

Atmospheric Thermodynamics



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Textbooks

The great Belgian tradition:
“Thermodynamique de l’atmosphere”
Dufour and v. Mieghem (1975)

Most recent:
Sam Miller (2015) “Applied thermodynamics for meteorologists”

Maarten Ambaum (2010) “Thermal physics of the atmosphere”

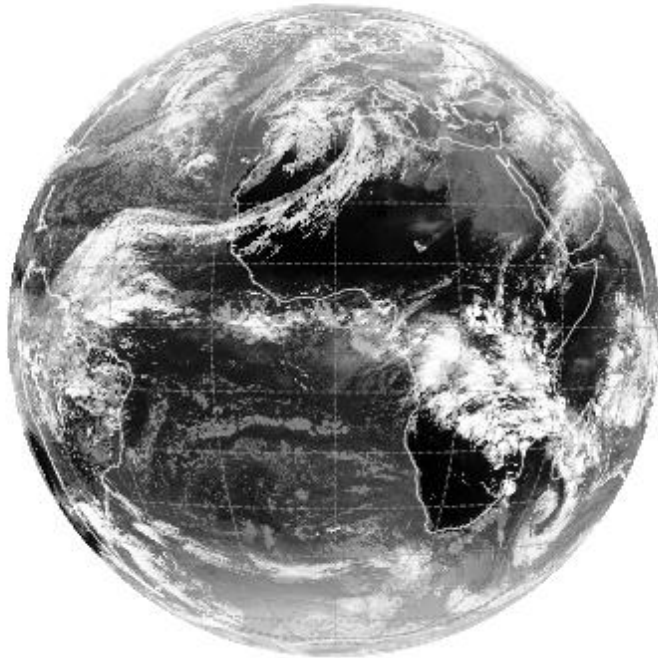
For simplified Overview:
Rogers and Yau (1989) “*A short course in cloud physics*”

Thermodynamics and Kinematics:
K. A. Emanuel (1994) “*Atmospheric Convection*”

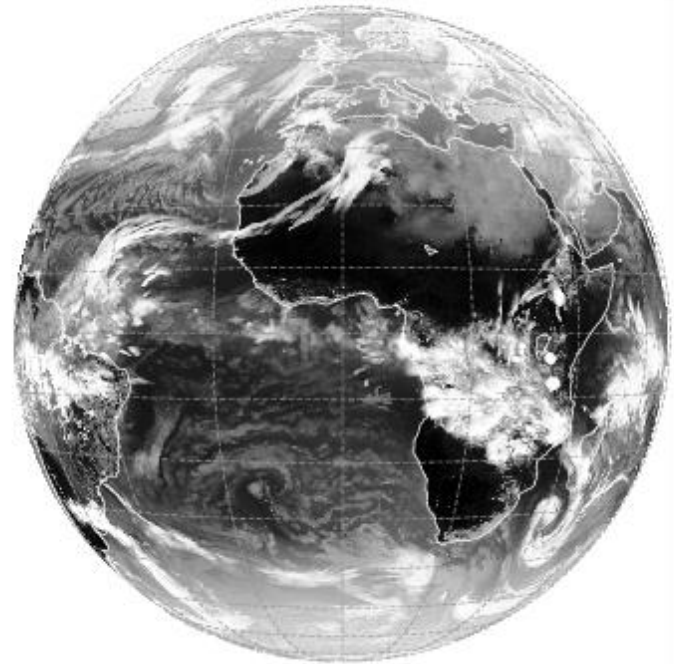
Challenge:

- define prognostic conservation equation for energy (T) & moisture
- compute transport and phase changes

IR10.8 20140110 15 UTC



ECMWF 1 Fc 20140110 00 UTC+15h:



Meteosat 9 and IFS T1279 forecast

The First and Second Law

The First Law of Thermodynamics:

Heat is work and work is heat.

Heat is work and work is heat

Very good! The Second Law of Thermodynamics:

Heat cannot of itself pass from one body to a hotter body,

Heat cannot of itself pass from one body to a hotter body

Heat won't pass from a cooler to a hotter,

Heat won't pass from a cooler to a hotter

You can try it if you like but you'd far better notter,

You can try it if you like but you'd far better notter

'Cos the cold in the cooler will get hotter as a ruler,

'Cos the cold in the cooler will get hotter as a ruler

'Cos the hotter body's heat will pass to the cooler,

'Cos the hotter body's heat will pass to the cooler

Good, First Law:

Heat is work and work is heat and work is heat and heat is work

Heat will pass by conduction,

Heat will pass by conduction

And heat will pass by convection,

Heat will pass by convection

And heat will pass by radiation,

Heat will pass by radiation

And that's a physical law.

Heat is work and work's a curse,

And all the heat in the universe,

Is gonna cooooool down

'Cos it can't increase,

Then there'll be no more work

And there'll be perfect peace

Really?

Yeah, that's entropy, maan!

The First and Second Law

And its all because of the Second Law of Thermodynamics, which lays down:

That you can't pass heat from a cooler to a hotter,
Try it if you like but you far better notter,
'Cos the cold in the cooler will get hotter as a ruler,
'Cos the hotter body's heat will pass to the cooler.
Oh you can't pass heat, cooler to a hotter,
Try it if you like but you'll only look a fooler
'Cos the cold in the cooler will get hotter as a ruler
And that's a physical Law!

Oh, I'm hot!
Hot? That's because you've been working!
Oh, Beatles - nothing!
That's the First and Second Law of Thermodynamics!

Authors: M. Flanders (1922-1975) & D. Swann (1923-1994)
From "At the Drop of Another Hat"

Moist Thermodynamics

Ideal Gas Law



What is definition of ideal gas?

- Assume that “moist air” can be treated as mixture of two ideal gases: “dry air” + vapour

Dry air equation of state:

$$p_d = \rho_d R_d T$$

Density

Temperature

Pressure

Gas Constant for dry air = $287 \text{ J Kg}^{-1} \text{ K}^{-1}$

Water Vapour equation of state:

$$e = \rho_v R_v T = \rho_v \frac{R_d}{\varepsilon} T$$

Vapour pressure

vapour density

Gas constant for Vapour = $461 \text{ J Kg}^{-1} \text{ K}^{-1}$

0.622

First law of thermodynamics

Energy conservation and Heat

$$de = dQ - dw = dQ - pd\alpha; \quad dQ = Tds; \quad \alpha = \frac{V}{m} = \frac{1}{\rho}$$

All quantities are per unit mass (specific)
 dQ is not a perfect differential, but ds
(change in entropie is !)

