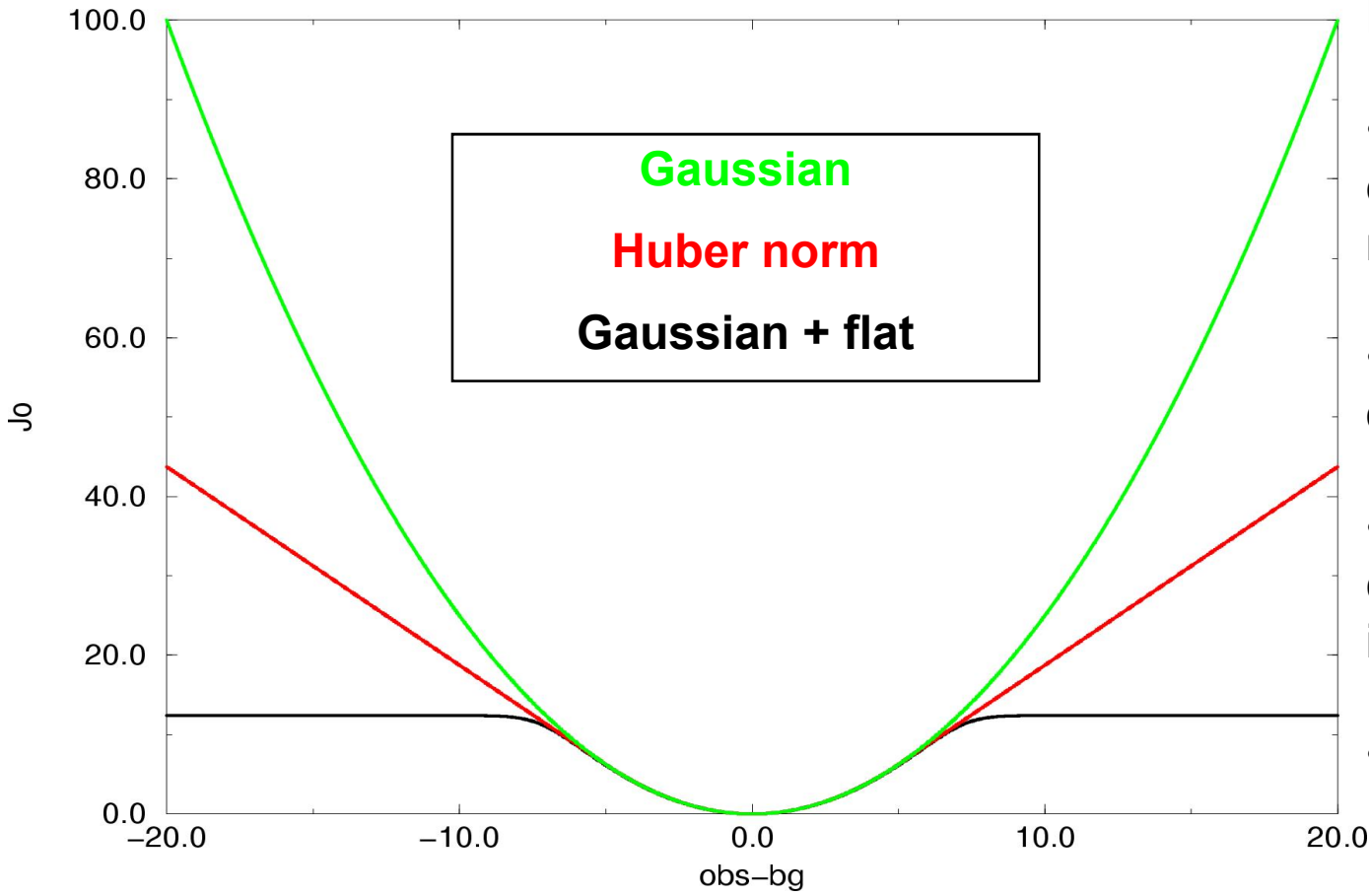
The shading reflects the value of P, the probability of gross error





Huber-norm as alternative for non-Gaussian Jo

A compromise between the I_2 and I_1 norms



Huber norm:

- Robust method: a few erroneous observations does not ruin analysis
- Adds some weight on observations with large departures
- A set of observations with consistent large departures will influence the analysis
- Concave cost function

