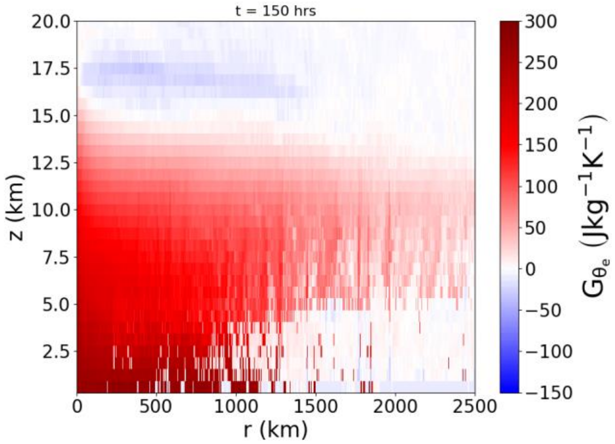
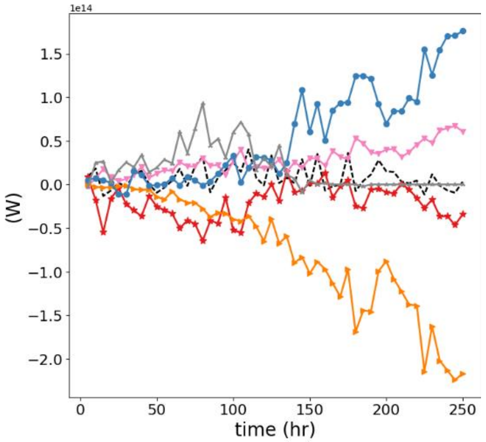
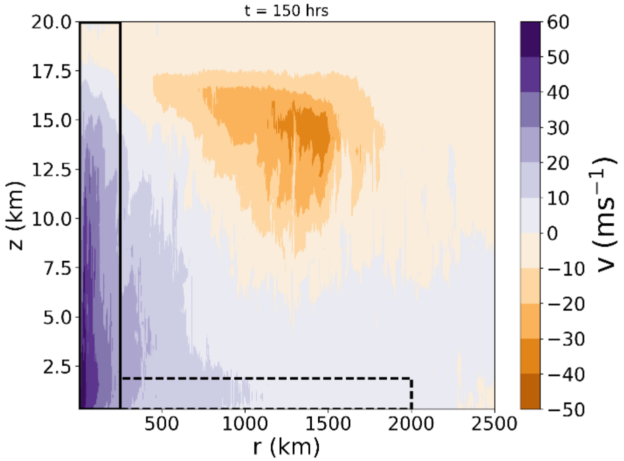
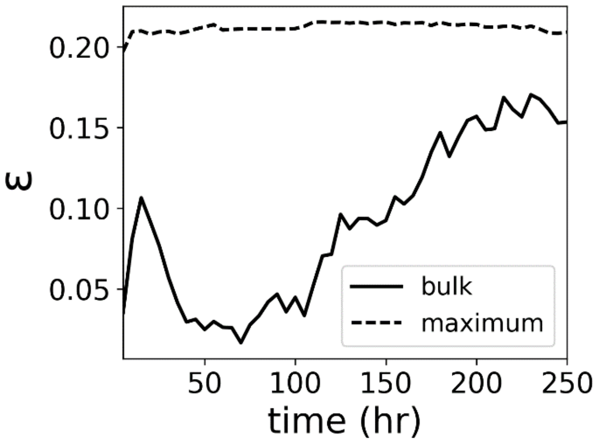
Department of Meteorology

AN AVAILABLE POTENTIAL ENERGY BUDGET FOR AN AXISYMMETRIC TROPICAL CYCLONE









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OVERVIEW

- What is Available Potential Energy (APE) density?
- What are the advantages of using APE theory to study TCs?
- APE budget for a simple axisymmetric model (*Rotunno & Emanuel, 1987)*
- Future development

APE DENSITY

- Energy available for reversible conversion to kinetic energy; local form derived by *Andrews (1981), Holliday and McIntyre (1981)*, generalised by *Tailleux (2018)*
- APE density e_a defined as:

$$\boxed{e_a = \int_z^{z_r} g \frac{\alpha\left(\theta_e, r_t, p_r\left(z'\right)\right) - \alpha_r\left(z'\right)}{\alpha_r\left(z'\right)} \mathrm{d}z'}$$

- Measured relative to a hydrostatic **reference state**
- Reference height z_r defined as the nearest Level of Neutral Buoyancy

WHY USE APE?

- Different climate models at similar horizontal resolutions can produce very different distributions of TC intensity *(Shaevitz et al. 2014)* – not well understood how model differences lead to differences in TCs

- Representation of moisture, convection and the coupling between them identified as important factors (*Kim et al. 2018*)

- APE budget offers opportunity to link moist processes and convection to intensification

WHY USE APE?

- Previous energetic studies of TCs have employed budgets of latent energy ($\mathrm{LE} = Lq_v$) and total potential energy ($\mathrm{TPE} = c_v T + gz$) (e.g. *Hack & Schubert 1986, Hogsett & Zhang 2009*)
- Most of the TPE is never converted into kinetic energy: APE tells us more about energy available for intensification/maintenance of TC

APE EVOLUTION

- APE density evolves as:

$$\boxed{\frac{\partial\left(\overline{\rho}e_a\right)}{\partial t} = \overline{\rho}G_{\theta_e}\frac{\mathrm{D}\theta_e}{\mathrm{D}t} + \overline{\rho}G_{r_t}\frac{\mathrm{D}r_t}{\mathrm{D}t} - \overline{\rho}bw + \ldots}$$

- θ_e and r_t are approximately conserved variables in model, but other pairs of variables can be equivalently used

APE EVOLUTION

$$\frac{\partial\left(\overline{\rho}e_a\right)}{\partial t} = \overline{\rho}G_{\theta_e}\frac{\mathrm{D}\theta_e}{\mathrm{D}t} + \overline{\rho}G_{r_t}\frac{\mathrm{D}r_t}{\mathrm{D}t} - \overline{\rho}bw + \ldots$$

generation/dissipation
by diabatic processes

- surface fluxes

APE EVOLUTION

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generation/dissipation
by diabatic processes