Change in internal
Energy

Heat supplied by
diabatic process

Work done
by Gas

Can write as

$$de = c_v dT = dQ - pd\alpha$$

Specific Heat at
constant volume

$$c_v = \frac{\partial e}{\partial T} = \left. \frac{\delta Q}{\delta T} \right|_{\alpha} = \frac{5}{2} R_d$$

First law of thermodynamics

Enthalpy and Legendre transformation

Changing variables

$$pd\alpha = d(\alpha p) - \alpha dp$$

$$dh = d(e + \alpha p) = c_p dT = dQ + \alpha dp$$

$$c_p = \frac{\partial h}{\partial T} = \left. \frac{\delta Q}{\delta T} \right|_p = c_v + R_d$$

Special processes: “Adiabatic Process” $dQ=0$ or better $ds=0$

$$c_p dT = \alpha dp$$

$$c_p d \ln T = R_d d \ln P$$

Special significance since many atmospheric **motions** can be approximated as adiabatic

Pressure, partial pressures and gas law for moist air

$$p = p_d + e; \quad p_d = p N_d; \quad e = p N_v$$

$$\Rightarrow \frac{dp}{p} = \frac{dp_d}{p_d} = \frac{de}{e}$$

Partial pressures add if both gases occupy same volume V . N_x are the mol masses

$$\left. \begin{array}{l} p_d V = m_d R_d T \\ e V = m_v R_v T \end{array} \right\} \Rightarrow p V = T (m_d R_d + m_v R_v)$$

Enthalpy and flow process

velocity

$$\frac{dU}{dt} = -\frac{1}{\rho} \nabla p = -\alpha \nabla p \quad \Rightarrow$$

$$\frac{dU}{dt} = -\nabla h + T \nabla s = -\nabla h$$

In isentropic (adiabatic) flow

Summary : Potentials and Maxwell relations

Internal Energy

$$de = Tds - pd\alpha$$

$$\left(\frac{\partial T}{\partial \alpha}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_\alpha$$

Enthalpy

$$dh = Tds + \alpha dp$$

$$\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial \alpha}{\partial s}\right)_p$$

Helmholtz free Energy

$$df = -sdT - pd\alpha$$

$$\left(\frac{\partial s}{\partial \alpha}\right)_T = \left(\frac{\partial p}{\partial T}\right)_\alpha$$

Gibbs free Energy

$$dg = -sdT + \alpha dp$$

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial \alpha}{\partial T}\right)_p$$

Conserved Variables

Using enthalpy equation and integrating, obtain Poisson's equation

$$\frac{T}{T_0} = \left(\frac{p}{p_0} \right)^{R_d/C_p} \quad \text{or}$$
$$\left(\frac{p}{p_0} \right) = \left(\frac{\rho}{\rho_0} \right)^{C_p/C_v}$$



What is the speed of sound?

Setting reference pressure to 1000hPa gives the definition of potential temperature for dry air

$$\theta = T \left(\frac{p_0}{p} \right)^{R_d/C_p}$$

Conserved in dry adiabatic motions, e.g. boundary layer turbulence

Humidity variables

1. Vapour Pressure Pa

e

$$\varepsilon = R_d / R_v = 0.622$$

2. Absolute humidity $kg\ m^{-3}$

$$\rho_v = \frac{m_v}{V}$$

3. Specific humidity $kg\ kg^{-1}$

$$q = \frac{m_v}{m_d + m_v} = \frac{\rho_v}{\rho} = \varepsilon \frac{e}{p - (1 - \varepsilon)e} \approx \varepsilon \frac{e}{p}$$

Mass of water vapour per unit moist air

4. Mixing ratio $kg\ kg^{-1}$

$$r = \frac{m_v}{m_d} = \frac{\rho_v}{\rho_d} = \varepsilon \frac{e}{p - e} \approx \varepsilon \frac{e}{p}$$

Mass of water vapour per unit dry air

5. Relative humidity

$$RH = \frac{e}{e_s} \quad (\text{or } \approx \frac{q}{q_s})$$

6. Specific liquid water content $kg\ kg^{-1}$

$$q_l = \frac{\rho_l}{\rho}$$

7. Total water content

$$q_t = q + q_l$$

Humidity variables

How to define “moist quantities” and how to switch from mixing ratio to specific humidity.

For any intensive quantity we have

$$(m_d + m_v) \chi = m_v \chi_v + m_d \chi_d \Rightarrow \chi = \frac{m_v}{m_d + m_v} \chi_v + \frac{m_d}{m_d + m_v} \chi_d$$

or dividing by m_d

$$\chi = q \chi_v + (1 - q) \chi_d$$

$$\chi = \frac{1}{1 + r} (r \chi_v + \chi_d)$$

$$q = \frac{r}{1 + r} \quad r = \frac{q}{1 - q}$$

Virtual temperature T_v

Another way to describe the vapour content is the *virtual temperature*, an artificial temperature.

It describes the temperature required for dry air, in order to have at the same pressure the same density as a sample of moist air

Definition:

$$\rho = \frac{P}{R_d T_v} = \frac{P}{RT}$$

$$R = q R_v + (1 - q) R_d = R_d (1 + q^{1 - \varepsilon / \varepsilon})$$

$$T_v \equiv T \left[1 + \frac{(1 - \varepsilon)}{\varepsilon} q \right] = T \left[1 + \frac{(1 - \varepsilon)r}{\varepsilon(1 + r)} \right] \approx T(1 + 0.608q)$$