Huber norm variational quality control

The pdf for the Huber norm is:

$$p(y \mid x) = \begin{cases} \frac{1}{\sigma_o \sqrt{2\pi}} \exp\left(\frac{a^2}{2} - |a\delta| \right) & \text{if } a < \delta \\ \frac{1}{\sigma_o \sqrt{2\pi}} \exp\left[-\frac{1}{2}\delta^2 \right] & a \le \delta \le b \\ \frac{1}{\sigma_o \sqrt{2\pi}} \exp\left(\frac{b^2}{2} - |b\delta| \right) & \text{if } \delta > b \end{cases}$$

$$\text{where } \delta = \frac{y - H(x)}{\sigma_o}$$

Equivalent to L_1 metric far from x, L_2 metric close to x.

With this pdf, observations far from x are given less weight than observations close to x, but can still influence the analysis.

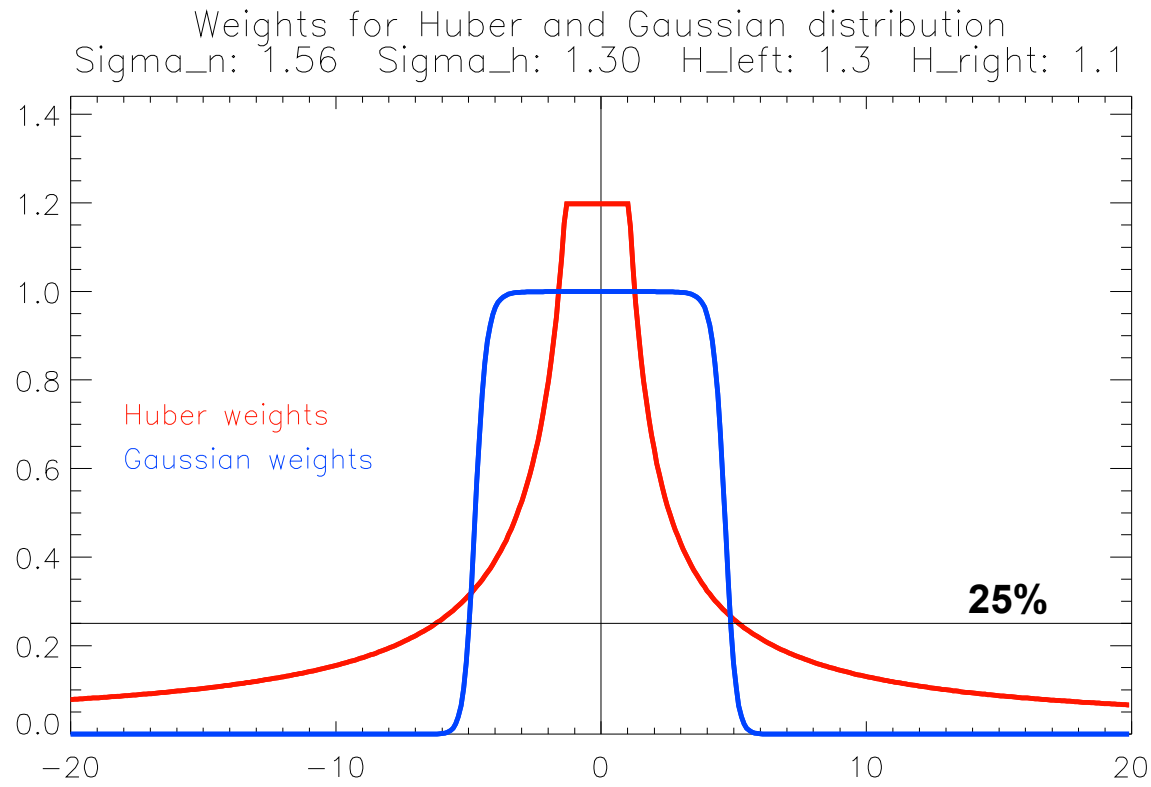
Many observations have errors that are well described by the Huber norm.