- surface fluxes
- mixing
- precipitation

APE EVOLUTION

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thermodynamic efficiencies

- functions of temperature & humidity in situ and at reference height

APE EVOLUTION

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thermodynamic
efficiencies

- functions of temperature & humidity in situ and at reference height

$$G_{\theta_e} = c_p\frac{T_0 - T_r}{\theta_e}$$

$$G_{r_t} = \frac{1}{\left(1 + r_t\right)^2}\left[\mu_0 - \mu_r - \left(T_0 - T_r\right)\frac{\partial\mu}{\partial T}\right]$$

APE EVOLUTION

$$\frac{\partial\left(\overline{\rho}e_a\right)}{\partial t}=\overline{\rho}G_{\theta_e}\frac{\mathrm{D}\theta_e}{\mathrm{D}t}+\overline{\rho}G_{r_t}\frac{\mathrm{D}r_t}{\mathrm{D}t}-\overline{\rho}bw+\ldots$$

thermodynamic efficiencies

- functions of temperature & humidity in situ and at reference height

- A process may be either a source or a sink of APE depending on sign of efficiency

APE EVOLUTION

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generation/dissipation
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exchange with
vertical kinetic energy

APE EVOLUTION

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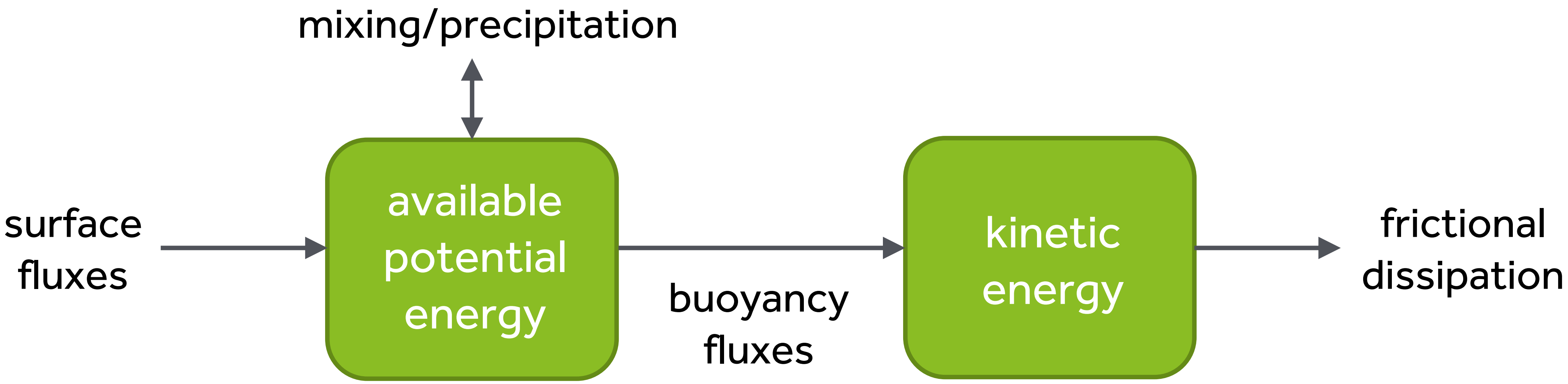
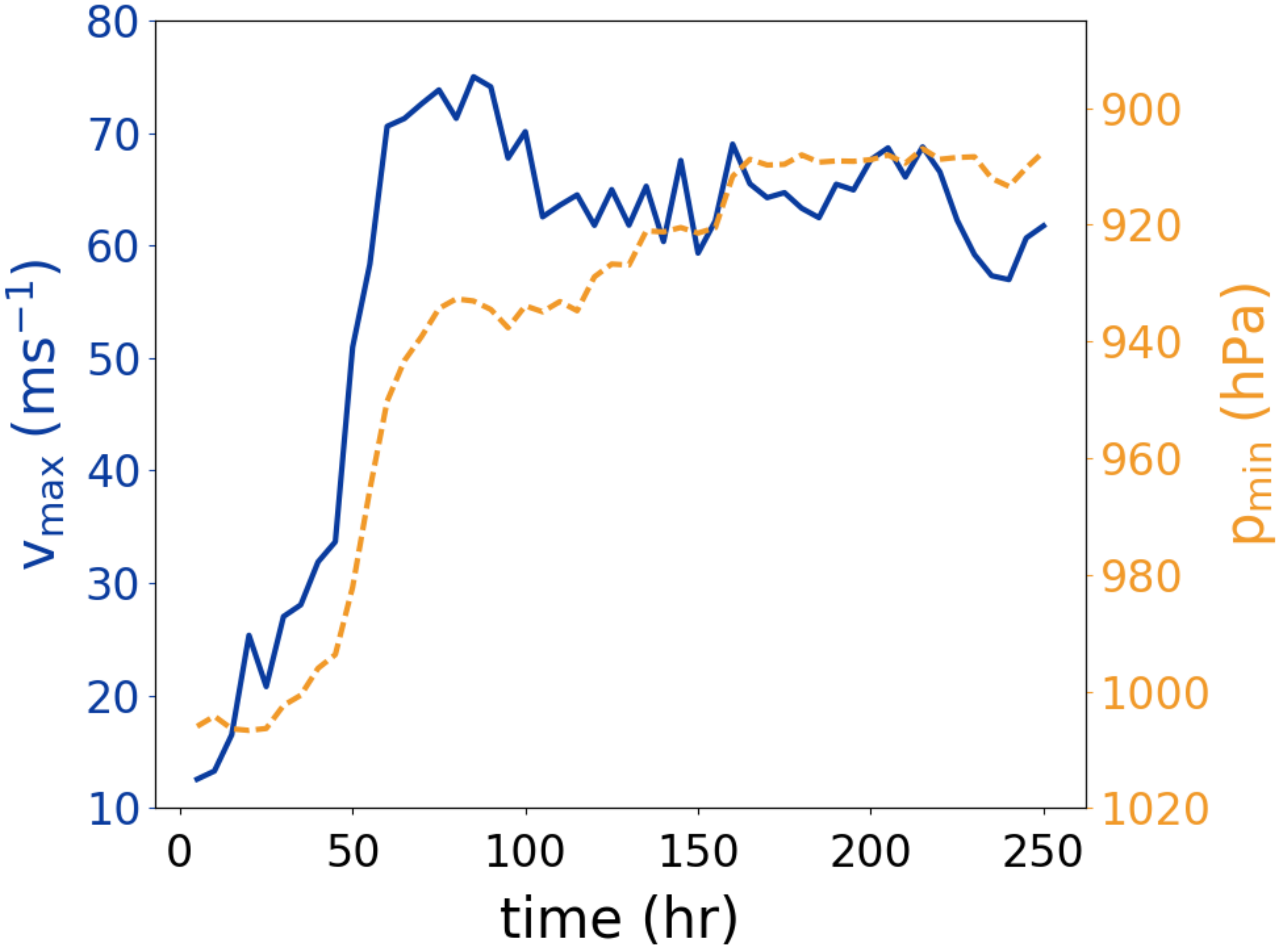