By extension, we define the *virtual potential temperature*, which is a conserved variable in unsaturated ascent, and related to density

$$\theta_v \equiv T_v \left(\frac{p_0}{p} \right)^{R_d / c_p}$$

Enthalpy and phase change

$$dh = c_p dT = Tds + \alpha dp - Ldq_s$$

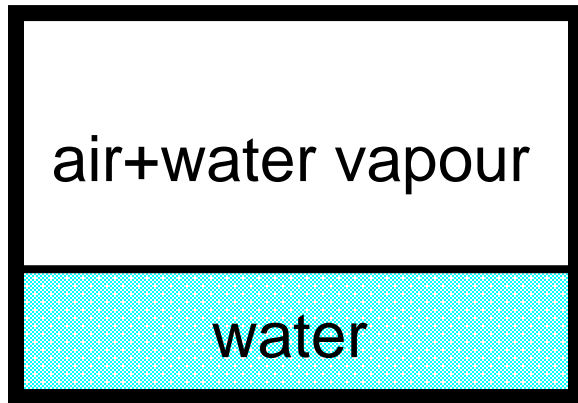
Have we been a bit negligiant ? Yes, more precisely

$$(m_d c_{pd} + m_v c_{pv}) dT = T(dS_d + dS_v) + (m_d R_d + m_v R_v) T \frac{dp}{p} + L dm_v$$

Divide by m_d (or $m_d + m_v$) and assume adiabatic process

$$(c_{pd} + rc_{pv}) dT = (R_d + rR_v) T \frac{dp}{p} - L dr_s$$

The Clausius-Clapeyron equation



Consider this closed system in equilibrium:
T equal for water & air, no net evaporation
or condensation
Air is said to be saturated

- For the phase change between water and water vapour the equilibrium pressure (often called saturation water vapour pressure) is a function of temperature only

$$\frac{de_s}{dT} = \frac{1}{T} \frac{L_v}{(\alpha_v - \alpha_w)}$$

- with $\alpha_v \gg \alpha_w$, and the ideal gas law $\alpha_v = R_v T / e_s$

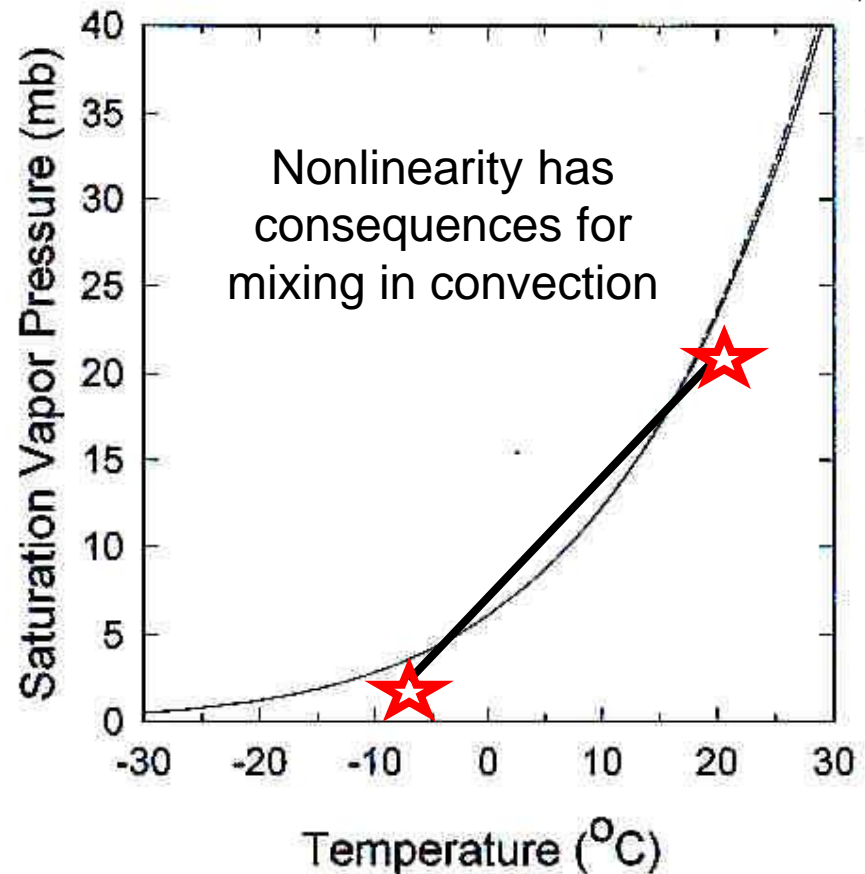
$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2}$$

Clausius-Clapeyron equation integrated

- The problem of integrating the Clausius-Clapeyron equation lies in the **temperature dependence of L_v** .
- Fortunately this dependence is only weak, so that approximate formulae can be derived.

$$\ln\left(\frac{e_s}{e_{s0}}\right) = \frac{L_v}{R_v} \left(\frac{1}{T_0} - \frac{1}{T} \right)$$

$$e_{s0} = 6.11 \text{ hPa at } T_0 = 273 \text{ K}$$



Ways of reaching saturation

Several ways to reach saturation:

- Diabatic Cooling (e.g. Radiation)
- Evaporation (e.g. of precipitation)
- Expansion (e.g. ascent/descent)

Dew point temperature T_d

Wet-bulb temperature T_w

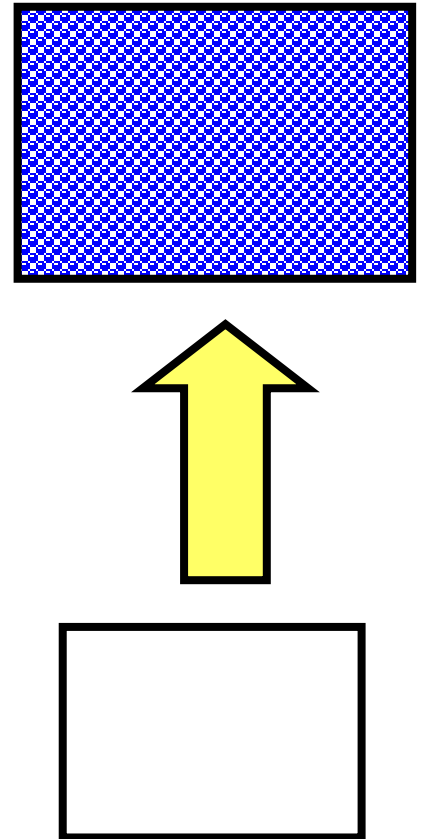
Isentropic condensation temperature

Ways of reaching saturation: Expansion: (Pseudo) Adiabatic Processes

As (unsaturated) moist air expands (e.g. through vertical motion), cools adiabatically conserving θ .

Eventually saturation pressure is reached at a level called “lifting condensation level”,
T,p are known as the “*isentropic condensation temperature and pressure*”,.

If expansion continues, condensation will occur thus the temperature will decrease at a slower rate.

