

A satellite view of Earth's clouds, showing a dense, swirling pattern of white and grey clouds against a dark blue background. The clouds are concentrated in the center and spread out towards the edges, creating a complex, textured appearance.

**Numerical Weather Prediction  
Parametrization of Subgrid Physical Processes**

**Clouds (3)**

**Sub-grid Cloud Cover**

(or “Sub-grid heterogeneity of cloud and humidity”)

**Richard Forbes**

(With thanks to Adrian Tompkins  
and Christian Jakob)

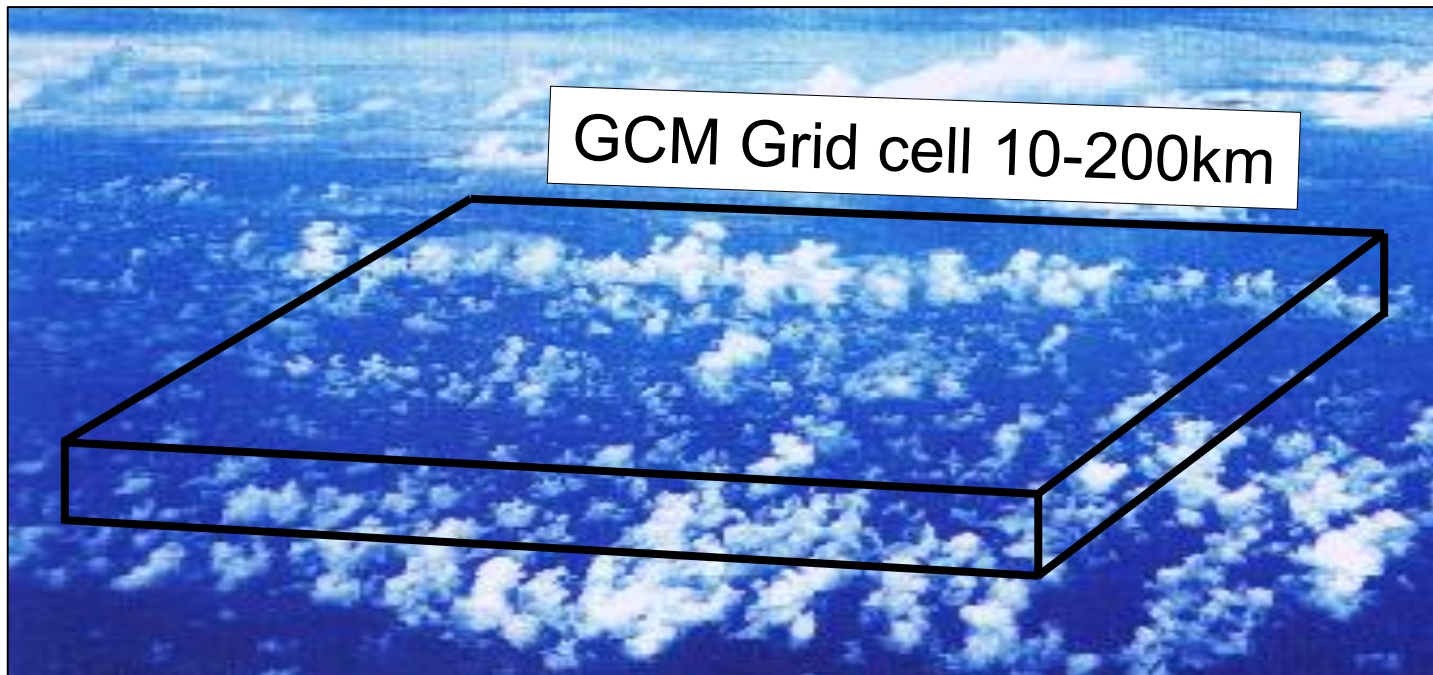
**forbes@ecmwf.int**

# Clouds in GCMs:

## Representing sub-grid heterogeneity



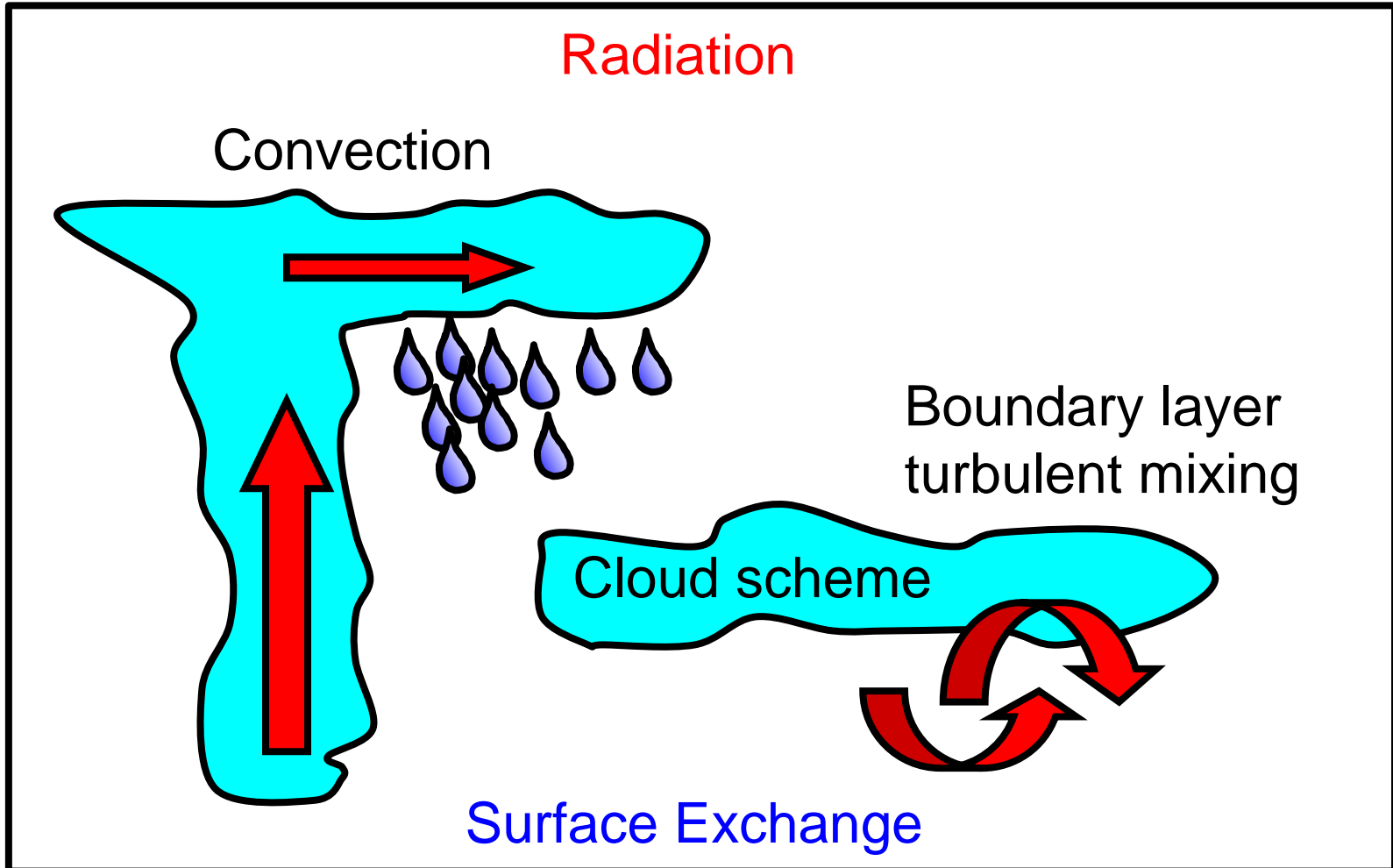
Many of the observed clouds and especially the processes within them are of **subgrid-scale size** (both horizontally and vertically)



# Clouds in GCMs: Representing sub-grid heterogeneity



Many heterogeneity assumptions across the model parametrizations...



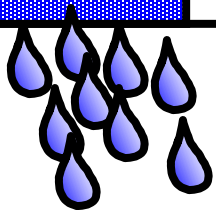
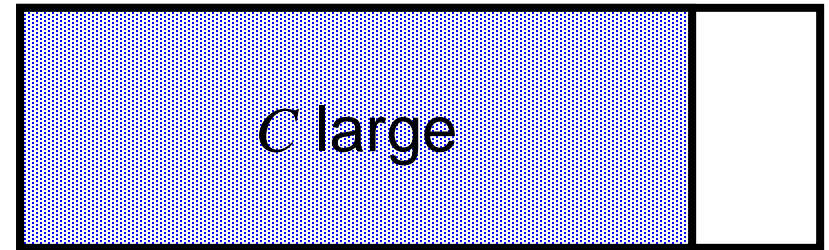
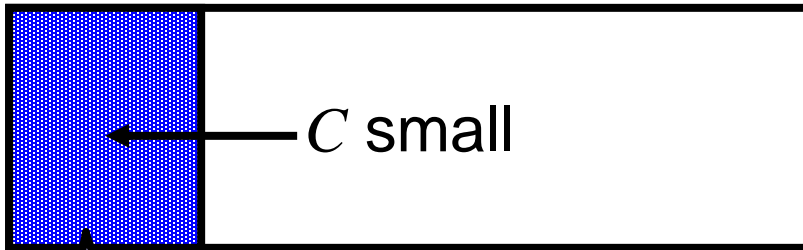
# Why represent heterogeneity?

Important for microphysics



Imagine a cloud with condensate mass  $q_l$  and cloud fraction  $C$   
The in-cloud mass mixing ratio is  $q_l/C$

GCM grid box



precipitation not equal in each case since  
cloud-to-rain autoconversion is **nonlinear**

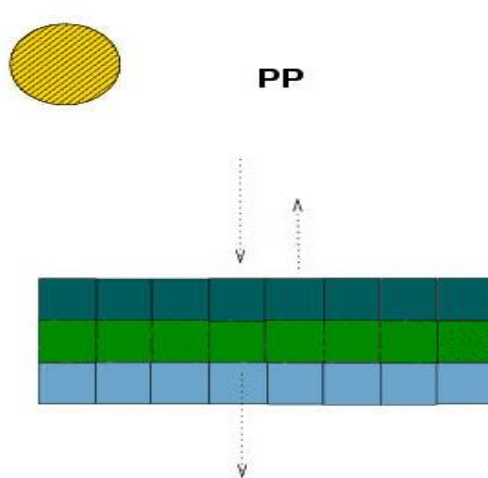
- Complex microphysics perhaps a wasted effort if assessment of cloud fraction  $C$  is poor!
- In addition, in-cloud condensate heterogeneity should also be represented, i.e. not all the cloud is precipitating?