Comparing observation weights:
Huber-norm (red) versus Gaussian+flat (blue)

- More weight in the middle of the distribution
- More weight on the edges of the distribution

- More influence of data with large departures

  -Weights: 0 – 25%



Departure statistics for radiosonde temperatures is well described by a Huber-norm distribution

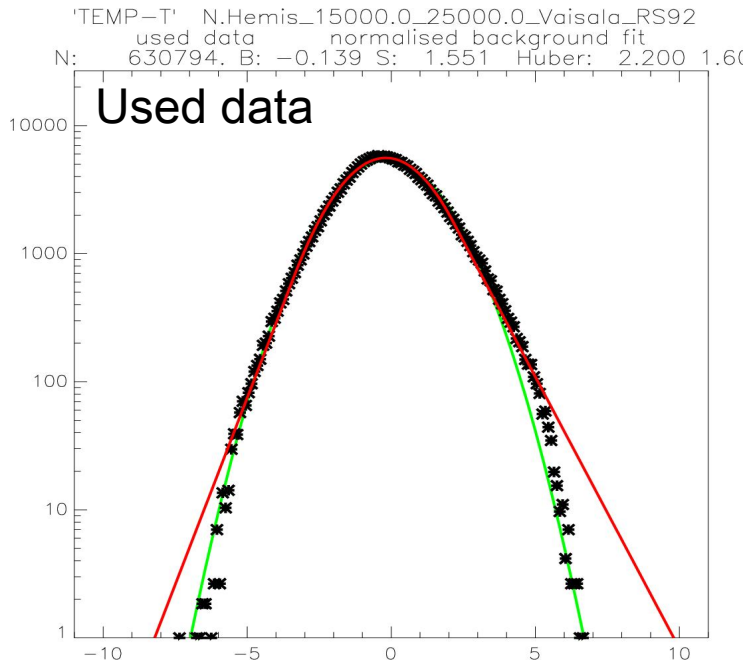
- Based on 18 months of data

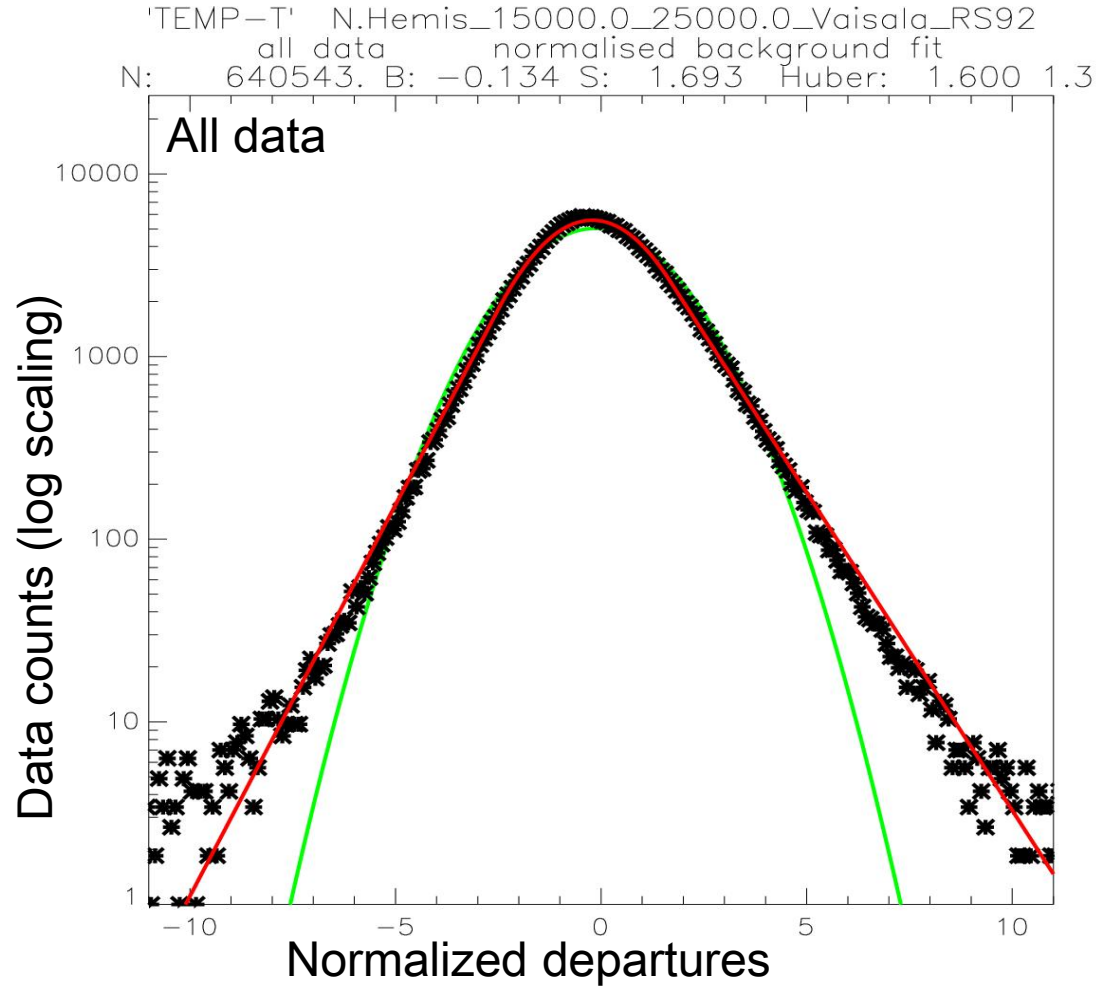
  Feb 2006 – Sep 2007

- Normalised fit of pdf to data

  - Best Gaussian fit

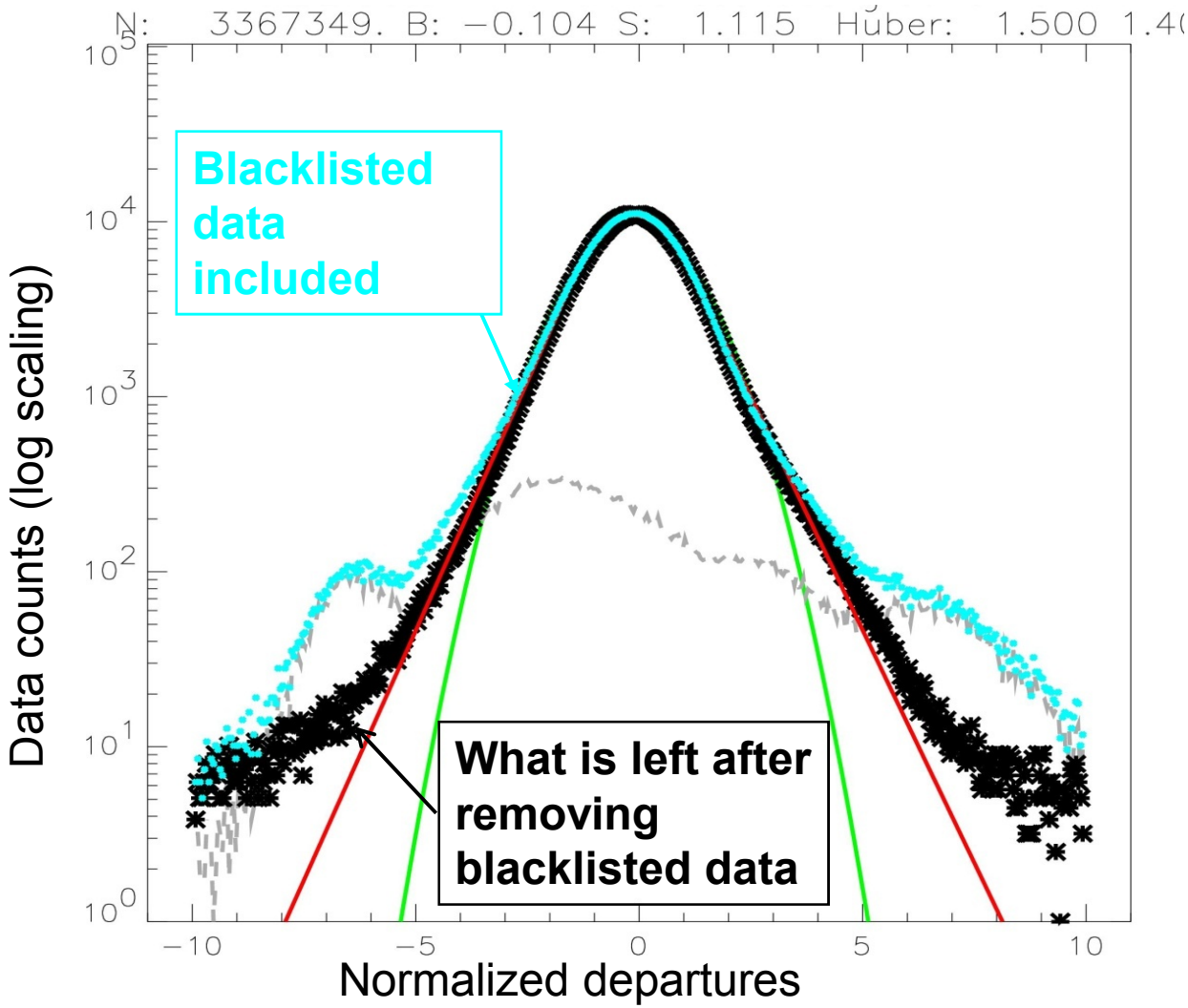
  - Best Huber norm fit





METAR surface pressure data (Tropics)