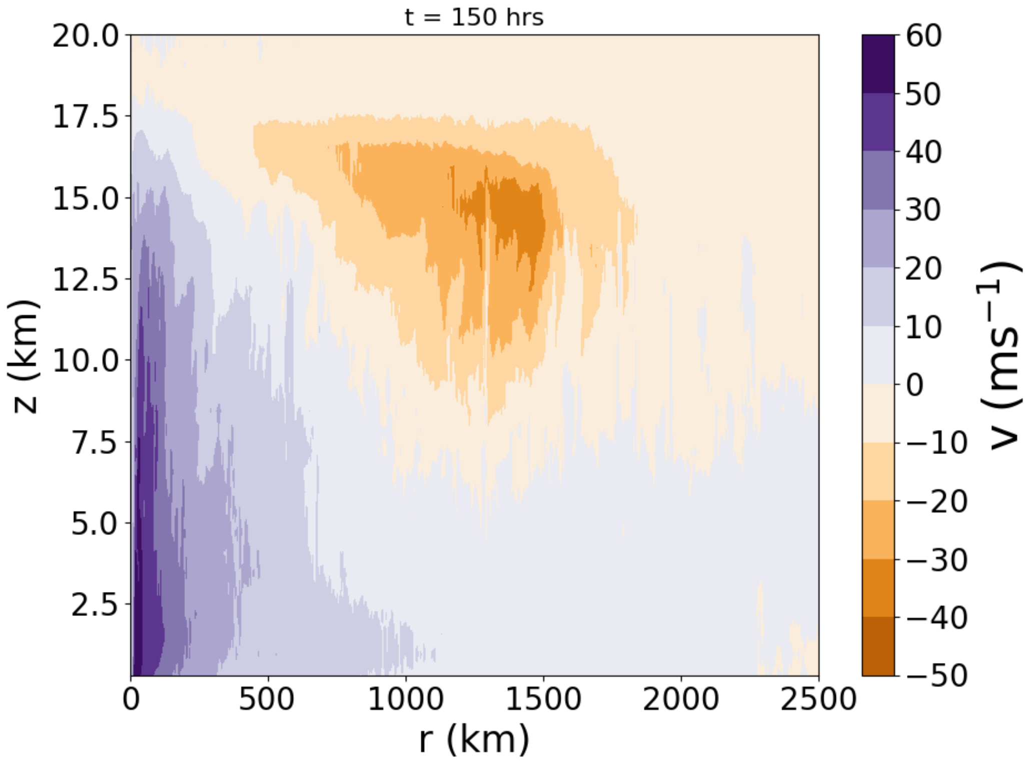
AXISYMMETRIC MODEL

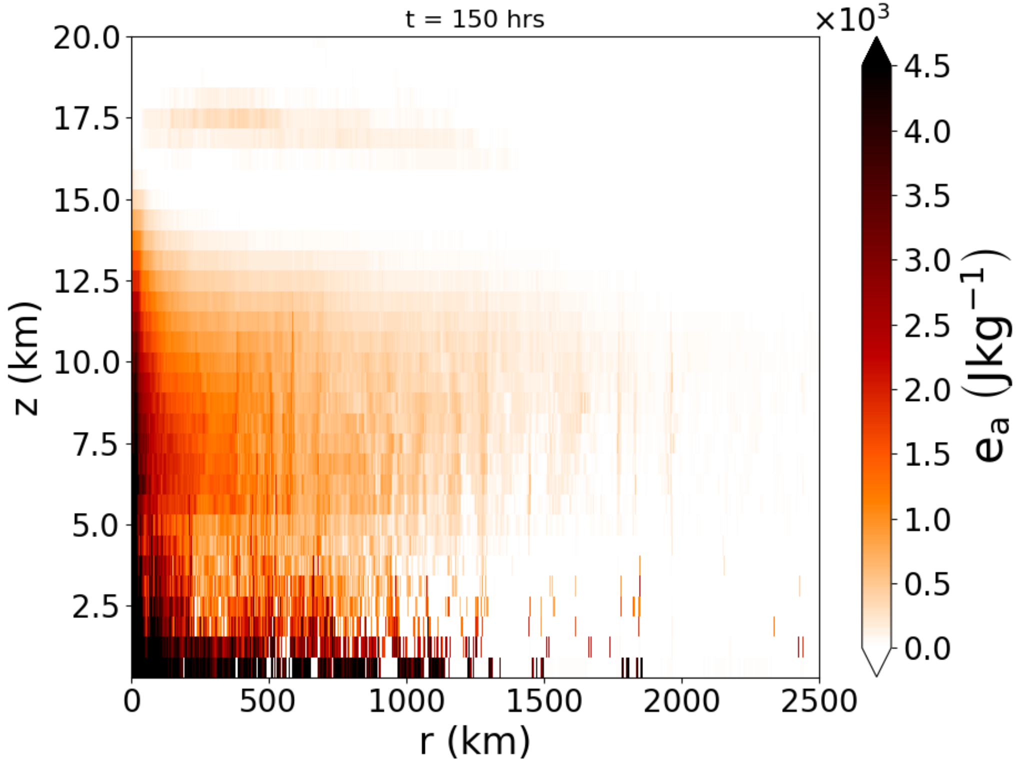


- *Rotunno & Emanuel (1987)*
- Microphysics modified by *Craig (1995, 1996)* – include ice in θ_e
- 2.5 x 0.625 km resolution
- Constant SST
- Prescribe initial vortex
- Model initialised with Jordan tropical mean sounding – use as reference state