# Why represent heterogeneity?

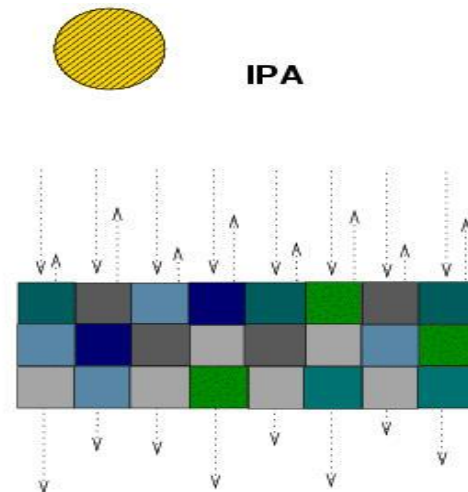


## Important for radiation

- Assuming homogeneity can lead to biased radiative calculations (e.g. Cahalan et al. 1994, Barker et al 1996).
- Monte Carlo Independent Column Approximation, for example, can treat the inhomogeneity of in-cloud condensate and vertical overlap in a consistent way between the cloud and radiation schemes



Traditional approach  
(homogeneous)



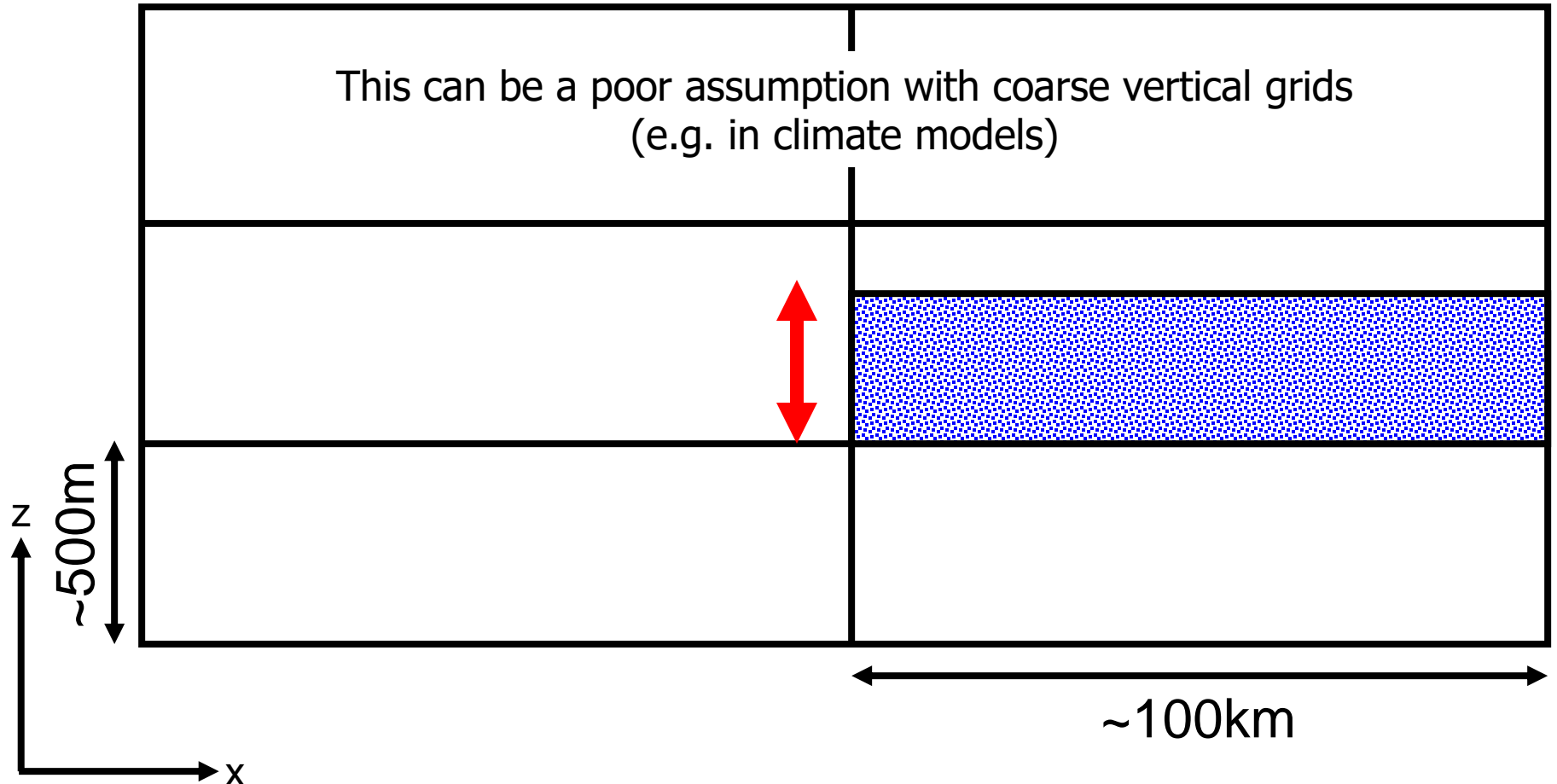
Independent Column  
Approximation, e.g.  
MCICA

# Macroscale Issues of Parameterization



## VERTICAL COVERAGE

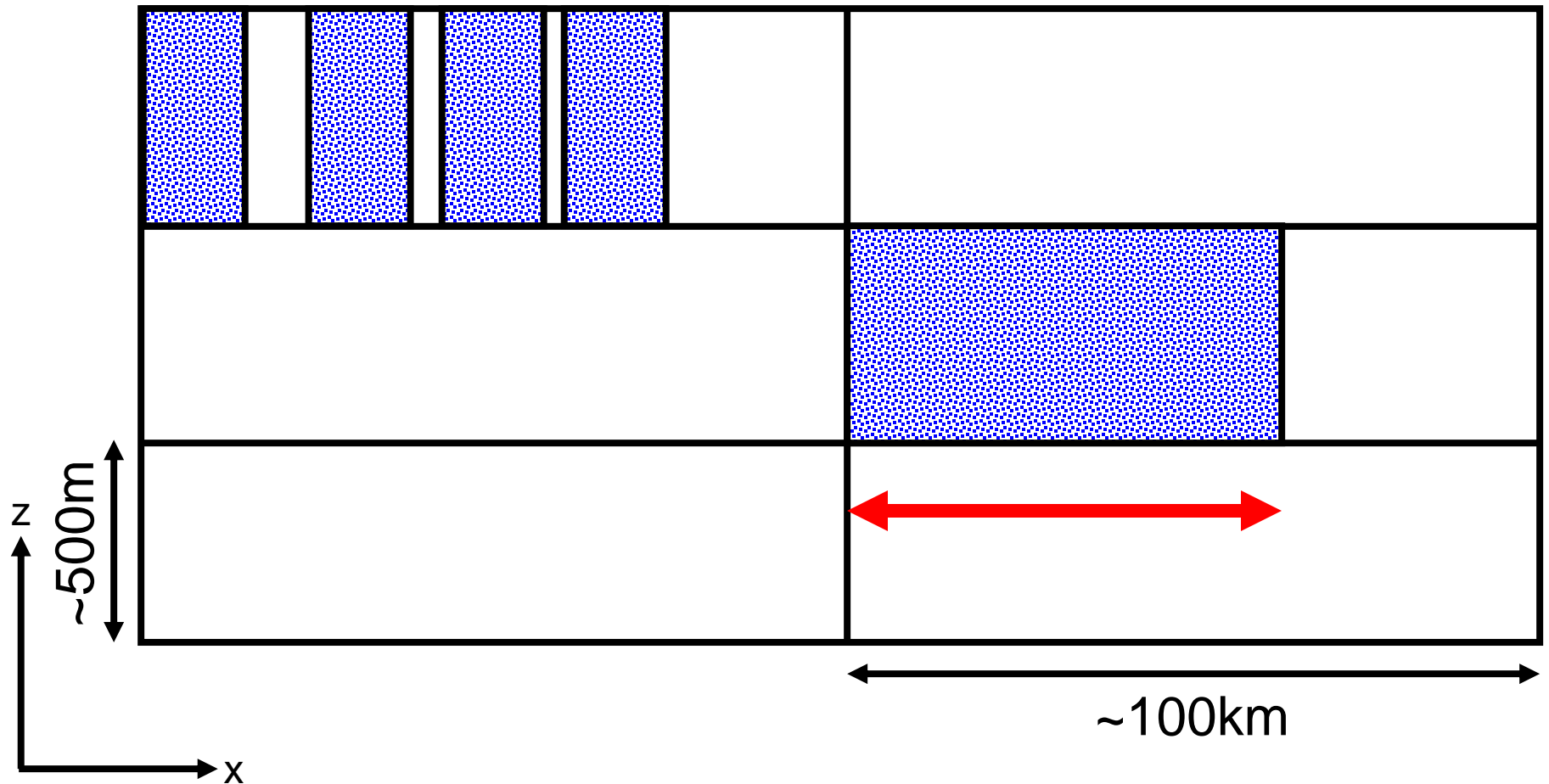
Most models assume that this is 1



# Macroscale Issues of Parameterization



HORIZONTAL COVERAGE,  $C$   
Spatial arrangement ?