Blacklisting data is sometimes enough to limit gross errors



After removing the blacklisted data the departures (black crosses) are well described by a Huber norm (red line)

27 Dec 1999 – French storm 18UTC

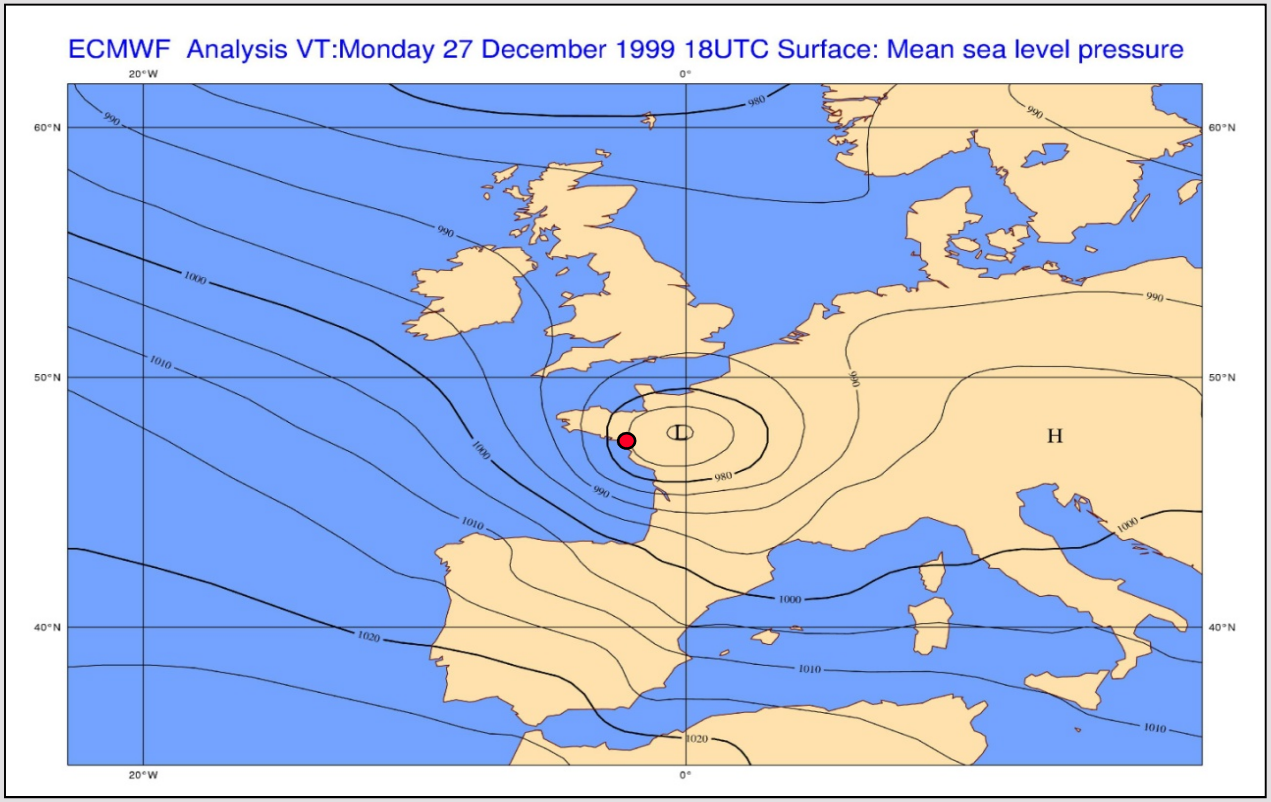
- Era interim analysis produced a low with min 970 hPa
- Lowest pressure observation (SYNOP: red circle)