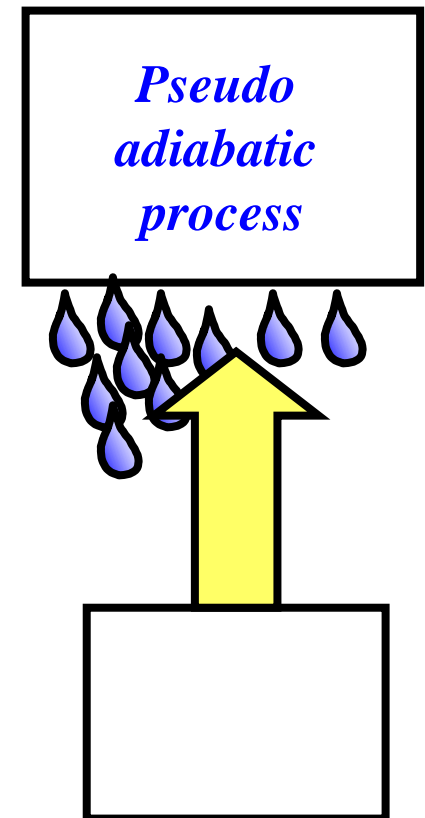


Ways of reaching saturation: Expansion: (Pseudo) Adiabatic Processes

Have to make a decision concerning the condensed water.

- Does it fall out instantly or does it remain in the parcel?
- Once the freezing point is reached, are ice processes taken into account? (complex)

We take the simplest case: all condensate instantly lost as precipitation, known as “*Pseudo adiabatic process*”



How to compute numerically saturation adjustment

given T, q

check if $q > q_s(T)$ then

solve for adjusted T^*, q^* so that

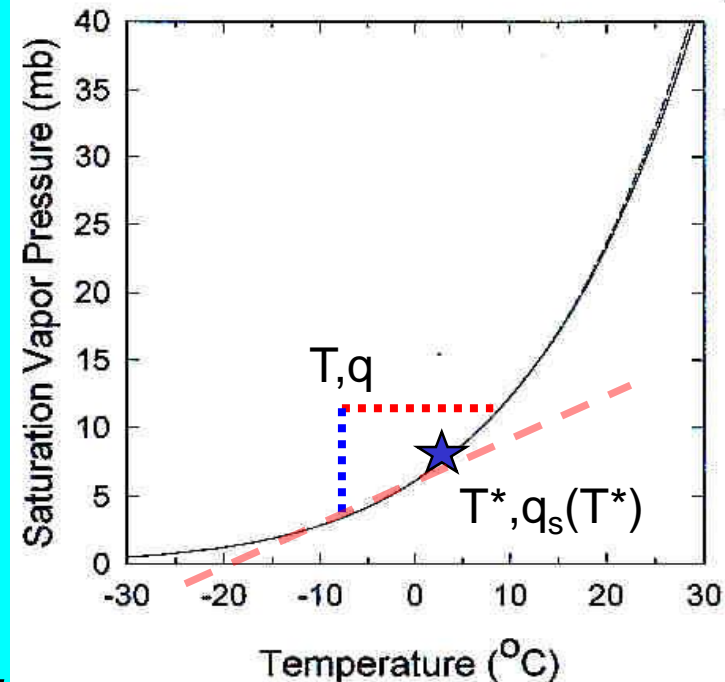
$$q^* = q_s(T^*)$$

$$q_l = q - q_s(T^*)$$

$$c_p dT = -L_v dq_s \Rightarrow$$

$$c_p (T^* - T) = L_v (q - q_s(T^*))$$

either numerically through iteration or with the aid of a linearisation of $q_s(T^*)$ (see Exercises !!)



$$c_p (T^* - T) = L_v \left(q - \left\{ q_s(T) + \left. \frac{dq_s}{dT} \right|_T (T^* - T) + O(2) \right\} \right)$$

Meteorological energy diagrams

Total heat added in cyclic process:

$$\oint dQ = \oint c_p T \left(\frac{dT}{T} - \frac{R_d dp}{C_p p} \right) = c_p \oint T d(\ln \theta)$$

Thus diagram with ordinates T versus $\ln \theta$ will have the properties of “equal areas”=“equal energy”
 Called a **TEPHIGRAM**