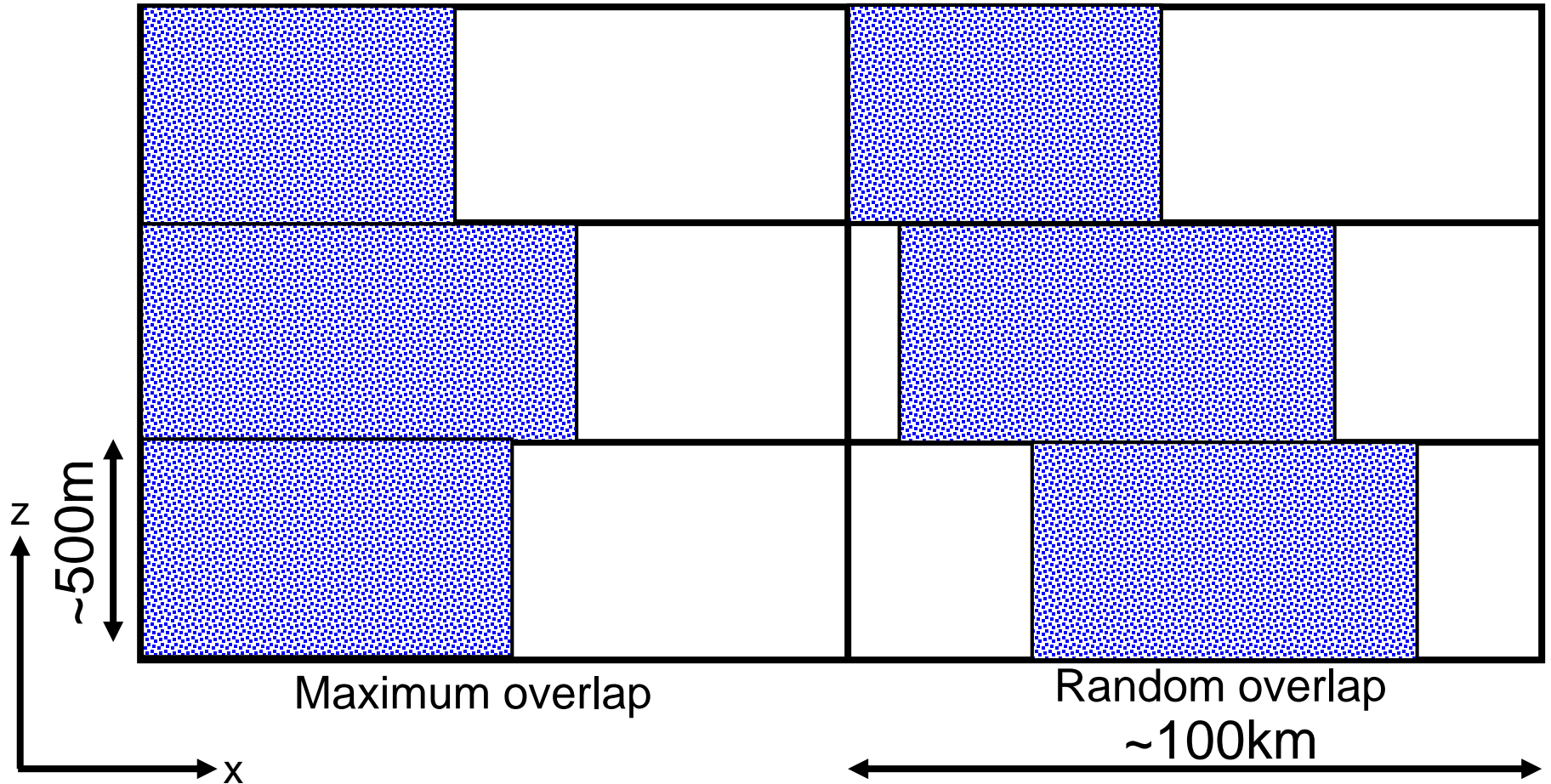


# Macroscale Issues of Parameterization



## Vertical overlap of cloud

Important for radiation and microphysics interaction

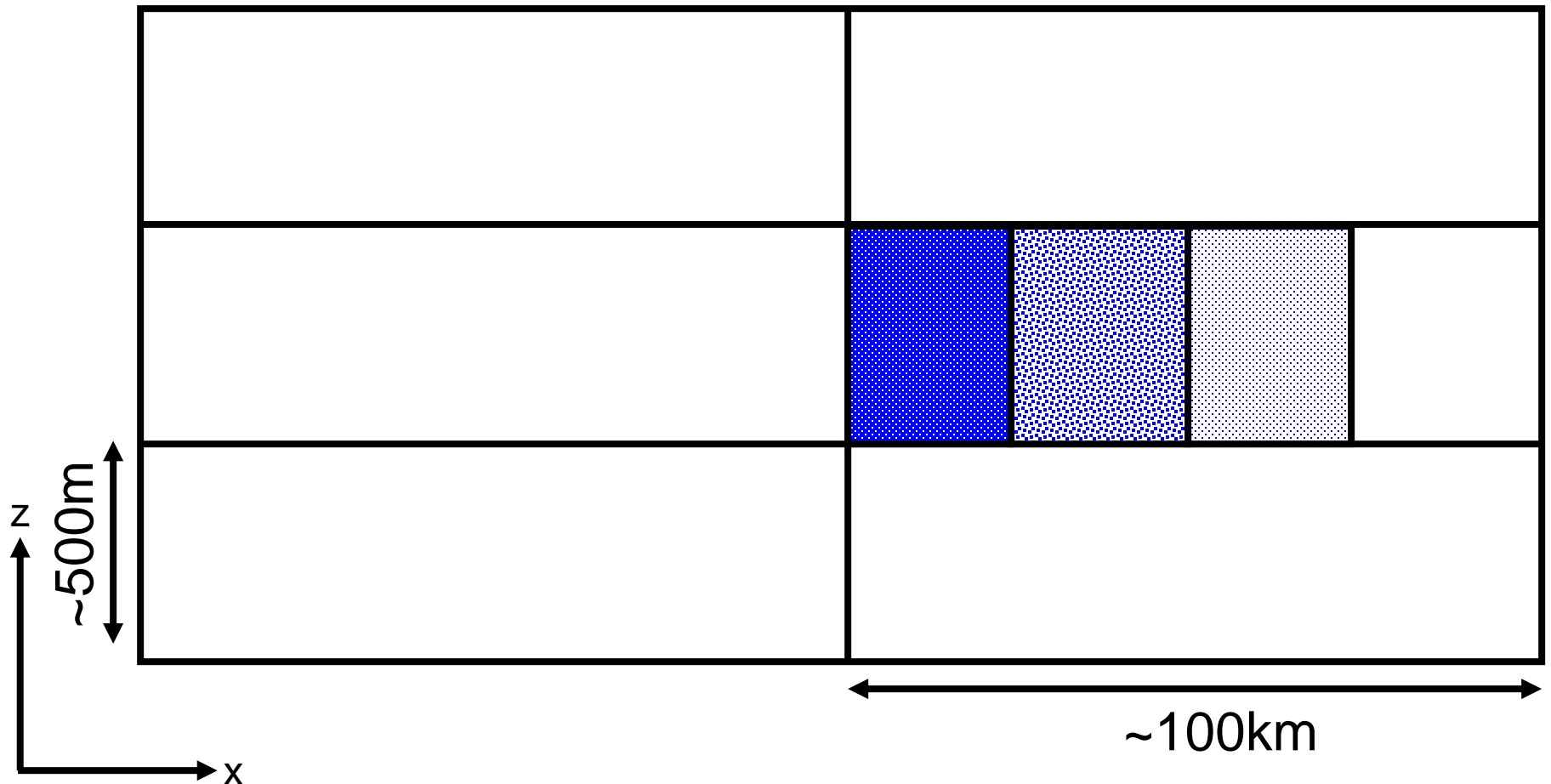




# Macroscale Issues of Parameterization



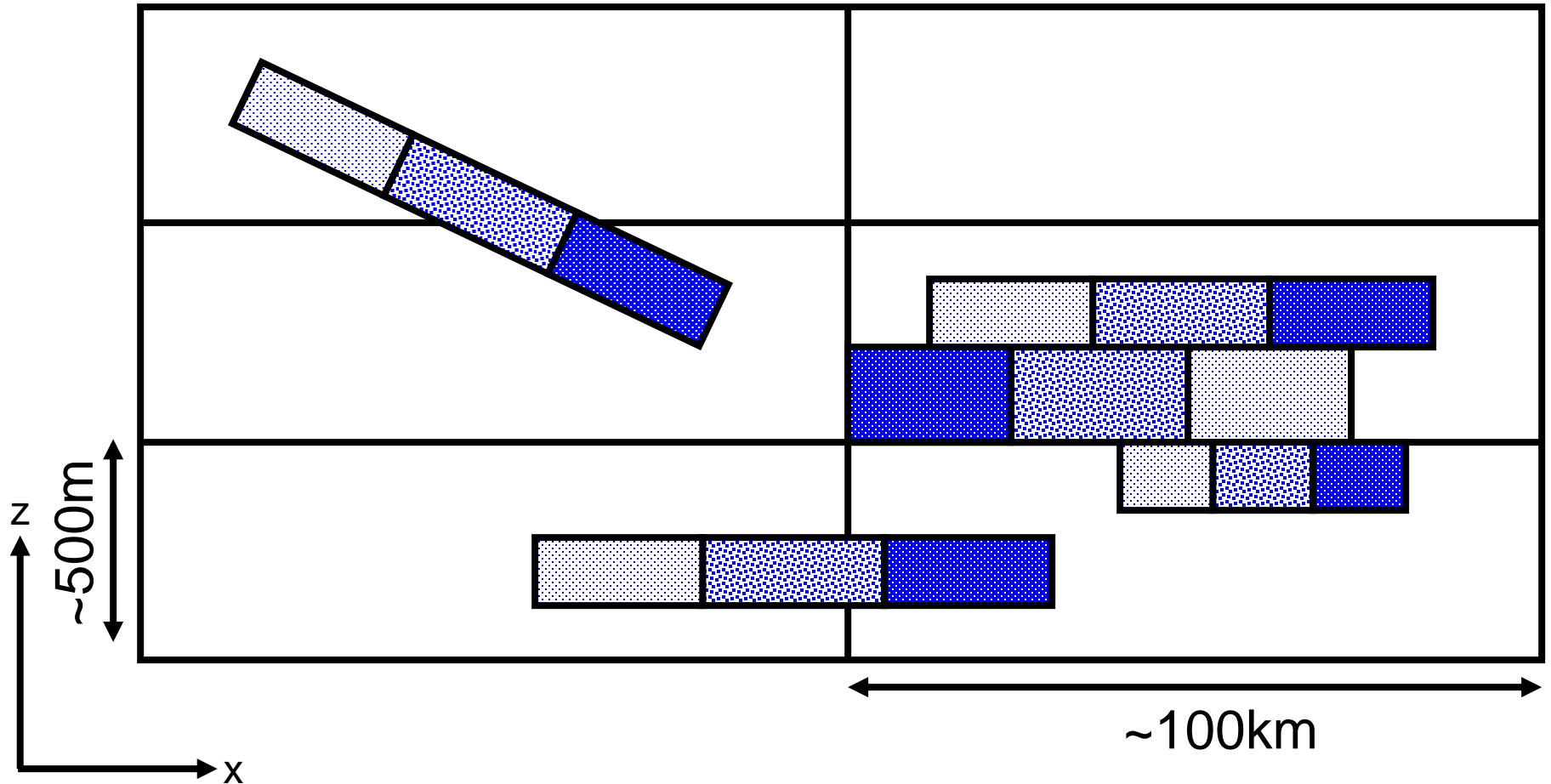
In-cloud inhomogeneity  
in terms of cloud water, particle size/number



# Macroscale Issues of Parameterization



Just these issues can become very complex!!!





# First: Some assumptions!

---

$q_v$  = water vapour mixing ratio

$q_c$  = cloud water (liquid/ice) mixing ratio

$q_s$  = saturation mixing ratio =  $F(T,p)$

$q_t$  = total water (vapour+cloud) mixing ratio

RH = relative humidity =  $q_v / q_s$

1. Local criterion for formation of cloud:  $q_t > q_s$

This assumes that no supersaturation can exist

2. Condensation process is fast (cf. GCM timestep)

$$q_v = q_s$$

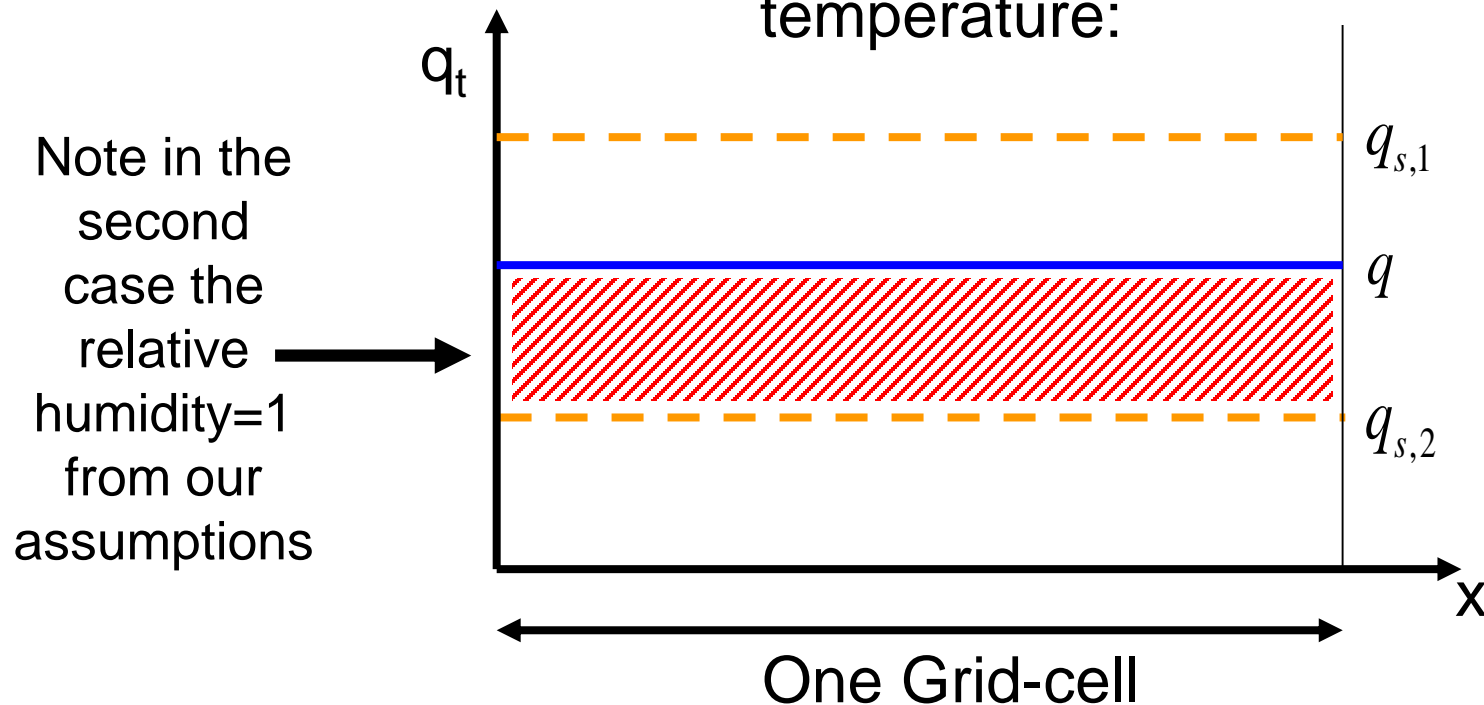
$$q_c = q_t - q_s$$

!!Both of these assumptions less applicable in ice clouds!!