  963.5 hPa (supported by neighbouring stations)

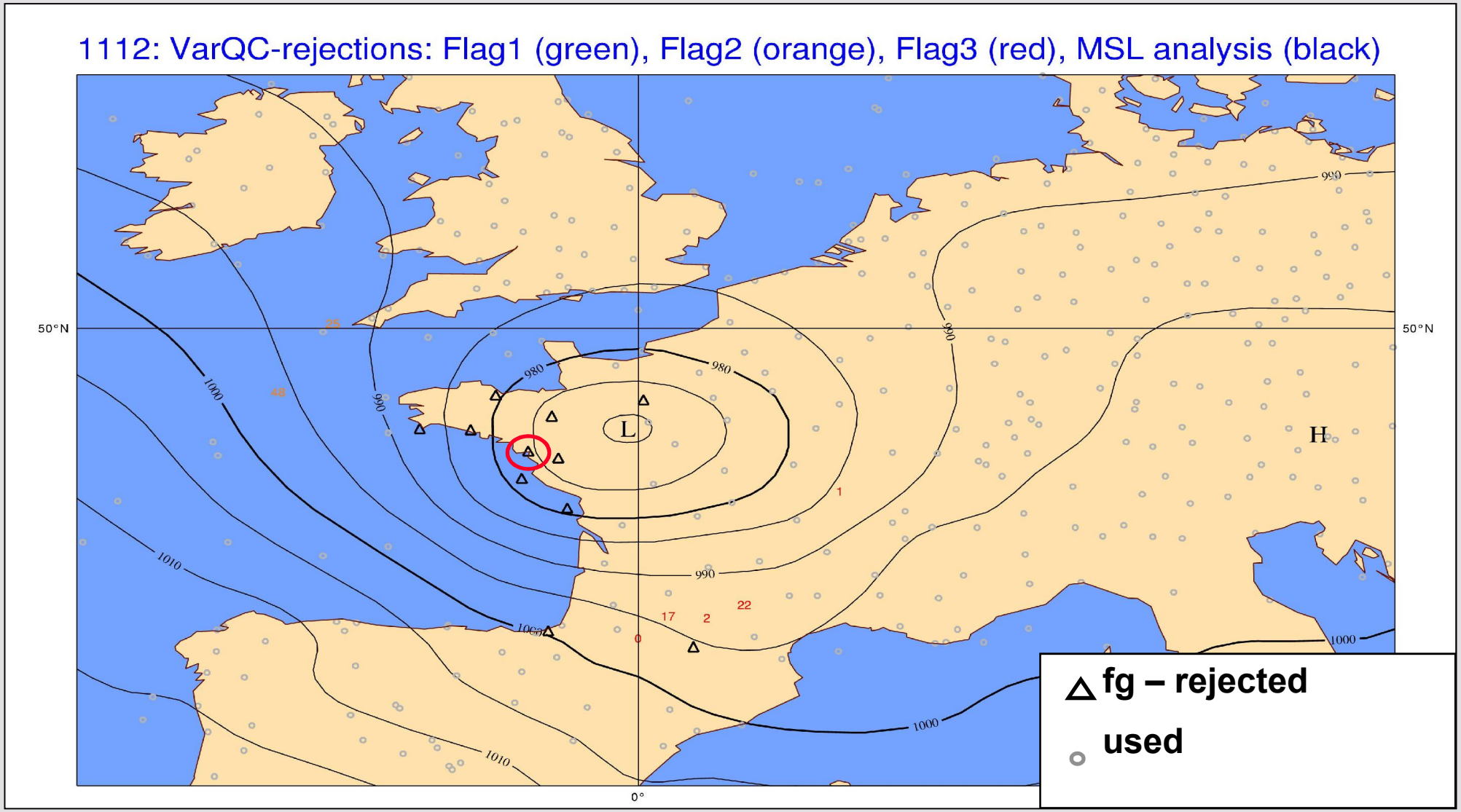
  At this station the analysis shows 977 hPa

  Analysis wrong by 16.5 hPa!