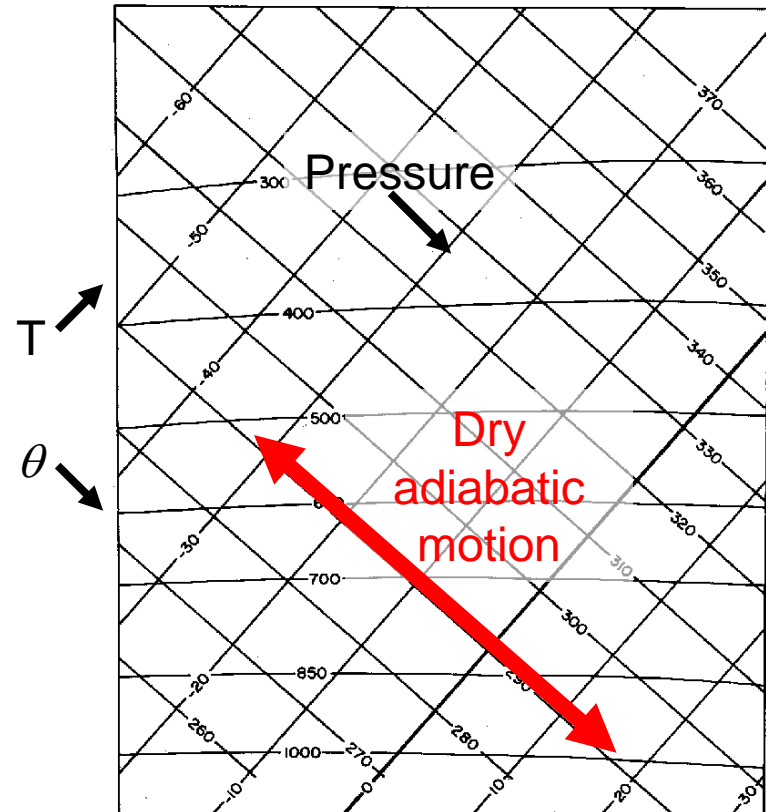
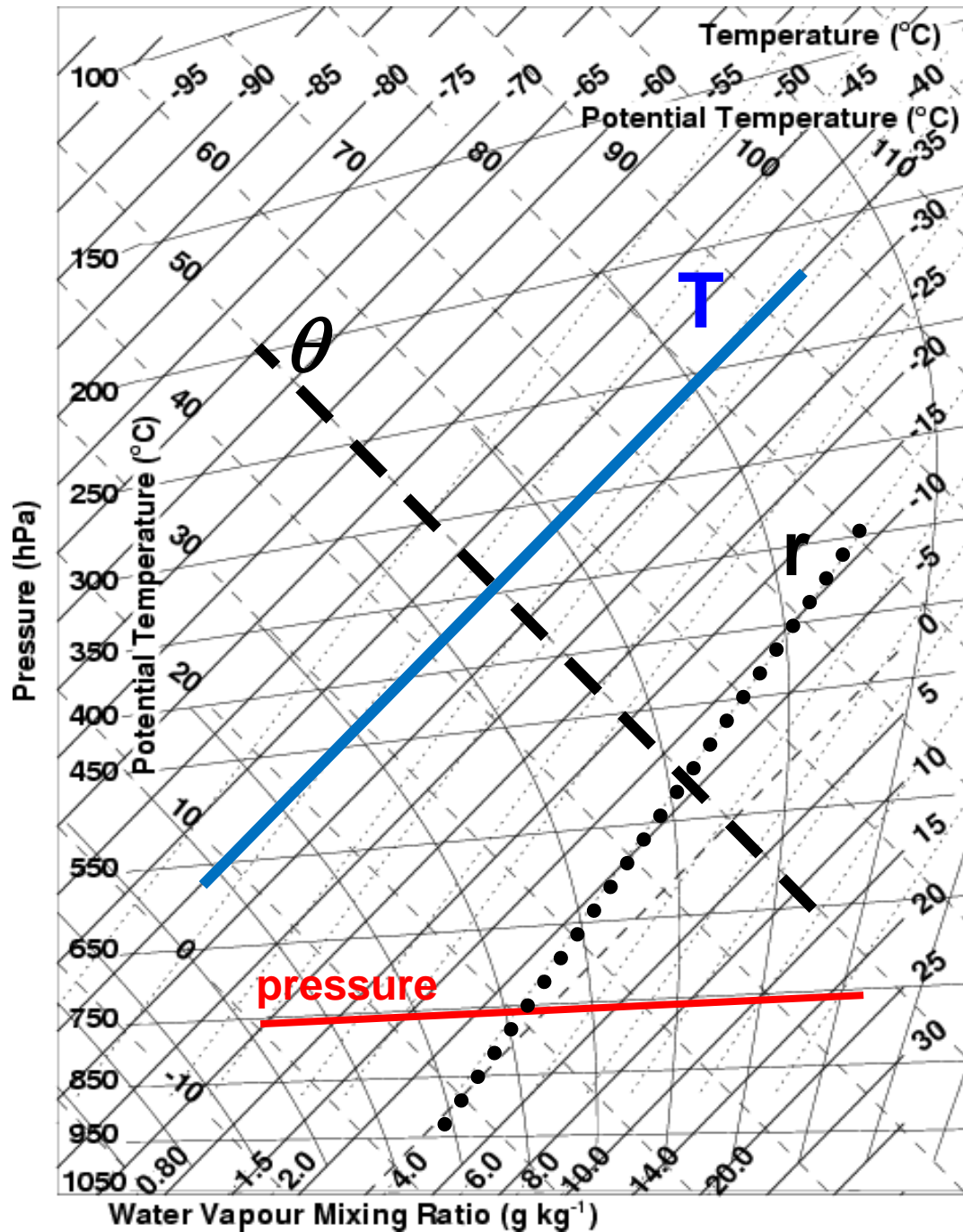


FIG. 1.7. Skeleton of a tephigram. Isobars are approximately horizontal, labeled in mb. Isotherms in deg C go upwards and to the right. Dry adiabats are