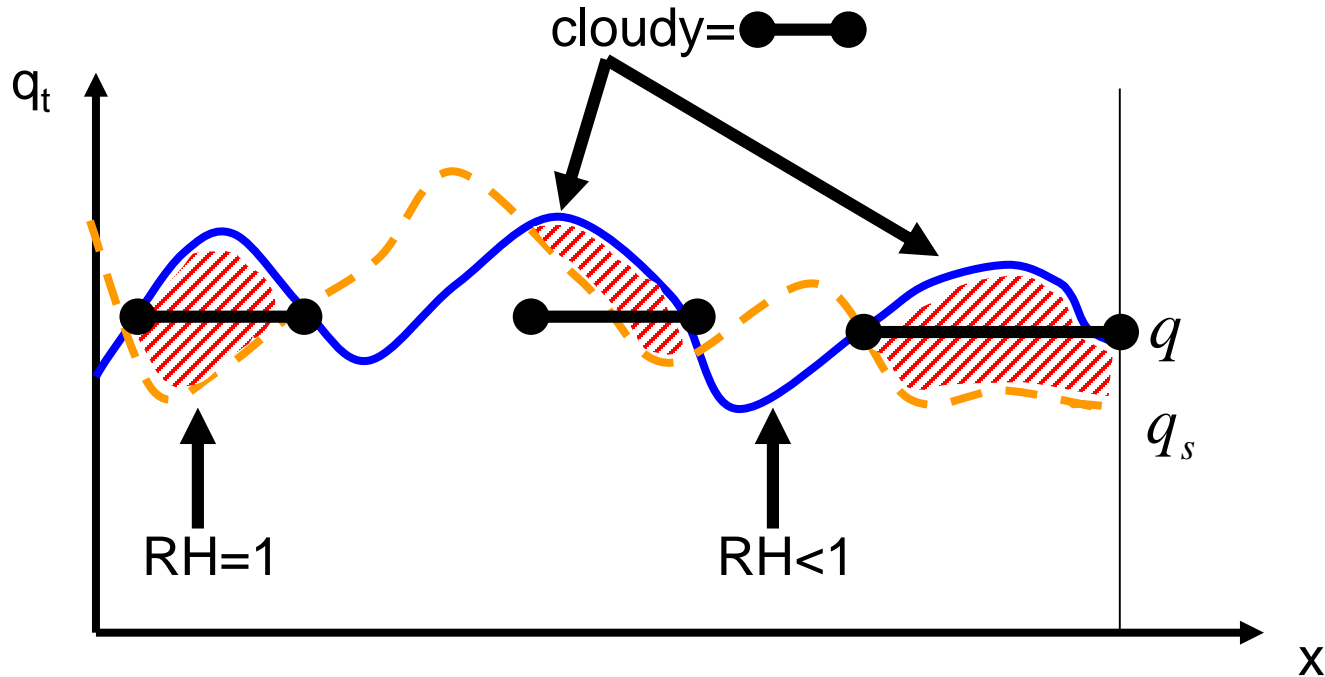
# Partial cloud cover

Homogeneous distribution  
of water vapour and  
temperature:



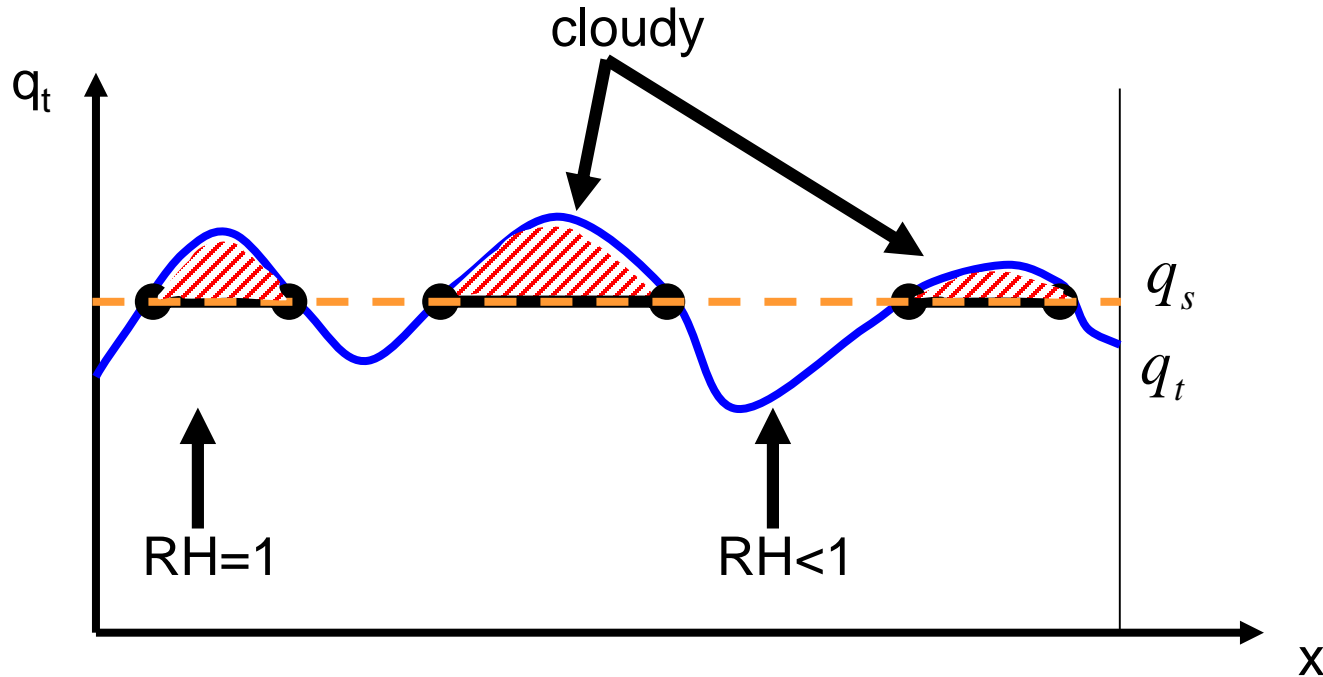
Partial coverage of a grid-box with clouds is only possible if there is an inhomogeneous distribution of temperature and/or humidity.

# Heterogeneous Distribution of $T$ and $q$



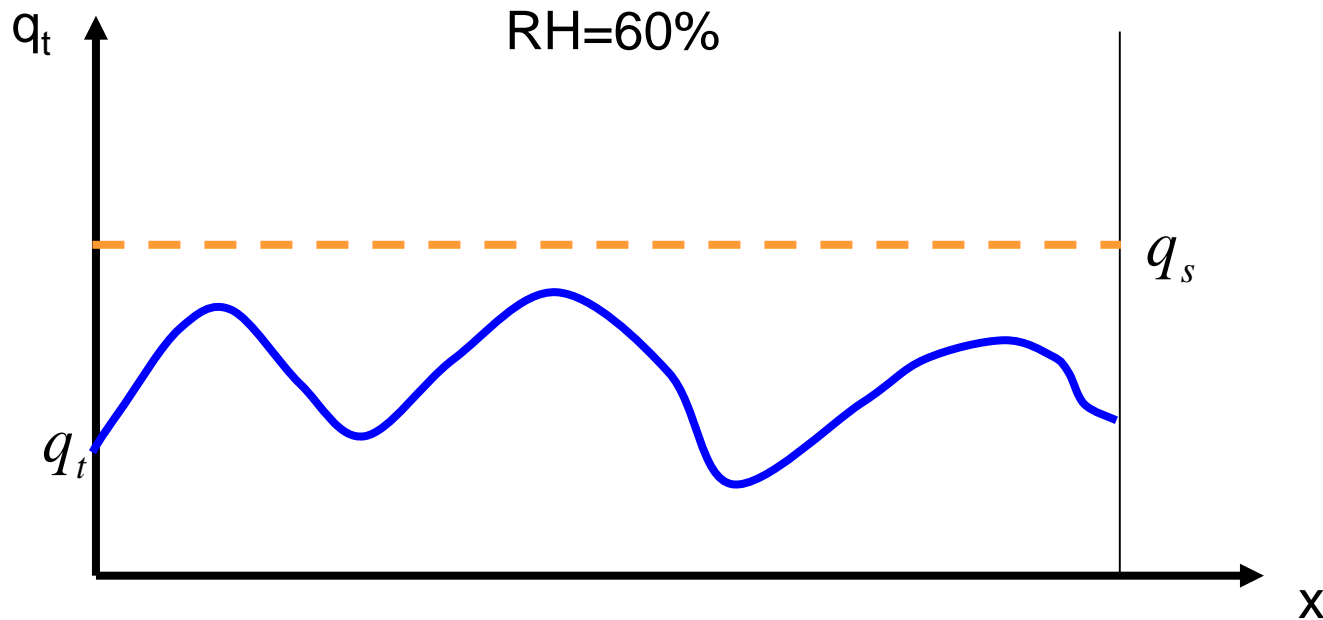
Another implication of the above is that clouds must exist before the grid-mean relative humidity reaches 1.

# Heterogeneous Distribution of $q$ only



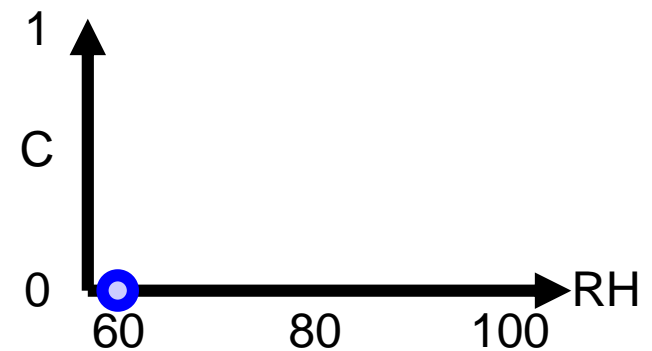
- The interpretation does not change much if we only consider humidity variability
- Throughout this talk I will neglect temperature variability
- Analysis of observations and model data indicates humidity fluctuations are more important most of the time.

# Simple Diagnostic Cloud Schemes: Relative Humidity Schemes

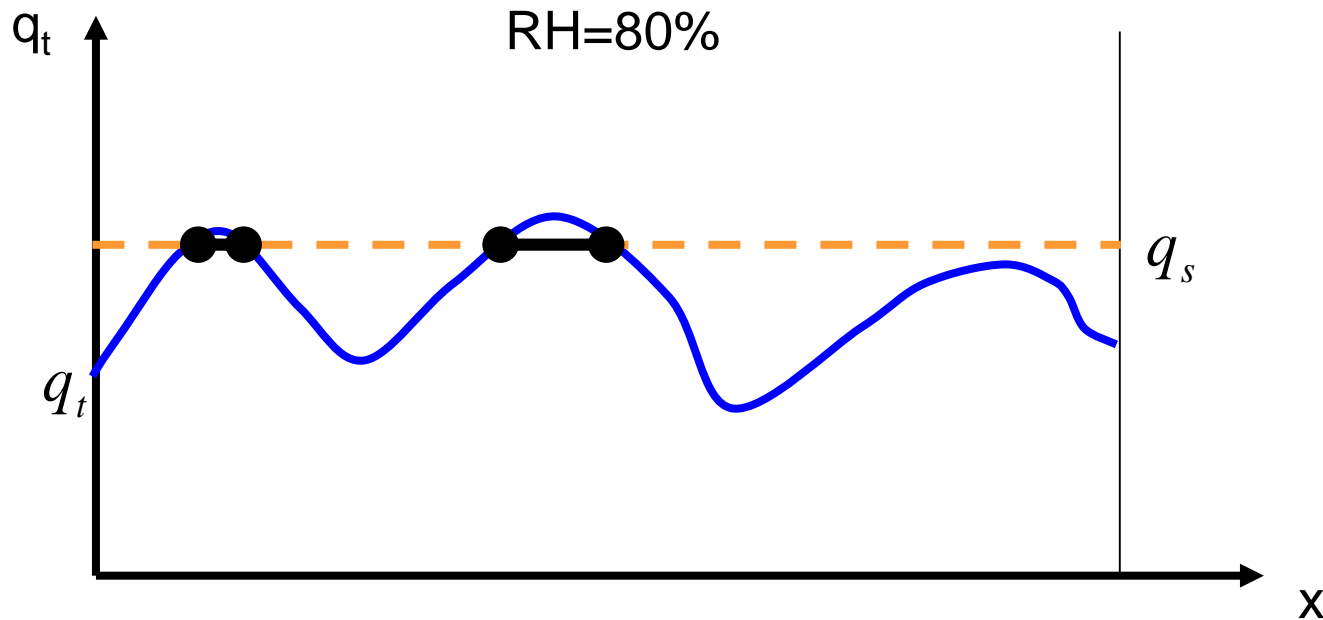


Take a grid cell with a certain (fixed) distribution of total water.