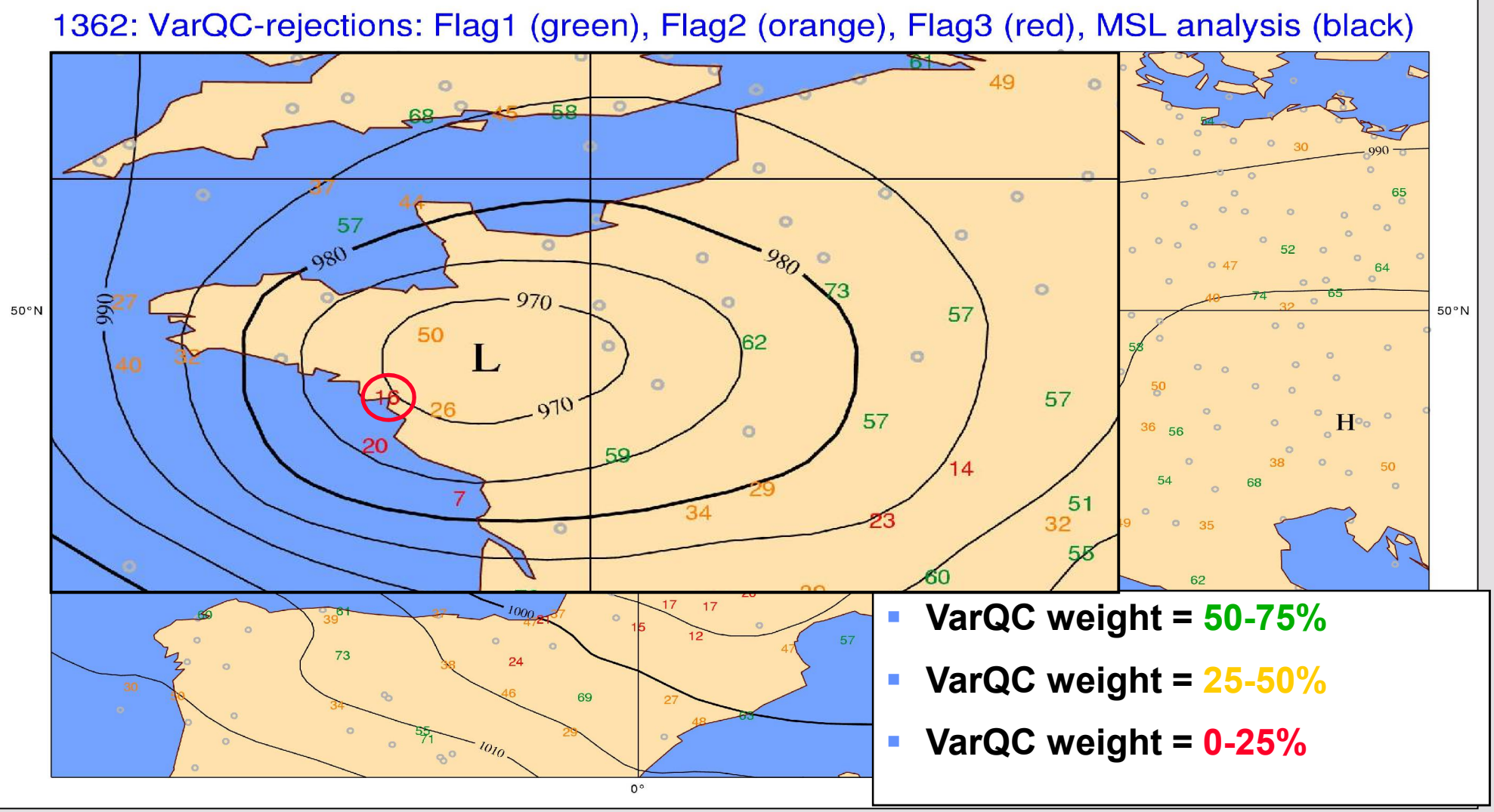
- High density of good quality surface data for this case