APE DENSITY

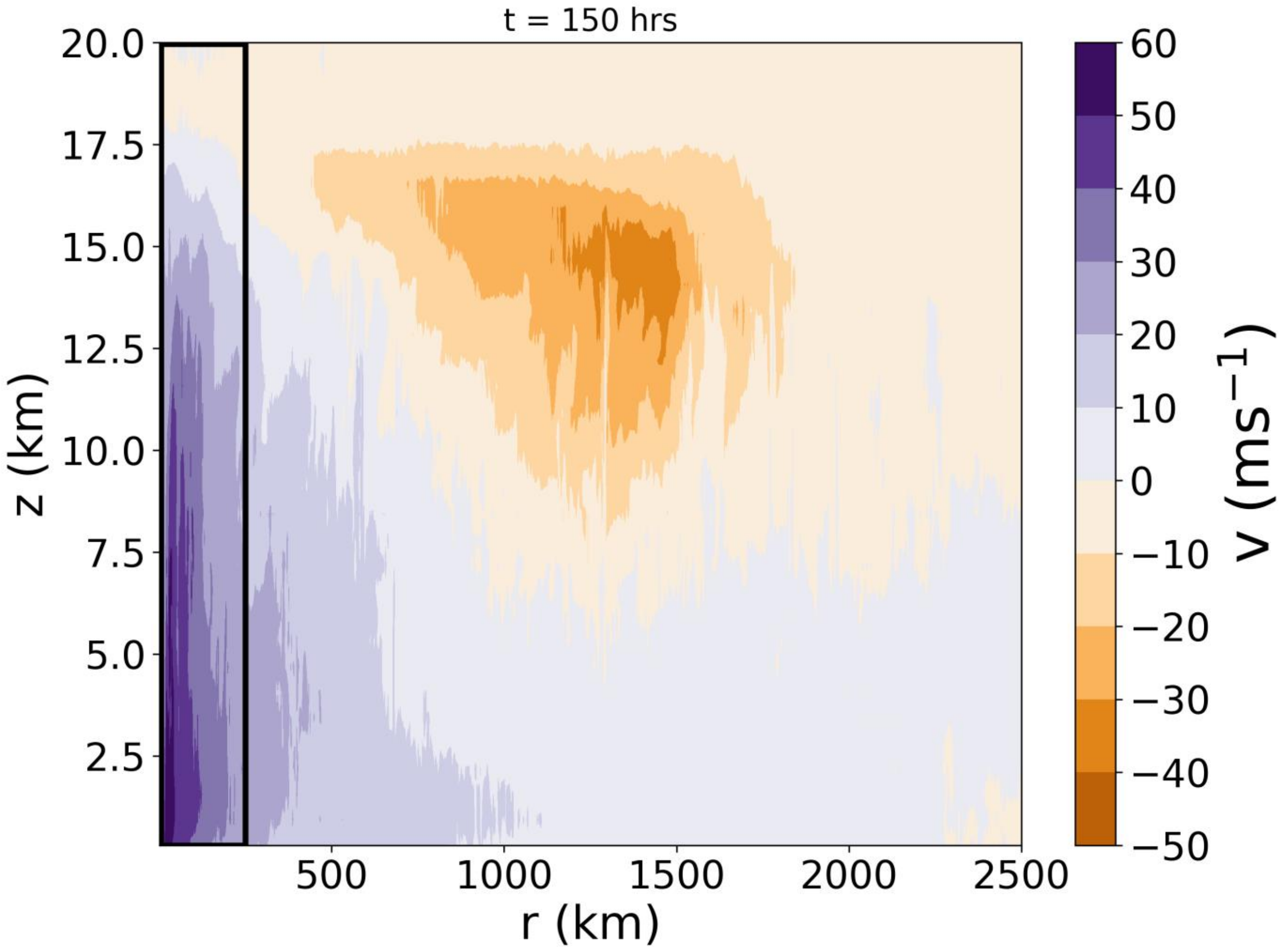
- Storage of APE in cyclone due to warm core
- High APE in eyewall due to warming by latent heat release
- High APE near surface due to heating by surface fluxes