normal to the isotherms and are labeled according to potential temperature (deg K).

Tephigram (II)



Saturation
specific humidity

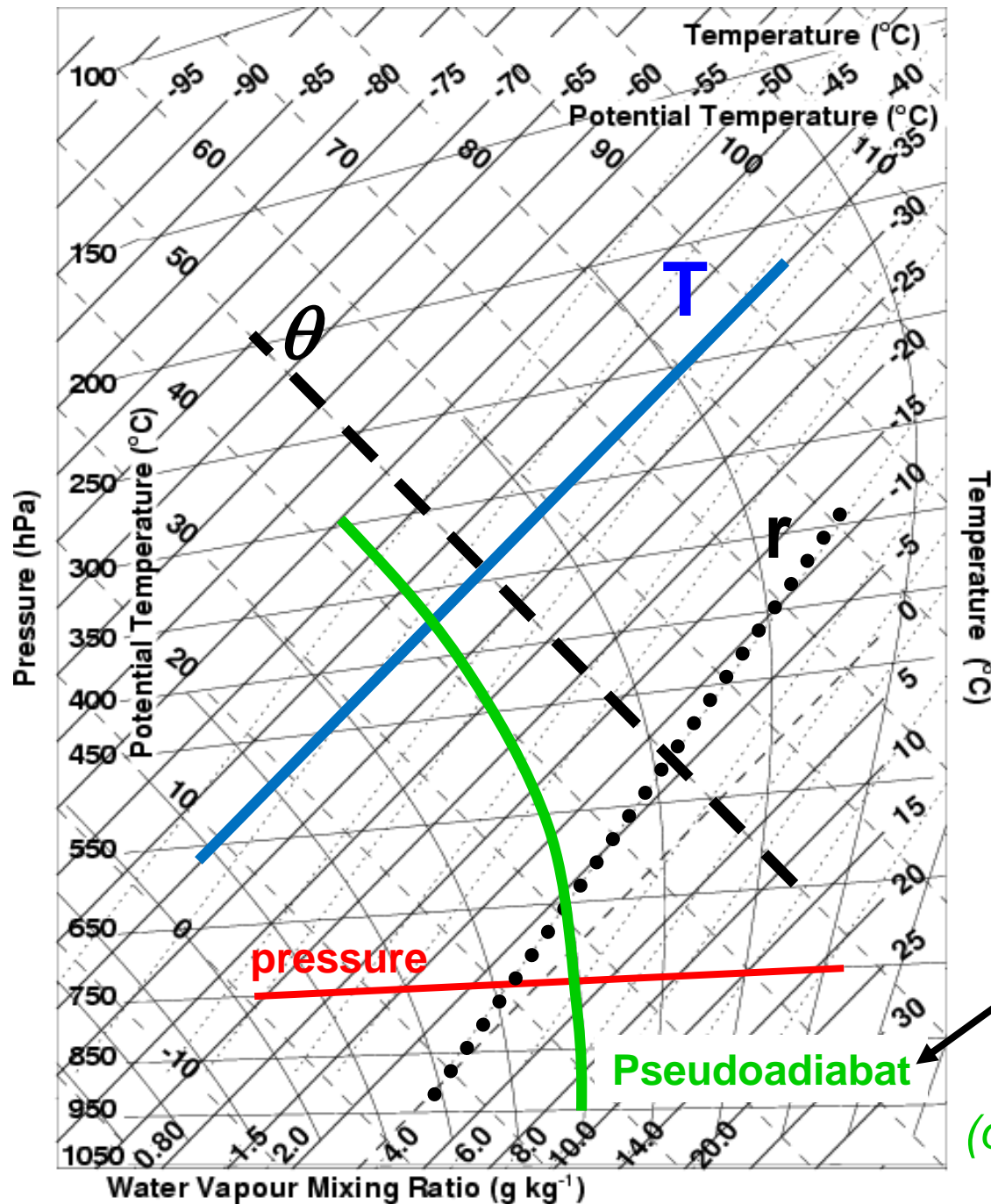
$$q_s = \varepsilon \frac{e_s}{p - (1 - \varepsilon)e_s}$$