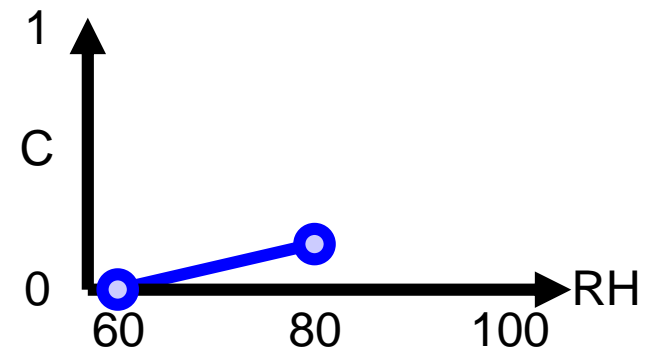
At low mean RH, the cloud cover is zero, since even the moistest part of the grid cell is subsaturated



# Simple Diagnostic Cloud Schemes: Relative Humidity Schemes

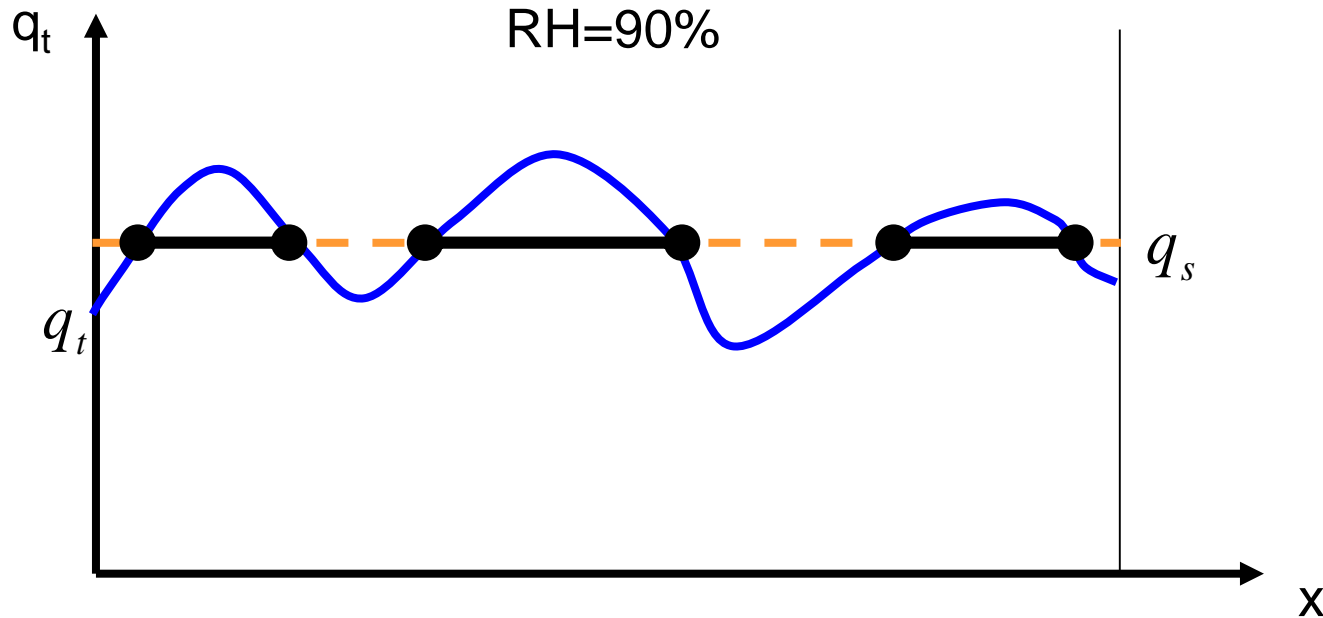


Add water vapour to the gridcell,  
the moistest part of the cell  
become saturated and cloud  
forms. The cloud cover is low.

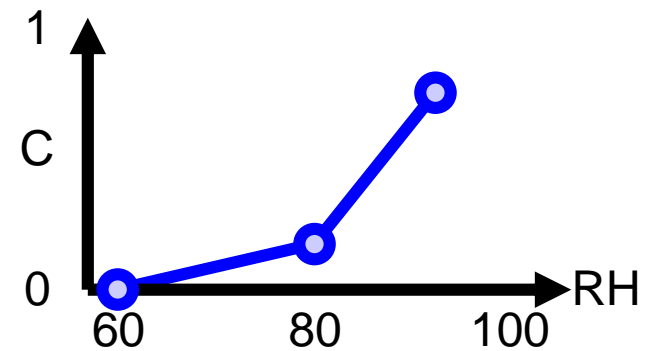




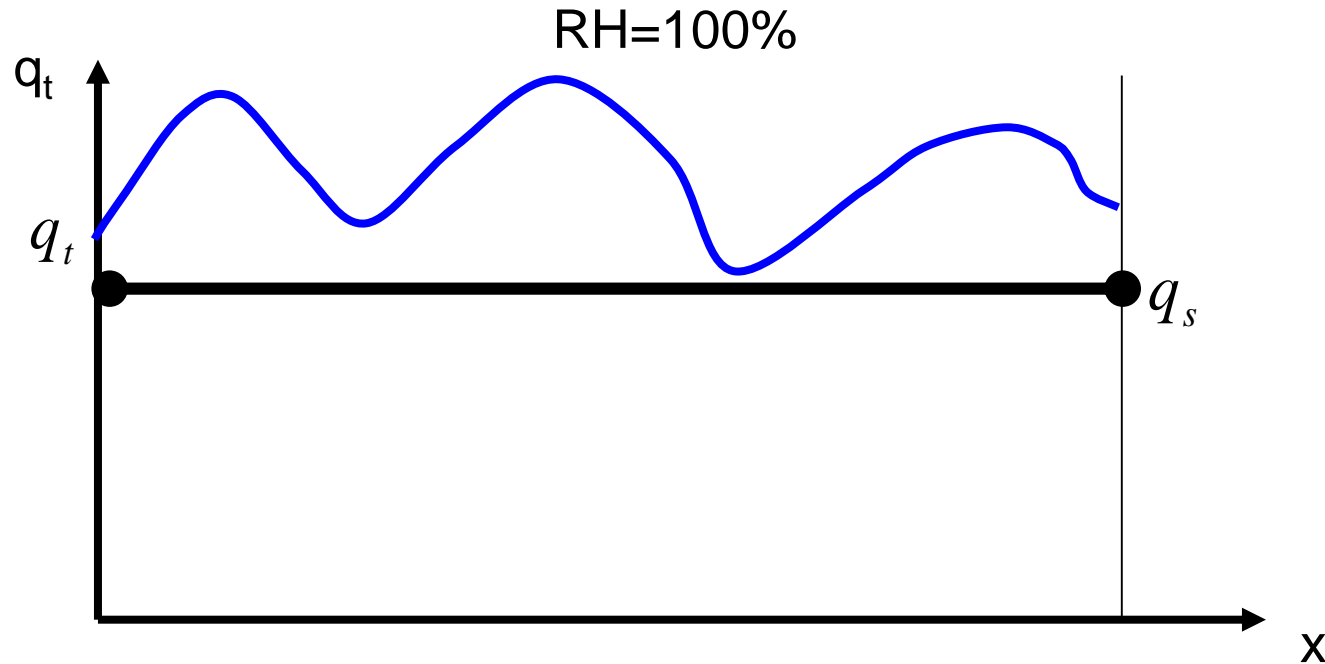
# Simple Diagnostic Cloud Schemes: Relative Humidity Schemes



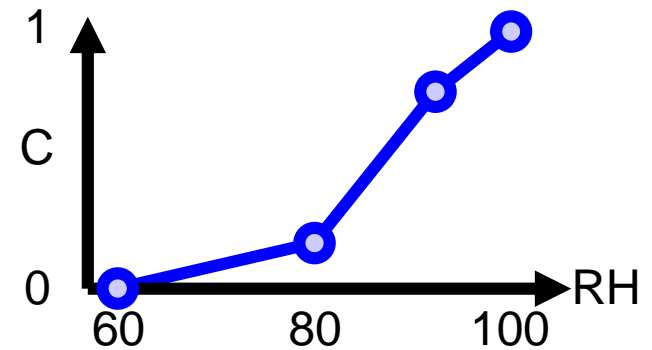
Further increases in RH  
increase the cloud cover



# Simple Diagnostic Cloud Schemes: Relative Humidity Schemes



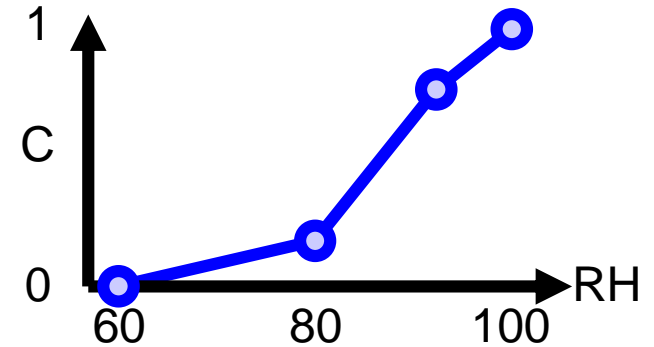
- The grid cell becomes overcast when  $RH=100\%$ , due to lack of supersaturation
- Diagnostic RH-based parametrization  $C = f(RH)$



# Diagnostic Relative Humidity Schemes



- Many schemes, from the 1970s onwards, based cloud cover on the relative humidity (RH)
- e.g. Sundqvist et al. MWR 1989:



$$C = 1 - \sqrt{\frac{1-RH}{1-RH_{crit}}}$$

 Remember this for later!

$RH_{crit}$  = critical relative humidity at which cloud assumed to form  
(= function of height, typical value is 60-80%)

# Diagnostic Relative Humidity Schemes



- Since these schemes form cloud when  $RH < 100\%$ , they implicitly assume subgrid-scale variability for total water,  $q_t$ , (and/or temperature,  $T$ ).
- However, the actual PDF (the shape) for these quantities and their variance (width) are often not known.
- They are of the form: “*Given a RH of  $X\%$  in nature, the mean distribution of  $q_t$  is such that, on average, we expect a cloud cover of  $Y\%$ ”.*”

# Diagnostic Relative Humidity Schemes

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- Advantages:
  - Better than homogeneous assumption, since clouds can form before grids reach saturation.
- Disadvantages:
  - Cloud cover not well coupled to other processes.
  - In reality, different cloud types with different coverage can exist with same relative humidity. This can not be represented.
- Can we do better?