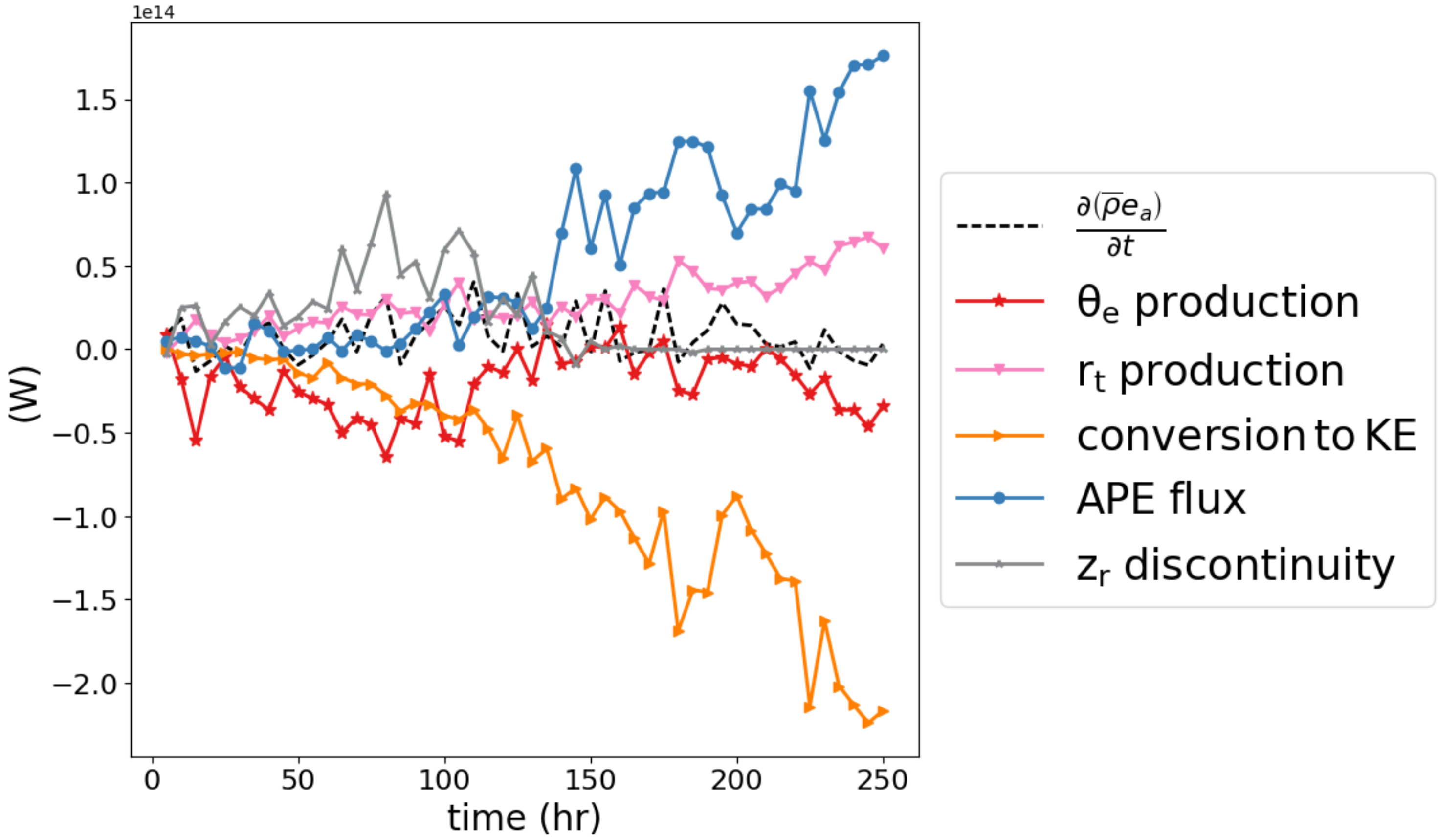
APE BUDGET: INNER REGION

- Integrate APE budget over all grid points with $r < 250$ km (solid box)
- What are the sources/sinks of APE in this region?



APE BUDGET: INNER REGION

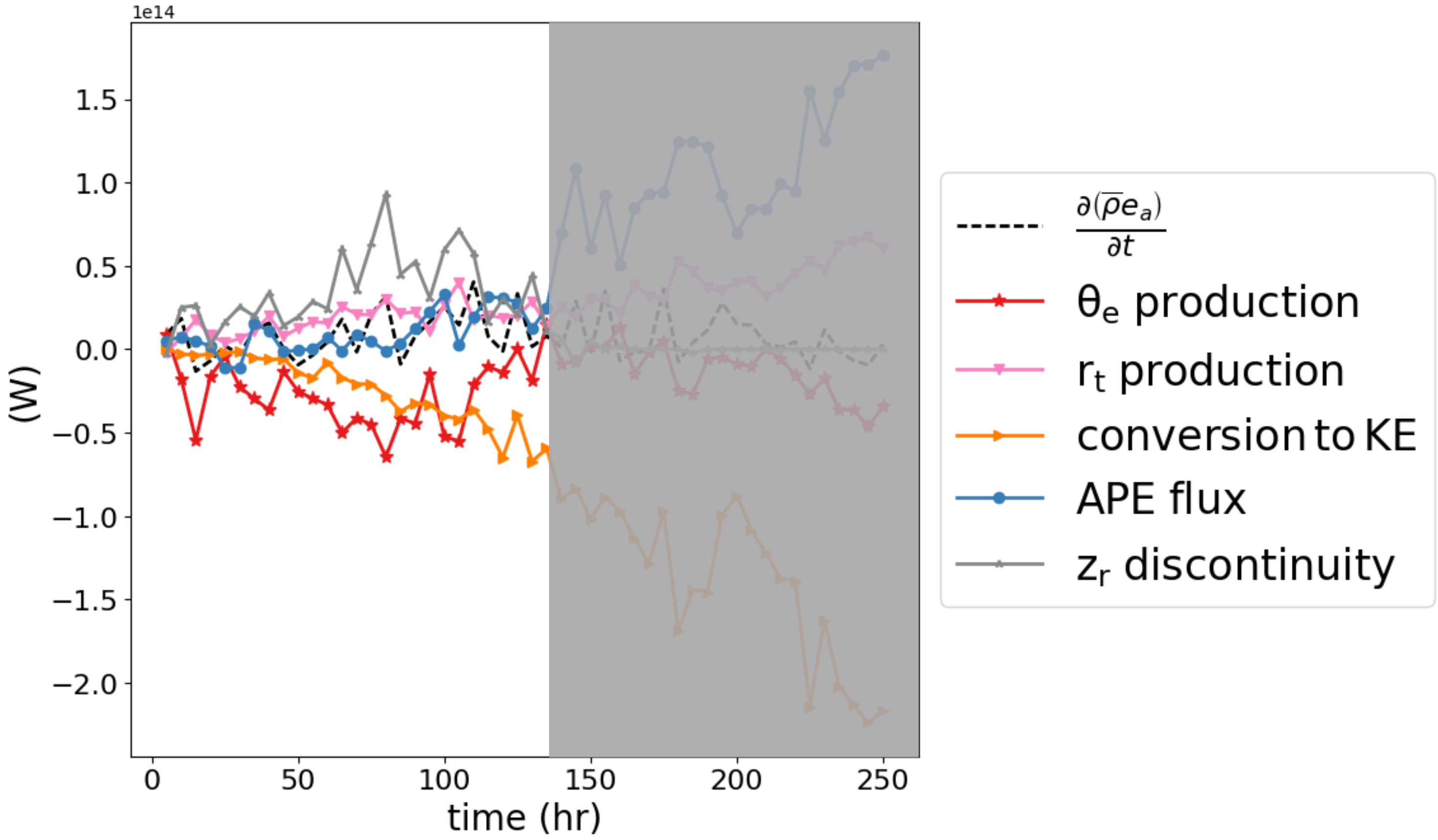
- Very different budget features in intensifying vs mature stages