Saturation
mixing ratio

$$r_s = \varepsilon \frac{e_s}{p - e_s}$$

Function of temperature and pressure only – tephigrams have isopleths of r_s

Tephigram (III)




Remember: Involves an arbitrary “cloud model”

(or moist adiabat)

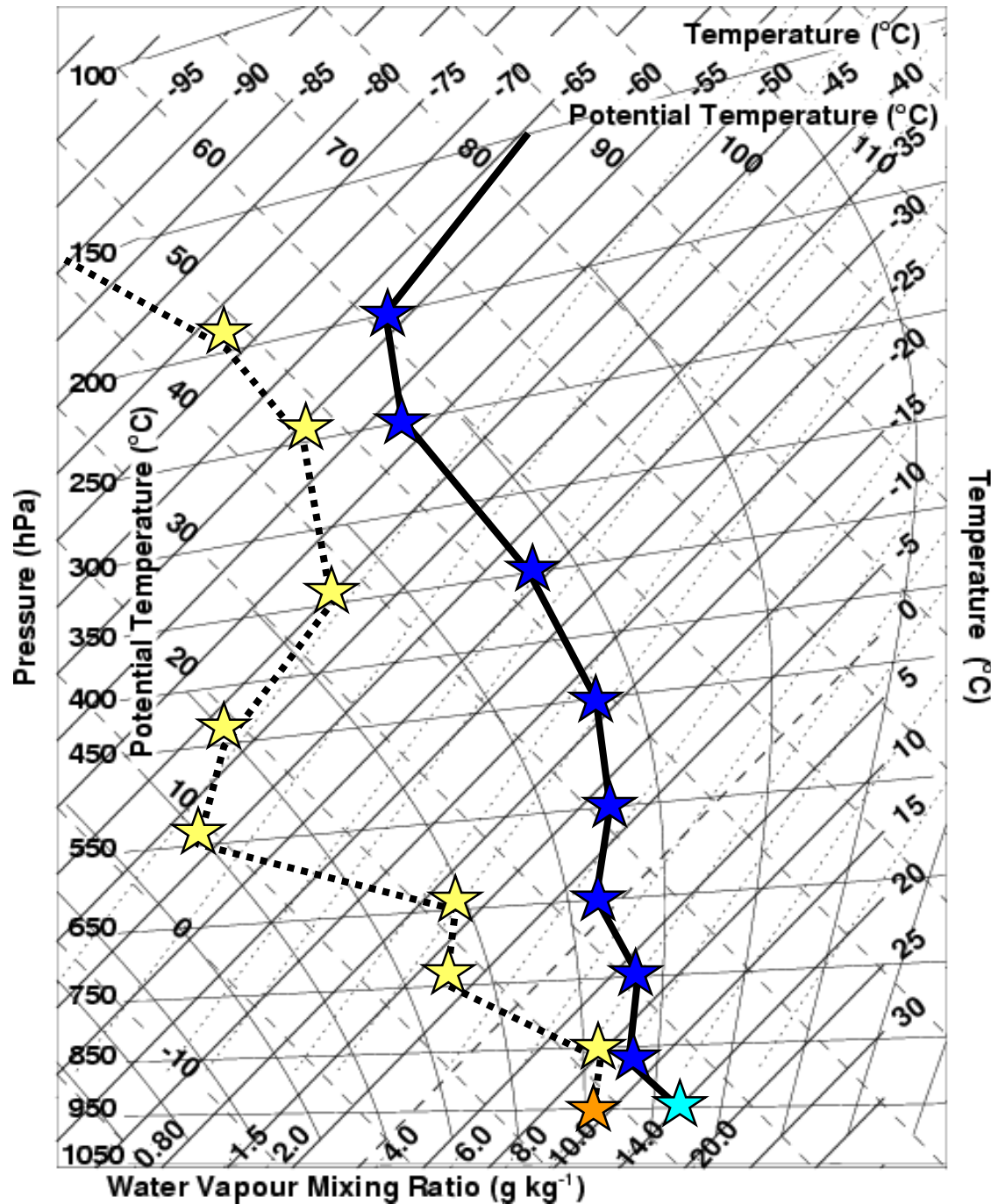
Using a Tephigram

At a pressure of
950 hPa

Measure

$T=20\text{ }^{\circ}\text{C}$ 

$r=10\text{ gkg}^{-1}$ 



plot a atmospheric
sounding

parcel mixing ratio=5g/kg

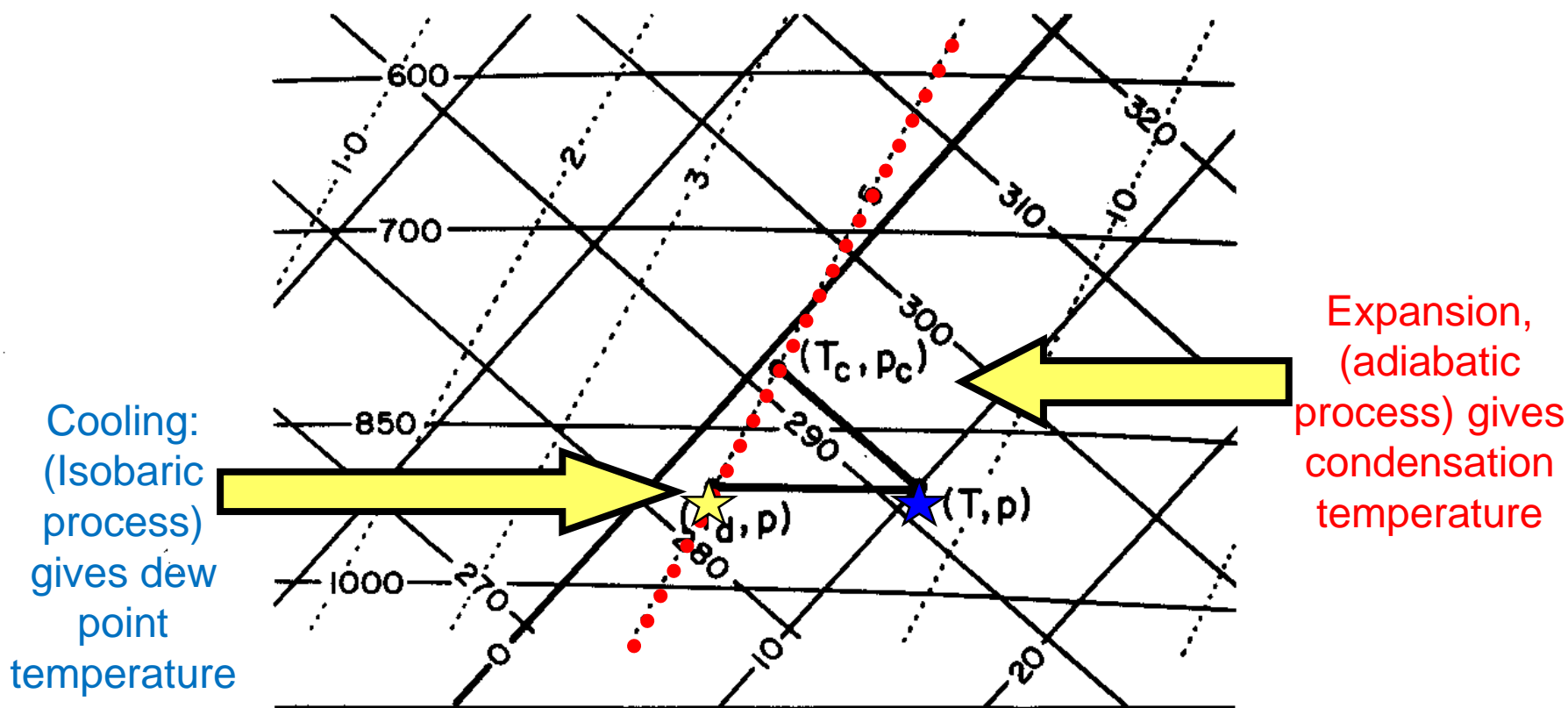


FIG. 2.2. Temperature, dew point, and isentropic condensation temperature, indicated on a tephigram. In the example shown the sample of air at 10°C, 900 mb, is assumed to have a mixing ratio of 5 g/kg. Its dew point, found from the intersection of the 900 mb isobar and the 5 g/kg vapor line, is 2.2°C. Its isentropic condensation point, found from the intersection of the adiabat through (T, p) with the 5 g/kg vapor line, is at 0.7°C and approximately 800 mb.