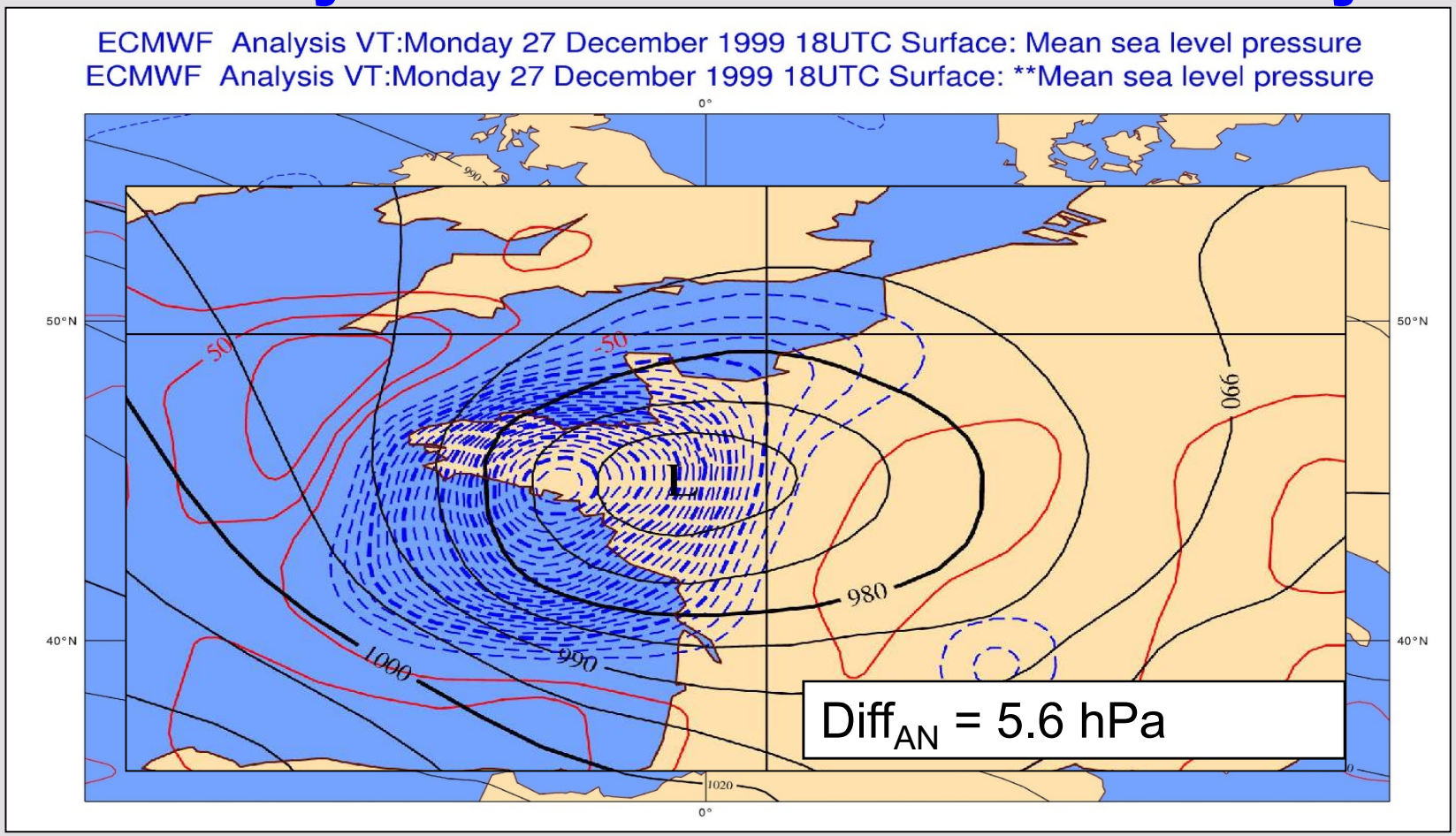
Data rejection and VarQC weights – Era interim reanalysis 27.12.99 18UTC +60min



Data rejection and VarQC weights – Huber exp.



MSL Analysis differences: Huber v. Reanalysis



- New min 968 hPa
- Low correctly shifted towards west and intensified in better agreement with surface pressure observations

VarQC general summary

- VarQC provides a satisfactory and very efficient quality control mechanism - consistent with 3D/4D-Var.
- The implementation can be very straight forward (multiply observation departures by a factor).
- VarQC does not replace the pre-analysis checks - the checks against the background for example. However, with Huber-norm these are relaxed significantly.
- All observational data from all data types are quality controlled simultaneously, as part of the general 3D/4D-Var minimisation.

A good description of background errors is essential for effective, flow-dependent QC: background error lecture.