APE BUDGET: INNER REGION

Intensification stage

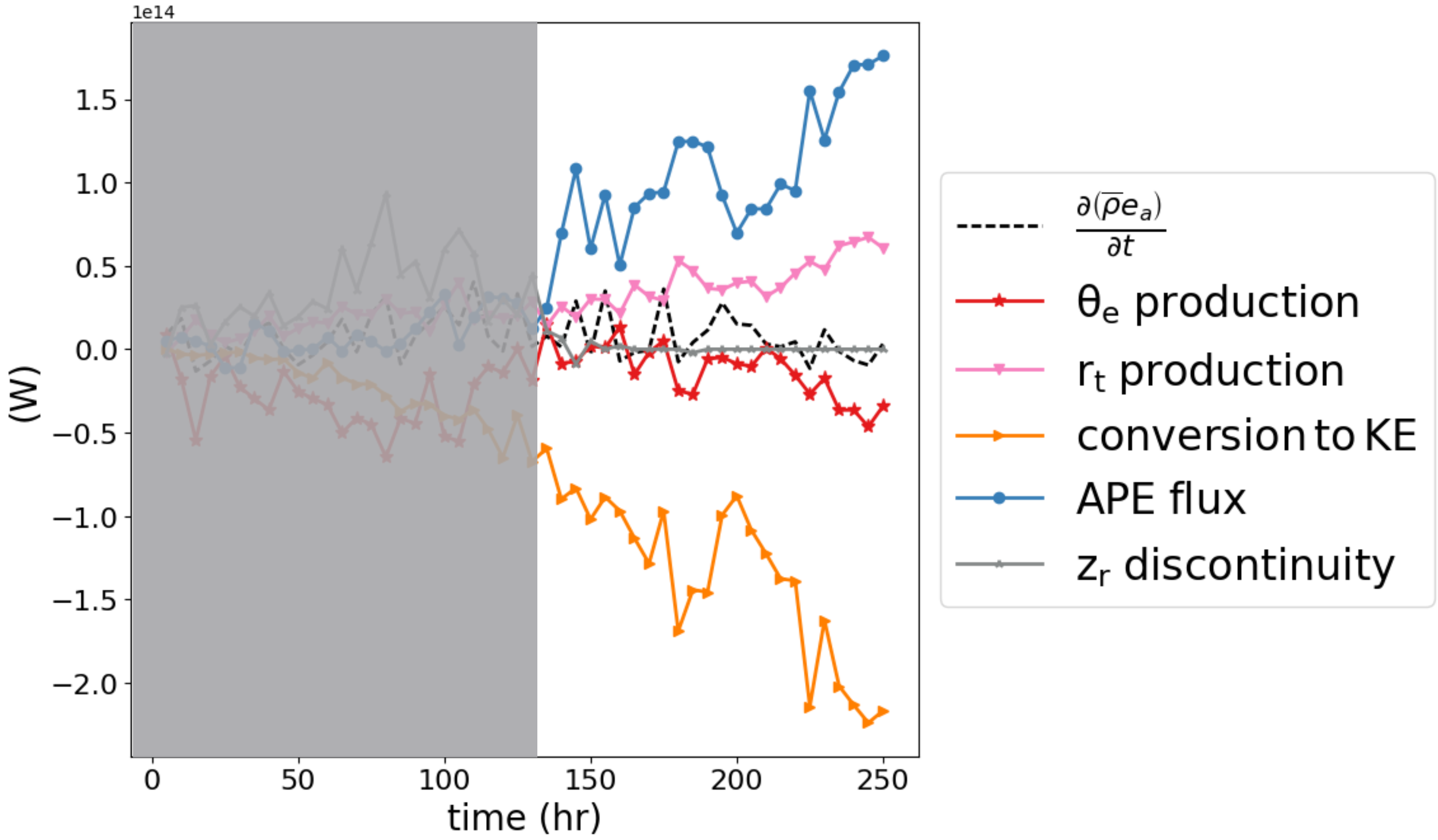
- Large contribution due to discontinuous jumps in reference heights
- Occurs when some parcels are buoyant and others aren’t
- Partitioning between APE and background potential energy changes
- Previously unrecognised behaviour of APE density



APE BUDGET: INNER REGION

Mature stage

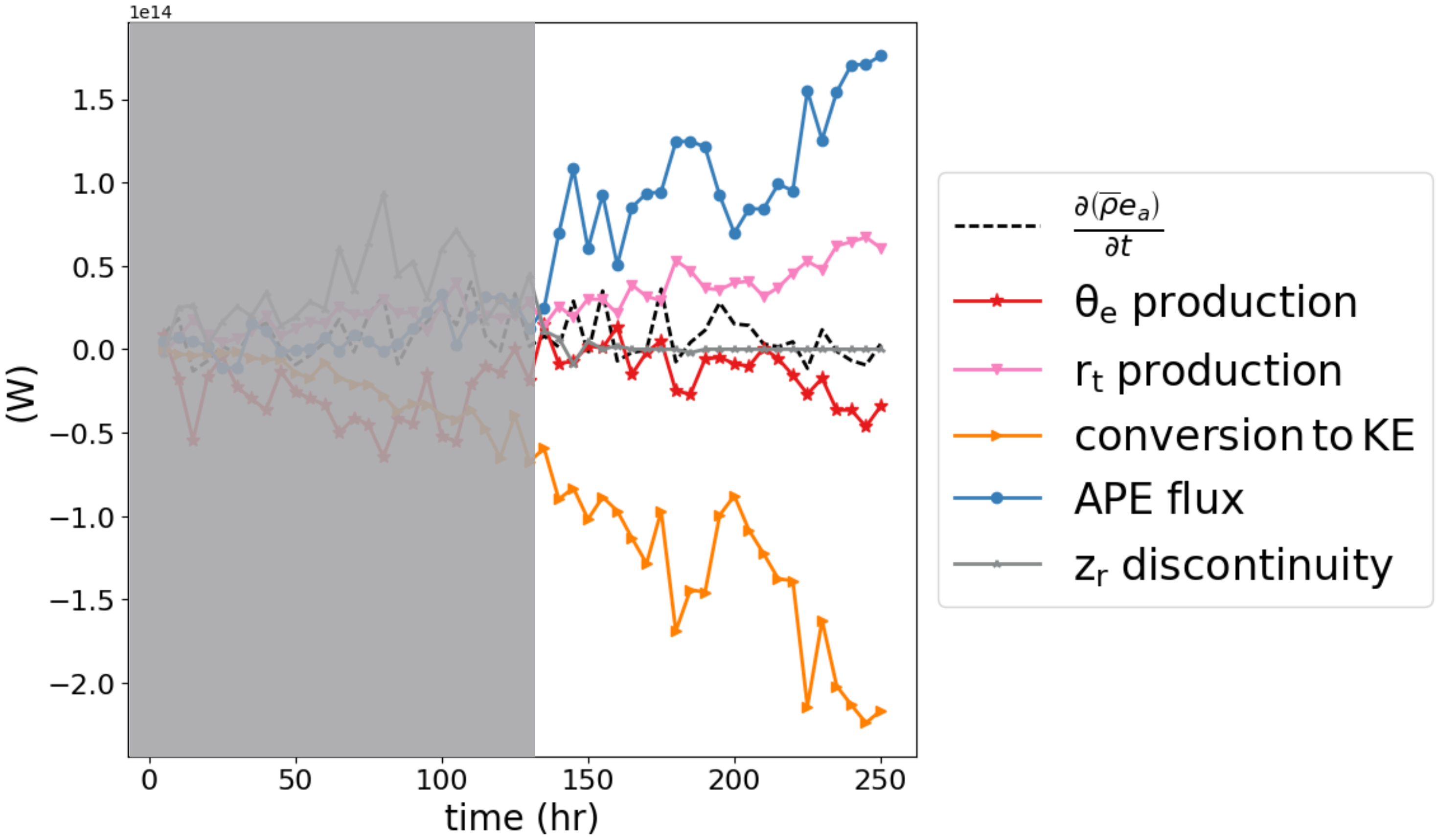
- Source of APE is flux entering region via radial inflow at low levels
- Agrees with previous latent energy budgets (*Kurihara 1975*)
- Import of APE balanced by conversion to KE
- Two production terms balance due to opposing effects of rainfall