# Diagnostic Relative Humidity Schemes



- Could add further predictors...

- e.g. Xu and Randall (1996) sampled cloud scenes from a 2D cloud resolving model to derive an empirical relationship with two predictors, relative humidity AND cloud condensate:

$$C = F(RH, q_c)$$

- e.g. scheme operational at ECMWF until 1995 (Slingo) adds dependence on vertical velocities,  $\omega$ , as well as RH for different cloud types:

$$C_m = \begin{cases} 0 & \omega \geq 0 \\ C_m^* \omega / \omega_{crit} & \omega_{crit} \leq \omega < 0 \\ C_m^* & \omega < \omega_{crit} \end{cases}$$

$$C_m^* = \left[ \max \left( \frac{RH - RH_{crit}}{1 - RH_{crit}}, 0 \right) \right]^2$$

- More predictors, more degrees of freedom = flexible
- But still do not know the form of the PDF (is cloud resolving model valid? Is it representative for all situations?)
- Can we do better?

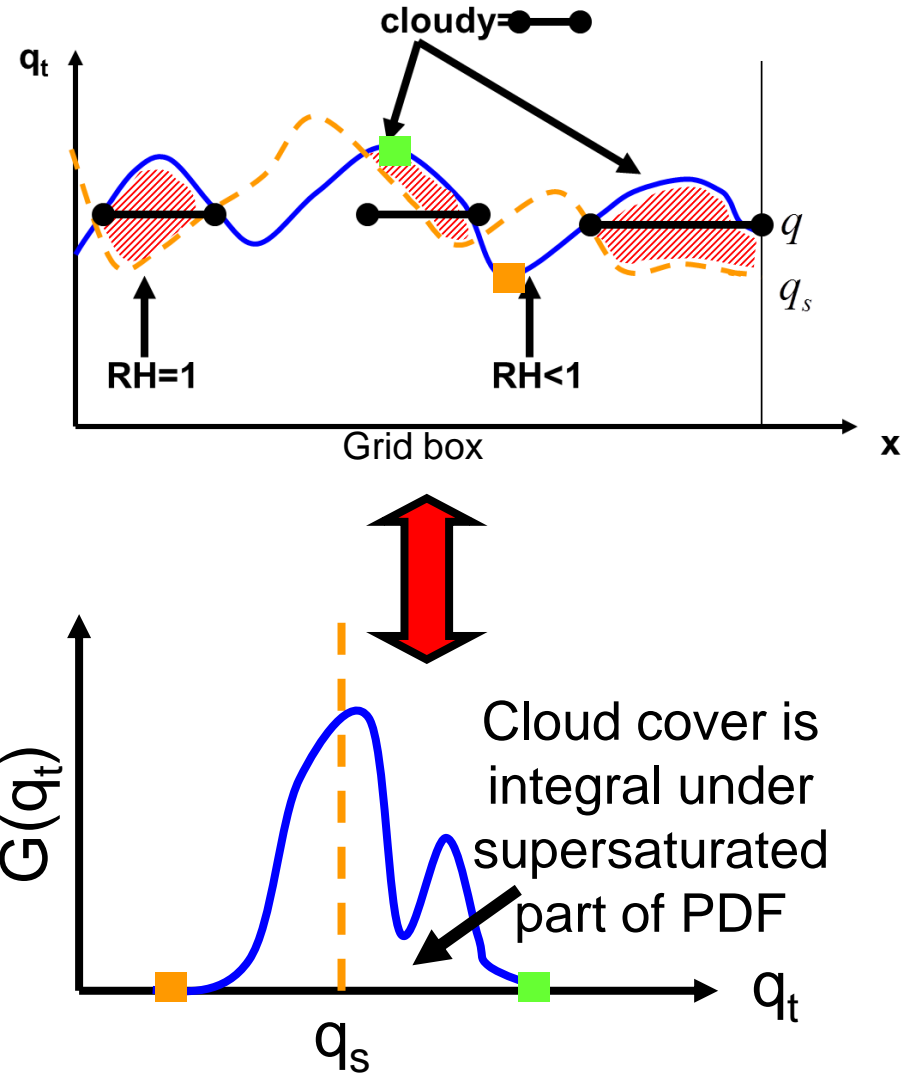


# Statistical PDF Schemes

- Statistical schemes explicitly specify the probability density function (PDF),  $G$ , for the total water  $q_t$  (and sometimes also temperature)

$$C = \int_{q_s}^{\infty} G(q_t) dq_t$$

$$q_c = \int_{q_s}^{\infty} (q_t - q_s) G(q_t) dq_t$$



# Statistical PDF Schemes

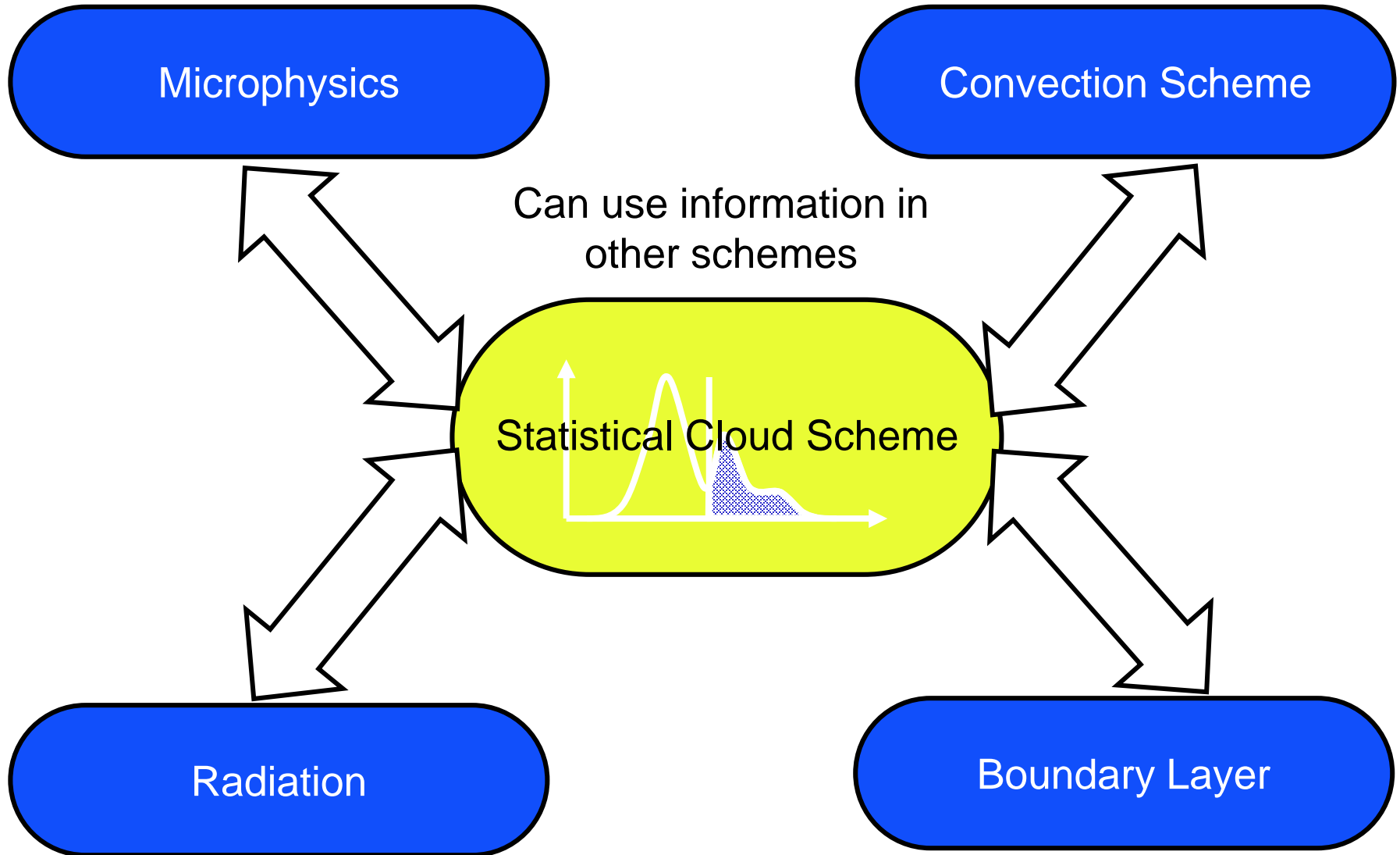


- Knowing the PDF has advantages:
  - Information concerning subgrid fluctuations of humidity and cloud condensate is available (for all parametrizations) , e.g.
    - More accurate calculation of radiative fluxes
    - Unbiased calculation of microphysical processes
  - Use of underlying PDF means cloud variables (condensate, cloud fraction) are always self-consistent.
  - Physically-based. Can evaluate with observations.

(Note, location of clouds within grid cell is still not known)



# Statistical PDF scheme: Consistency across parametrizations



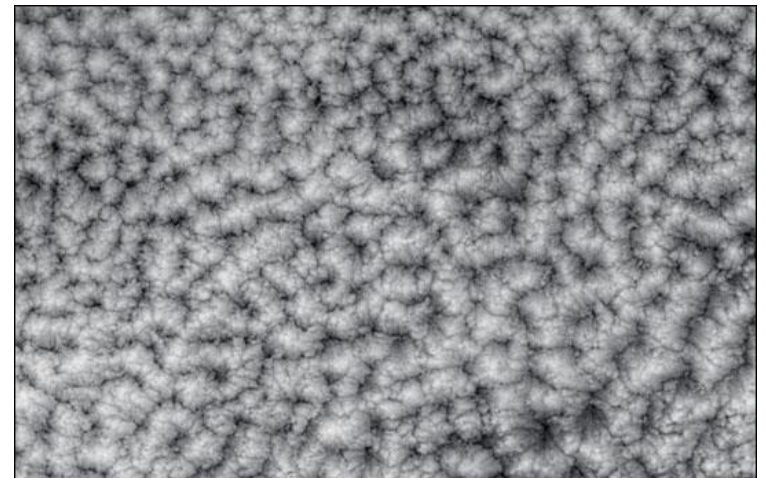
# Building a statistical cloud scheme



## What do we observe?

- Limited observations to determine  $q_t$  PDF
  - Aircraft data
    - limited coverage
  - Tethered balloon
    - boundary layer only
  - Satellite
    - difficulties resolving in vertical
    - no  $q_t$  observations
    - poor horizontal resolution
  - Ground-based radar/Raman Lidar
    - one location
- Cloud Resolving models have also been used
  - realism of microphysical parametrization?