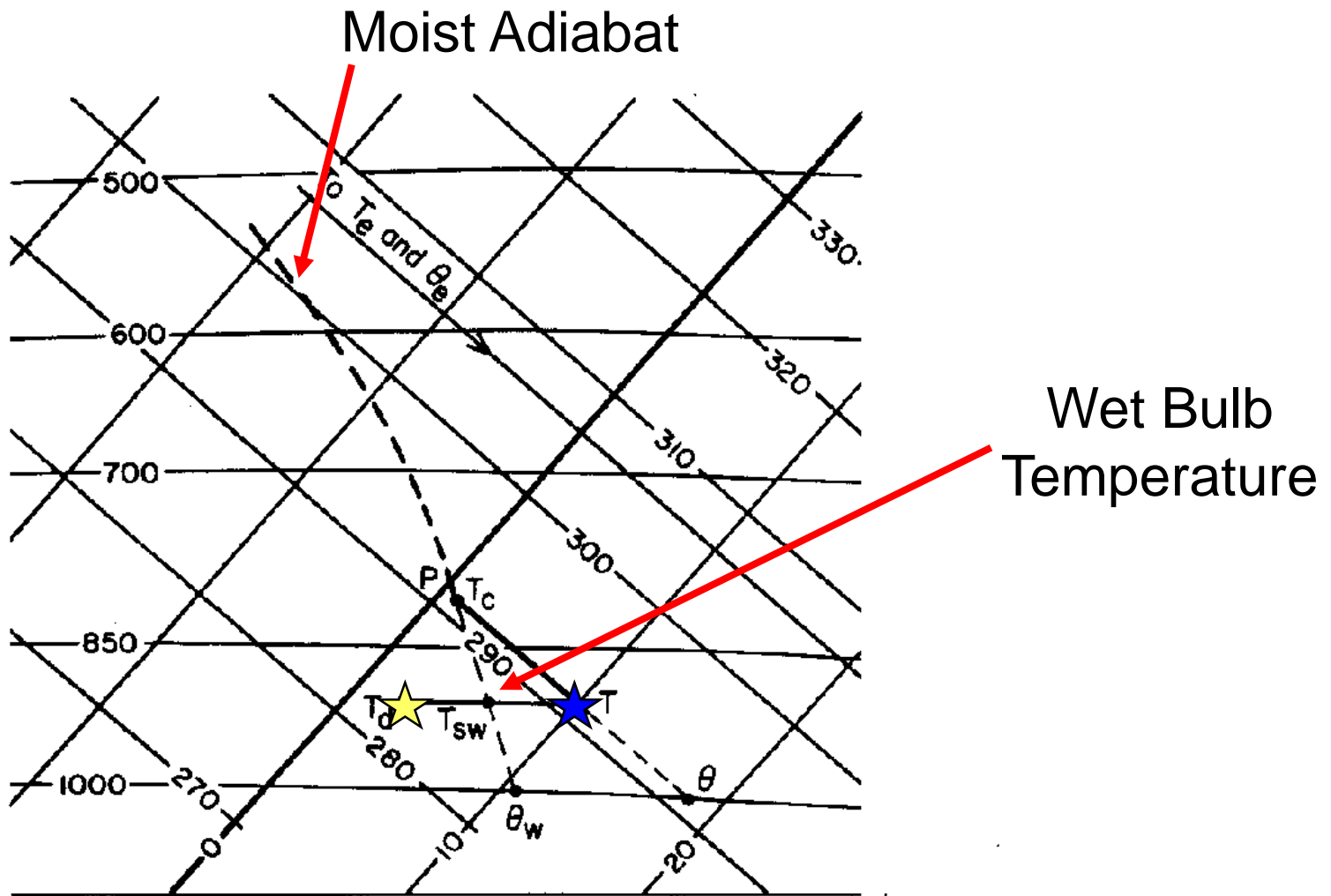
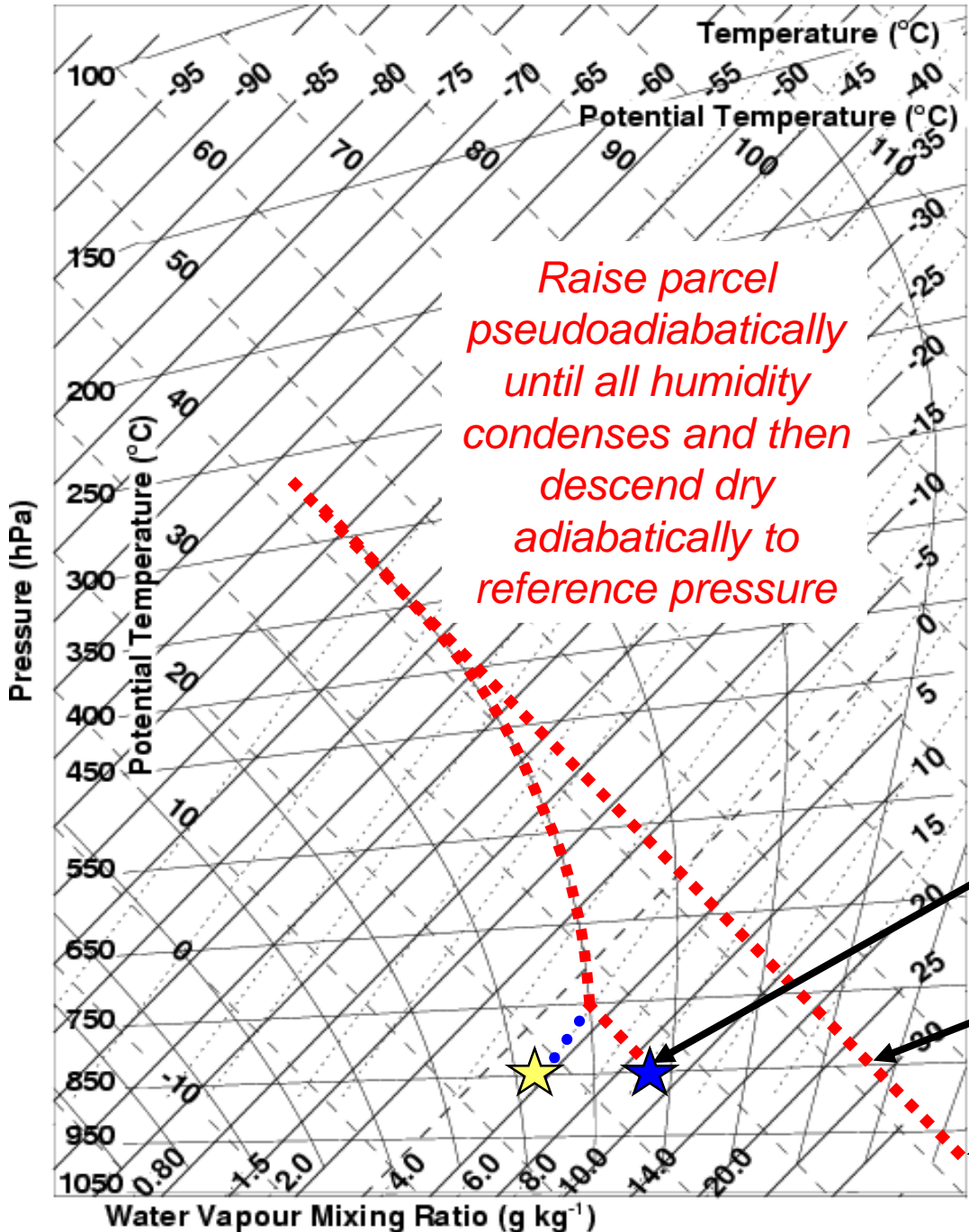


FIG. 2.3. Continued expansion of the air sample of Fig. 2.2 beyond point P , the isentropic condensation point. The dashed line is the pseudoadiabat through P . This diagram indicates the graphical determination of some of the important theoretical temperatures that characterize an air sample.



Raise parcel pseudoadiabatically until all humidity condenses and then descend dry adiabatically to reference pressure

Equivalent Potential temperature

$$\theta_e \approx \theta e^{\left(\frac{L_v r_v}{c_p T}\right)}$$

conserved in adiabatic motions

Parcel at 850 hPa,
T=12.5°C
r=6 g/kg

T_e

θ_e (=315K)

Summary: Conserved Variables

Dry adiabatic processes

$$\theta = T \left(\frac{p_0}{p} \right)^{\frac{R_d}{c_p}} ; \quad s = c_p T - \alpha p = c_p T + gz; \quad q$$

Potential temperature

dry static energy
(hydrostatic atmosphere)

water vapor

Moist adiabatic processes

$$\theta_e \approx \theta e^{\left(\frac{L_v r_v}{c_p T} \right)} \quad h = c_p T + gz + L q \quad q_t = q + q_l$$

Equivalent potential temperature

moist static energy

total water