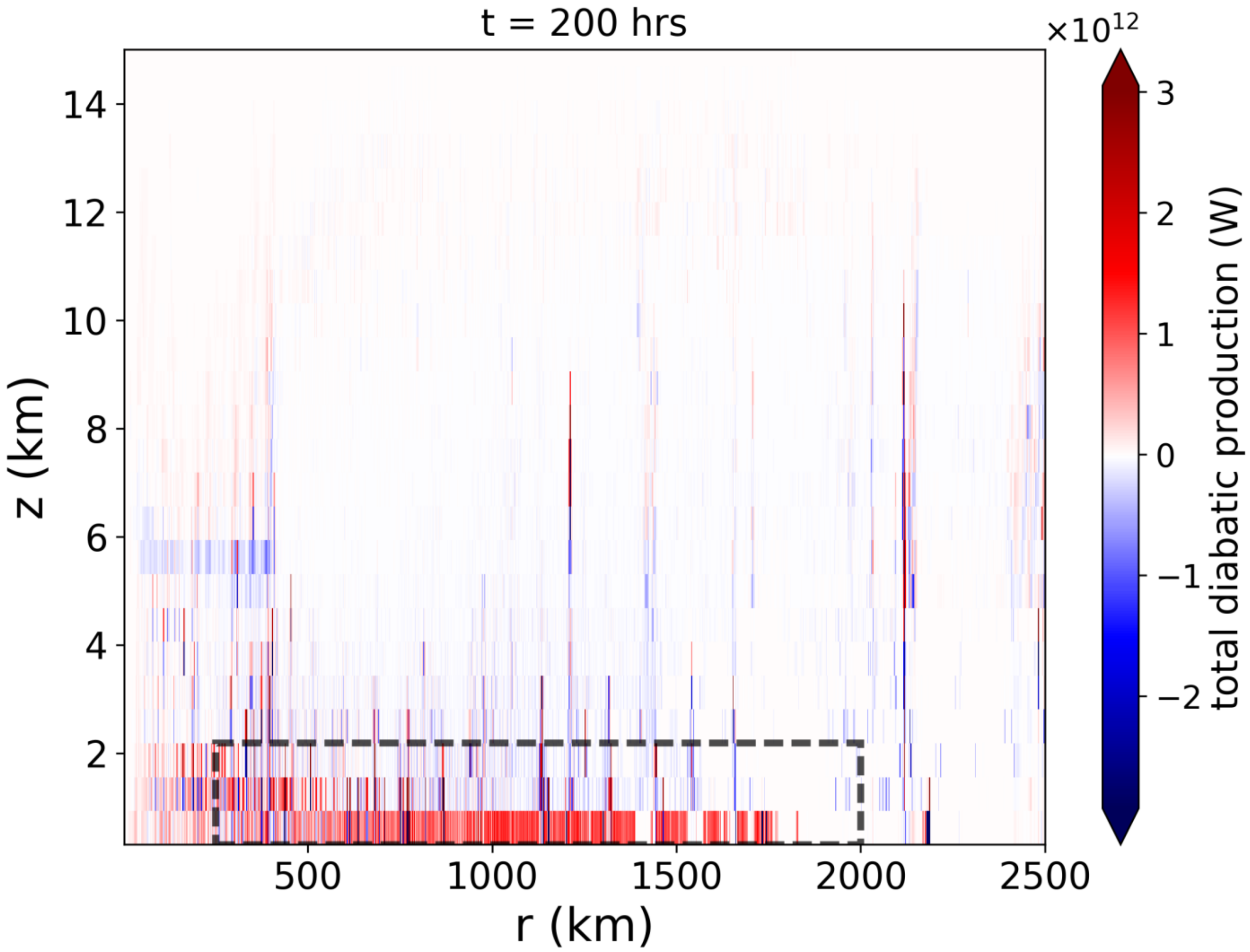
APE BUDGET: INNER REGION

Mature stage

- Production in regions of highest wind less important than transport by secondary circulation
  - Where and how is APE produced?