Modis image from NASA website

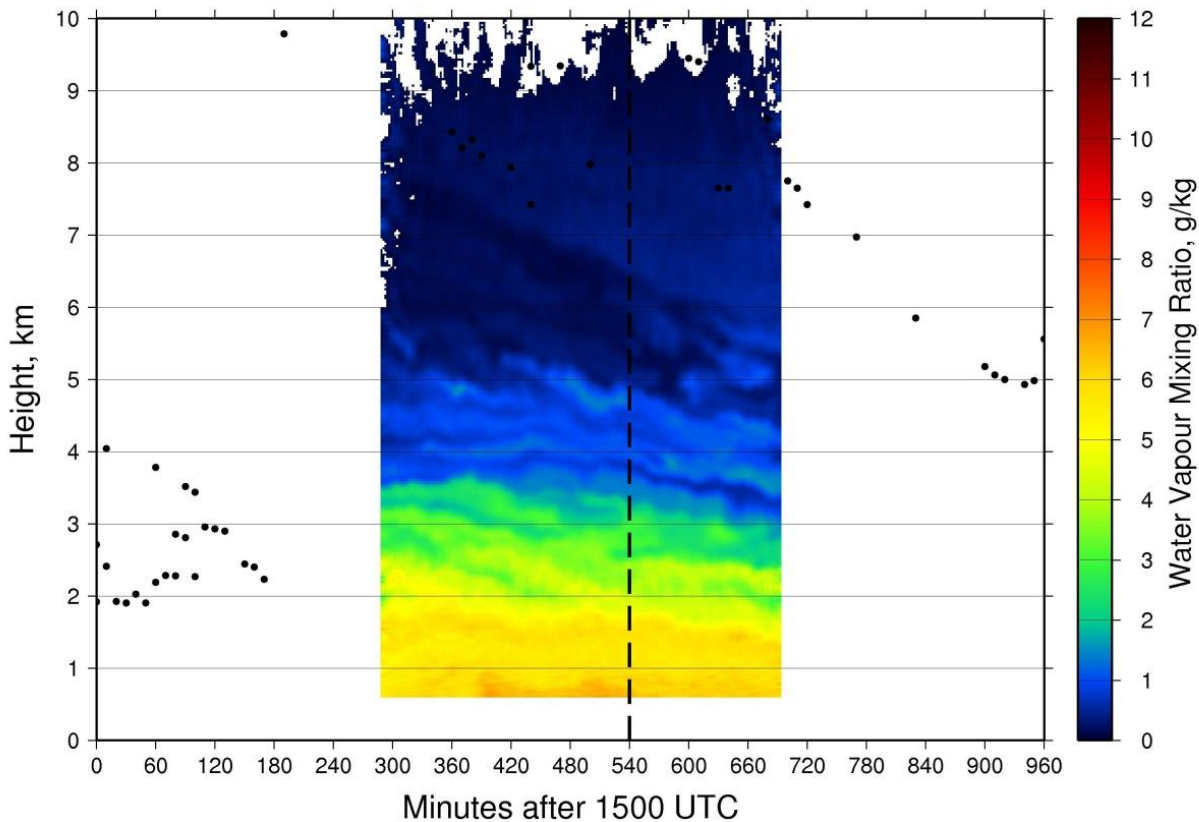


# Building a statistical cloud scheme

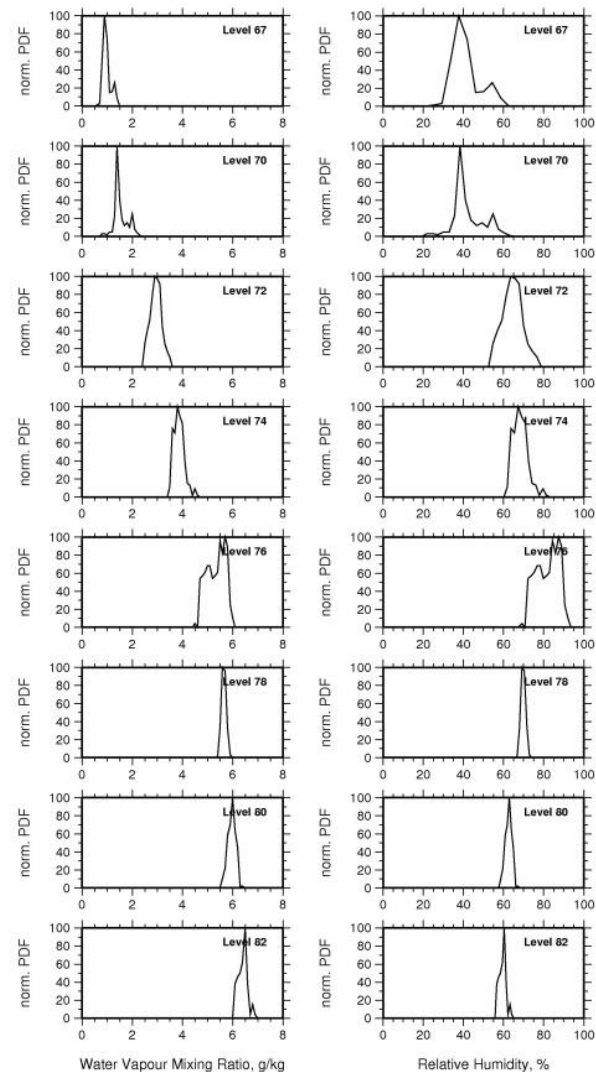


## Observed PDF of water vapour/RH Raman Lidar

Raman-Lidar RAMSES – 15./16. May 2008



16. May 2008 00:00 UTC



# Building a statistical cloud scheme



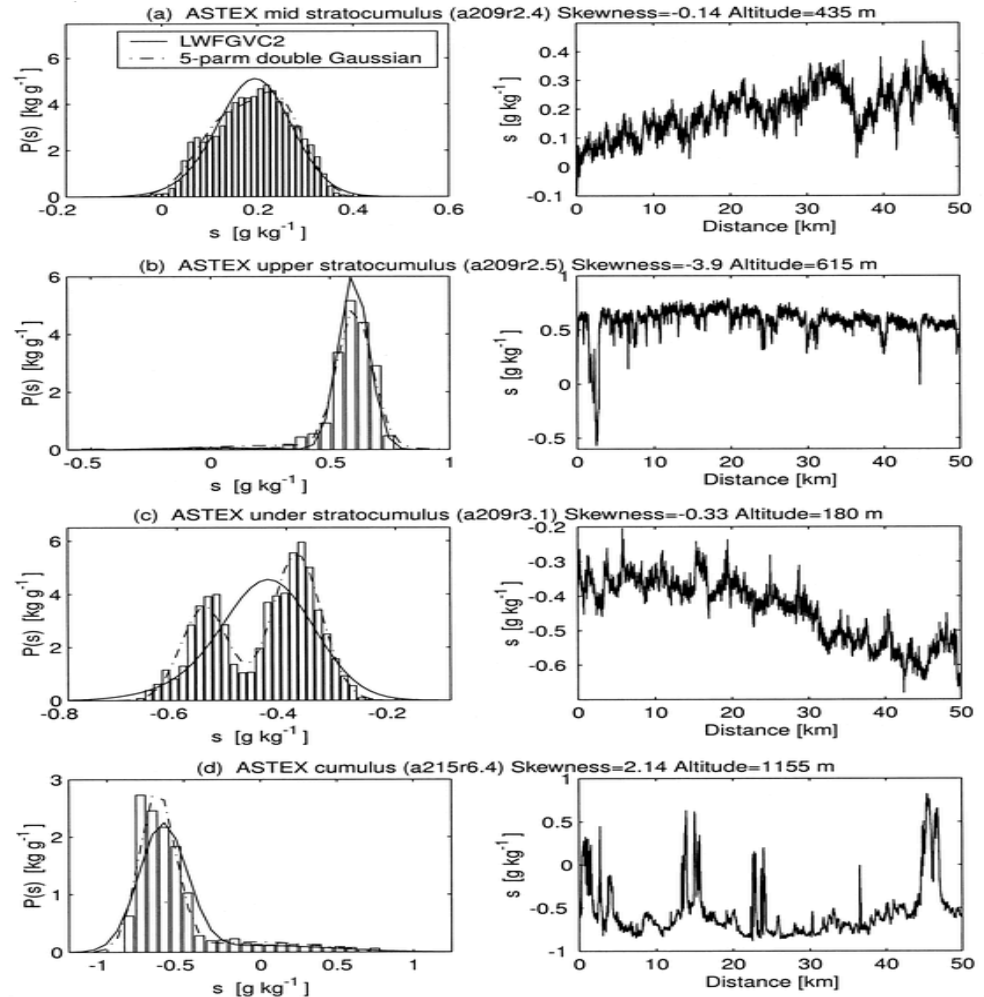
## Observed PDF example from aircraft

Example, aircraft data from Larson et al. 01/02

PDFs are mostly approximated by uni or bi-modal distributions, describable by a few parameters

PDF

Data

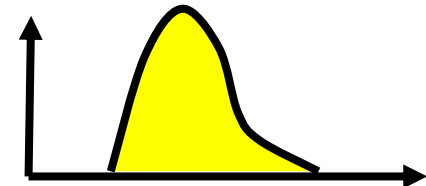
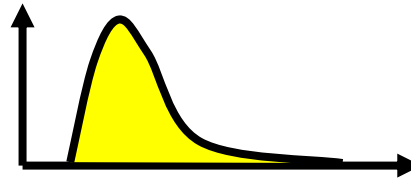
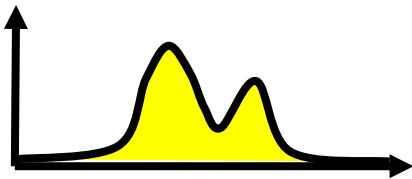
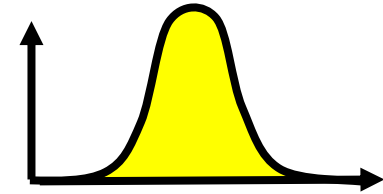
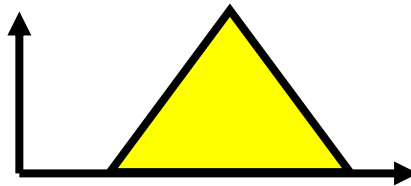


# Building a statistical cloud scheme



## Functional form

- Need to represent with a functional form, specify the:
  - (1) **PDF shape** (unimodal, bimodal, symmetrical, bounded?)
  - (2) **PDF moments** (mean, variance, skewness?)
  - (3) **Diagnostic or prognostic** (how many degrees of freedom?)

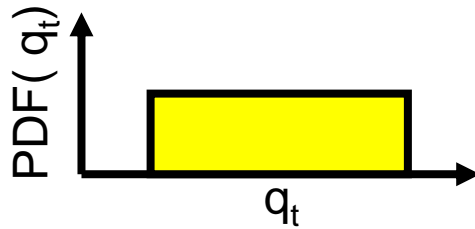


# Building a statistical cloud scheme



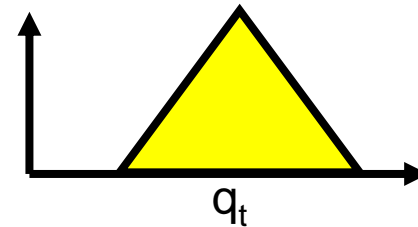
## (1) Specification of PDF shape

Many function forms have been used  
*symmetrical distributions:*



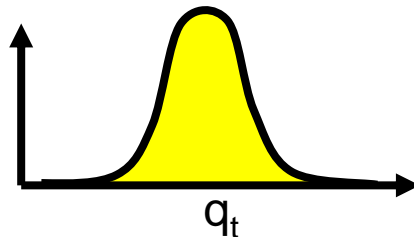
Uniform:

Letreut and Li (91)



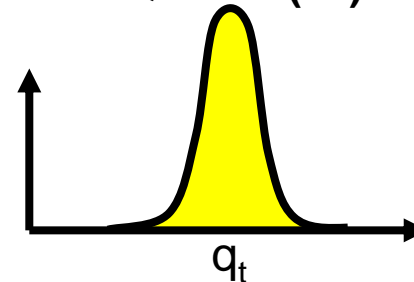
Triangular:

Smith QJRMS (90)



Gaussian:

Mellor JAS (77)



$s^4$  polynomial:

Lohmann et al. J. Clim (99)

Bounded

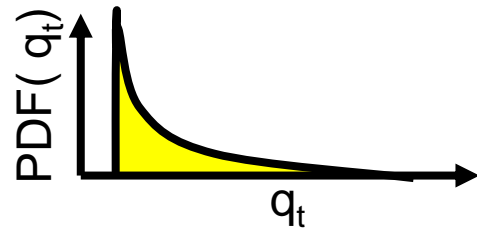
Unbounded:  
Can clip, but need  
additional  
parameters