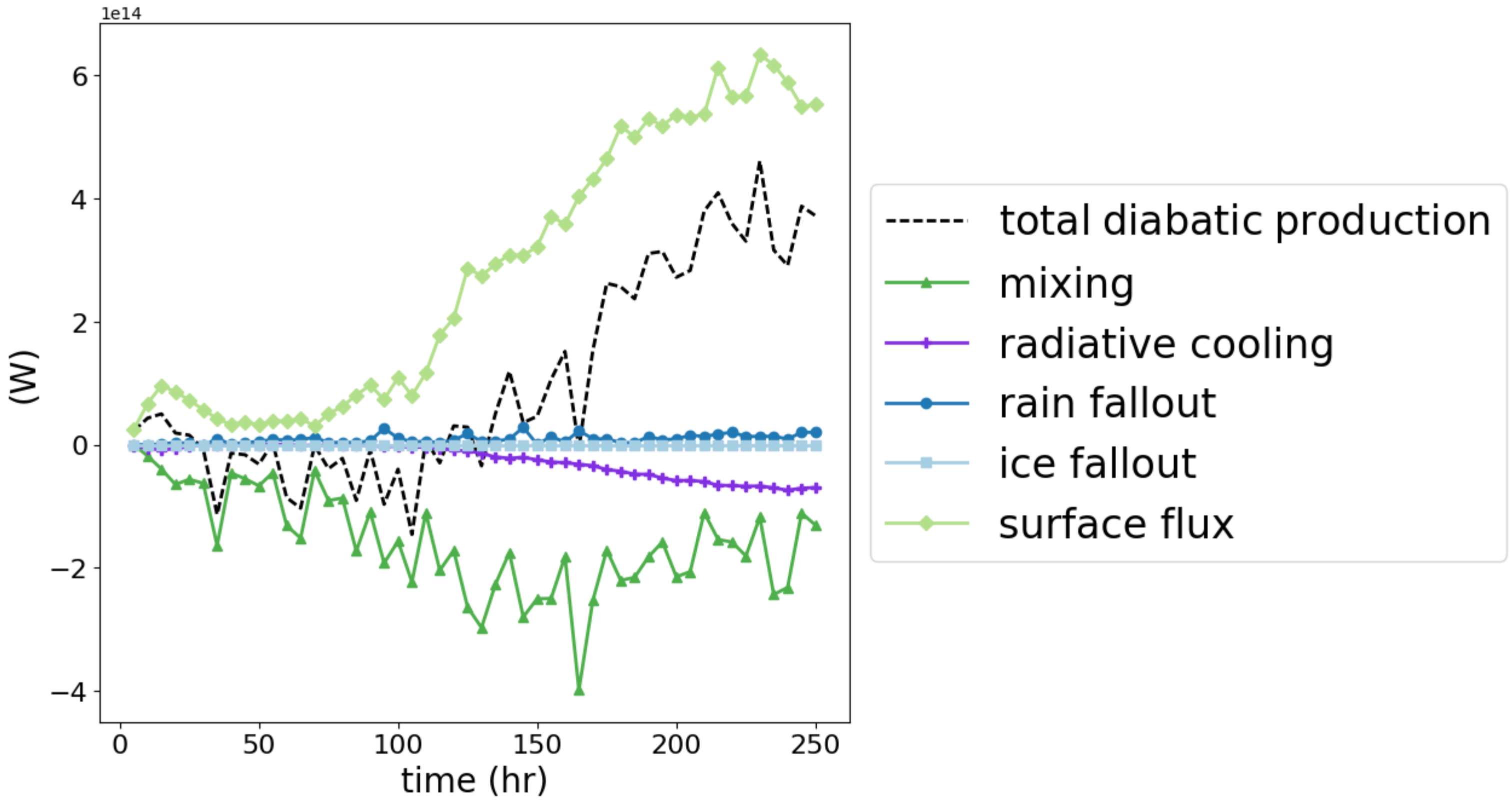


APE PRODUCTION



- Total diabatic production (both θ_e and r_t) chiefly in surface parcels
- Budget diabatic production in dashed box (*inflow* region)

APE PRODUCTION BUDGET: INFLOW

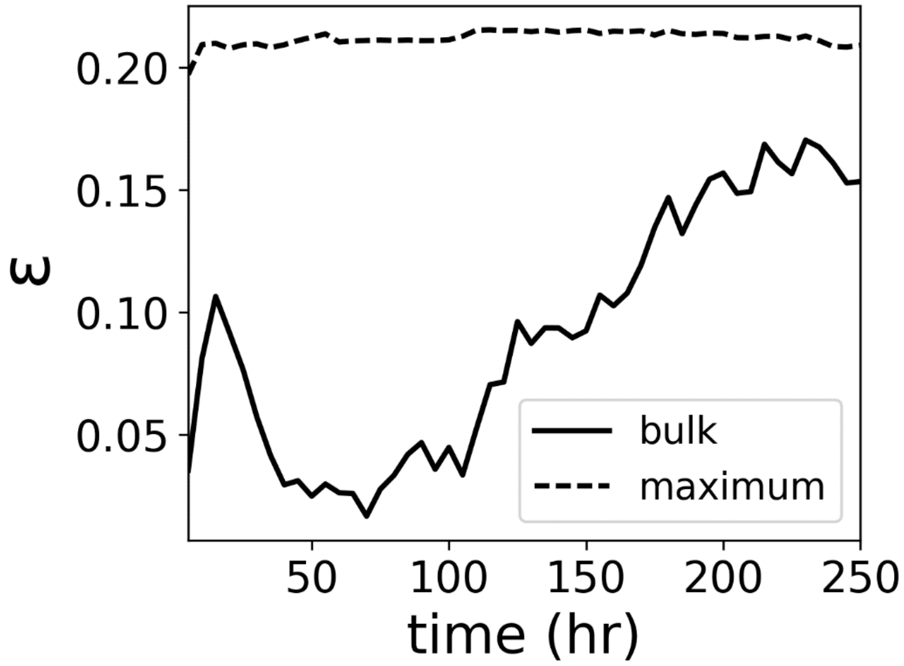


- APE is mostly generated by surface fluxes (primarily latent heat flux) in inflow region
- This is transported to provide the main APE source for the inner regions of the TC
- Subgrid diffusive mixing acts to reduce surface source of APE

SURFACE APE EFFICIENCY

- Define efficiency of surface parcels:

$$\varepsilon = \frac{\text{total APE production by surface fluxes}}{\text{total surface enthalpy flux}}$$



- Maximum efficiency 20% ≈ Carnot efficiency of TC
- Compare with 1% efficiency when using total potential energy budget *(Hack & Schubert, 1986)*
- Bulk efficiency varies with time as more parcels become buoyant

SUMMARY

- Possible to construct a closed budget of Available Potential Energy density for a TC
- Main energy supply for TC is the generation of Available Potential Energy via surface fluxes
- Surface enthalpy fluxes generate APE with a maximum efficiency of approximately 20%, and an overall efficiency that varies in time
- For inner regions, inward transport of APE more important than local production
- Import of APE balanced by conversion to KE

FUTURE DIRECTIONS

- Does the temporal variation of APE efficiency play an important role in governing intensification?

- Can APE budget provide physical explanations for how different treatments of moisture and convection in models lead to differences in TCs?

    - Apply budget to convection-permitting Met Office regional NWP models, and high-resolution climate model output

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