# Building a statistical cloud scheme

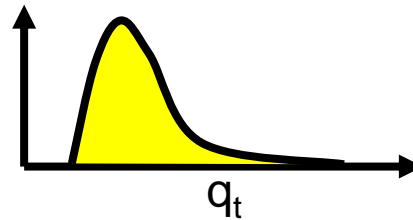


## (1) Specification of PDF shape

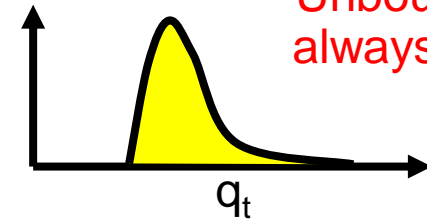
### *skewed distributions:*



Exponential:  
Sommeria and Deardorff JAS  
(77)

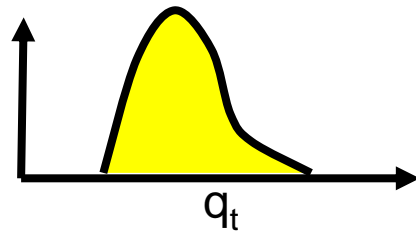


Lognormal:  
Bony & Emanuel  
JAS (01)

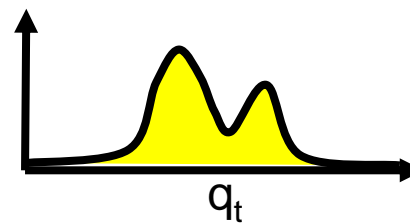


Gamma:  
Barker et al. JAS (96)

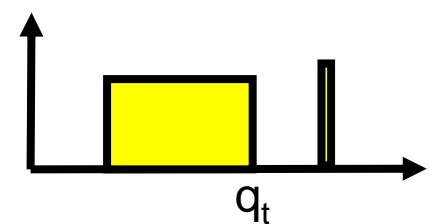
Unbounded,  
always skewed



Beta:  
Tompkins JAS (02)



Double Gaussian:  
Lewellen and Yoh JAS (93),  
Golaz et al. JAS 2002  
(CLUBB)



Uniform-delta:  
Tiedtke (1993)  
(ECMWF)

Bounded, symmetrical or skewed

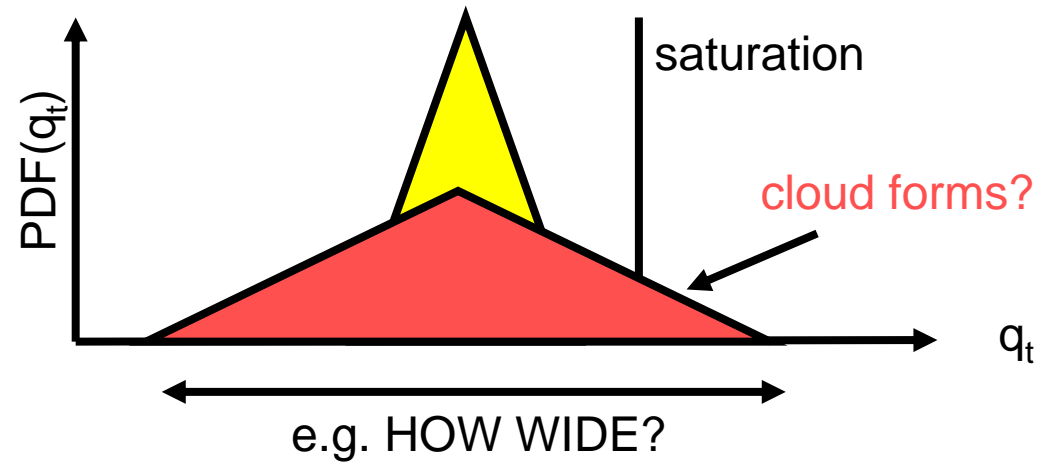
# Building a statistical cloud scheme



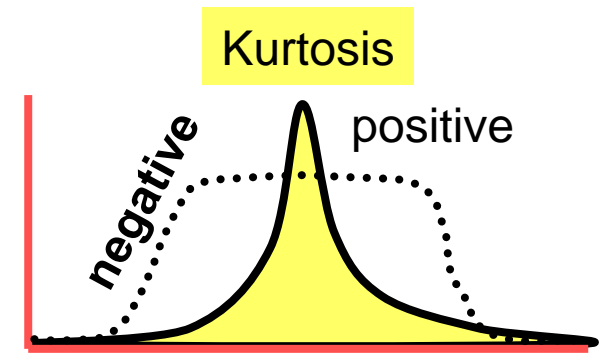
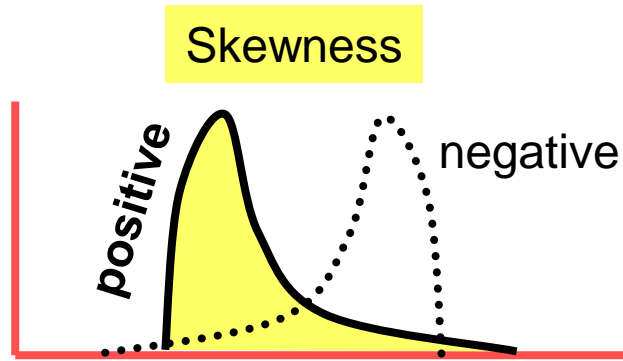
## (2) Specification of PDF moments

Need also to determine the moments of the distribution:

- Variance (Symmetrical PDFs)
- Skewness (Higher order PDFs)
- Kurtosis (4-parameter PDFs)



Moment 1=MEAN  
Moment 2=VARIANCE  
Moment 3=SKEWNESS  
Moment 4=KURTOSIS



Functional form – needs to fit data but be sufficiently simple

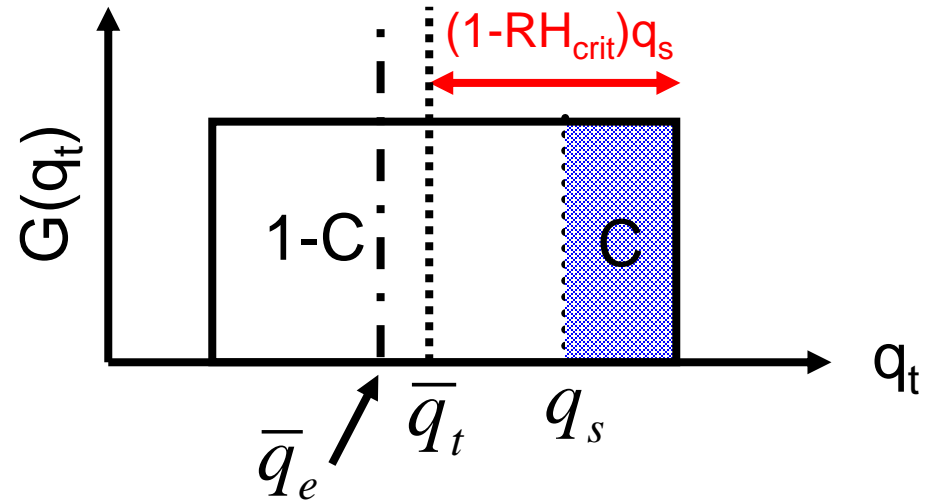


# Building a statistical cloud scheme



## (3) Diagnostic or prognostic PDF moments

- Some schemes fix the moments (diagnostic e.g. Smith 1990) based on critical RH at which clouds assumed to form.
- Some schemes predict the moments (prognostic, e.g. Tompkins 2002). Need to specify sources and sinks.
- If moments (variance, skewness) are fixed, then statistical schemes are identically equivalent to a RH formulation
- e.g. uniform  $q_t$  distribution = Sundqvist formulation



$$\bar{q}_e = q_s (1 - (1 - RH_{crit})(1 - C))$$

$$\bar{q}_v = Cq_s + (1 - C)\bar{q}_e$$

$$RH = \frac{\bar{q}_v}{q_s} = 1 - (1 - RH_{crit})(1 - C)^2$$

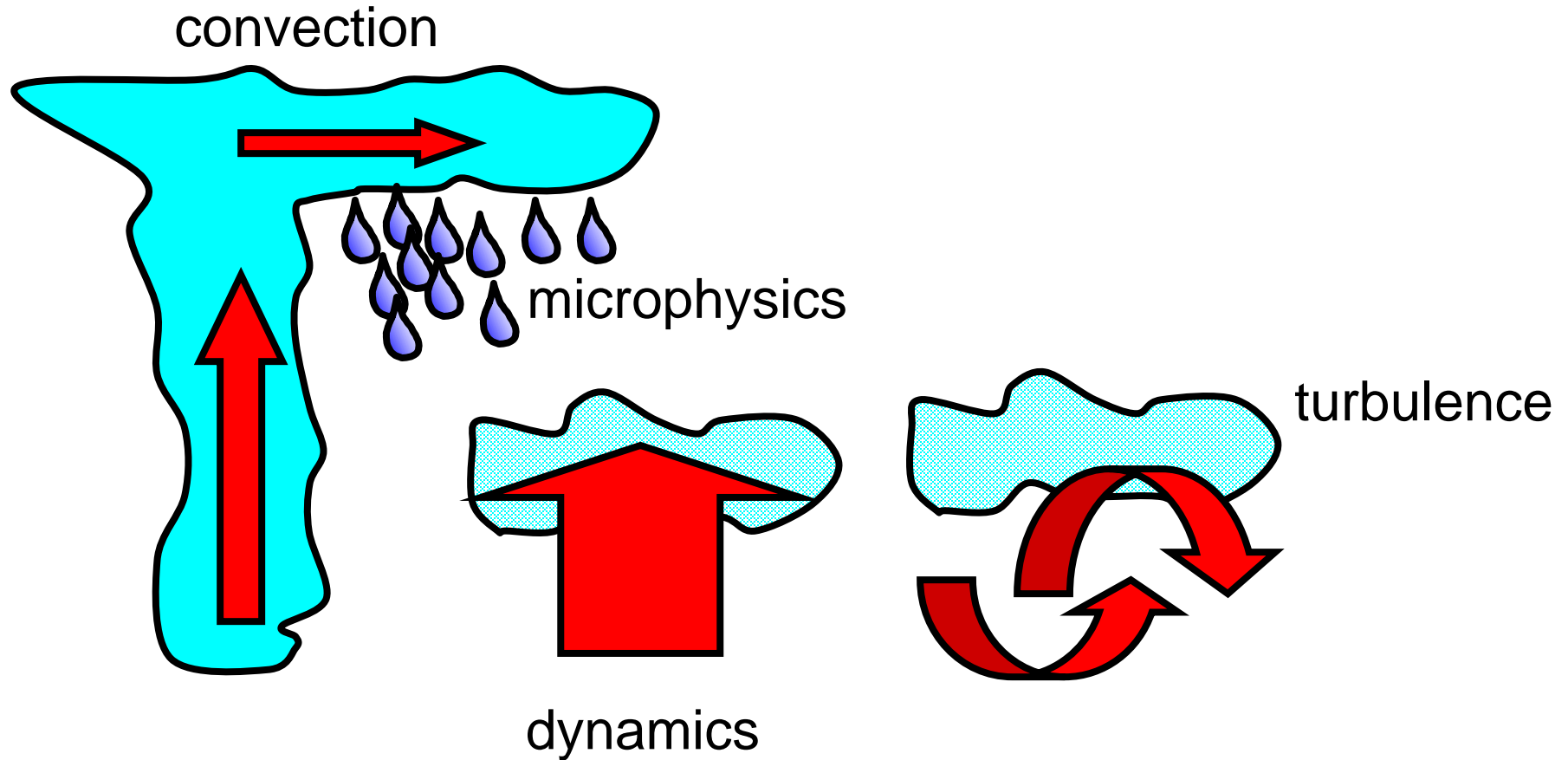
Sundqvist formulation!!!

$$\therefore C = 1 - \sqrt{\frac{1 - RH}{1 - RH_{crit}}}$$

# Building a statistical cloud scheme

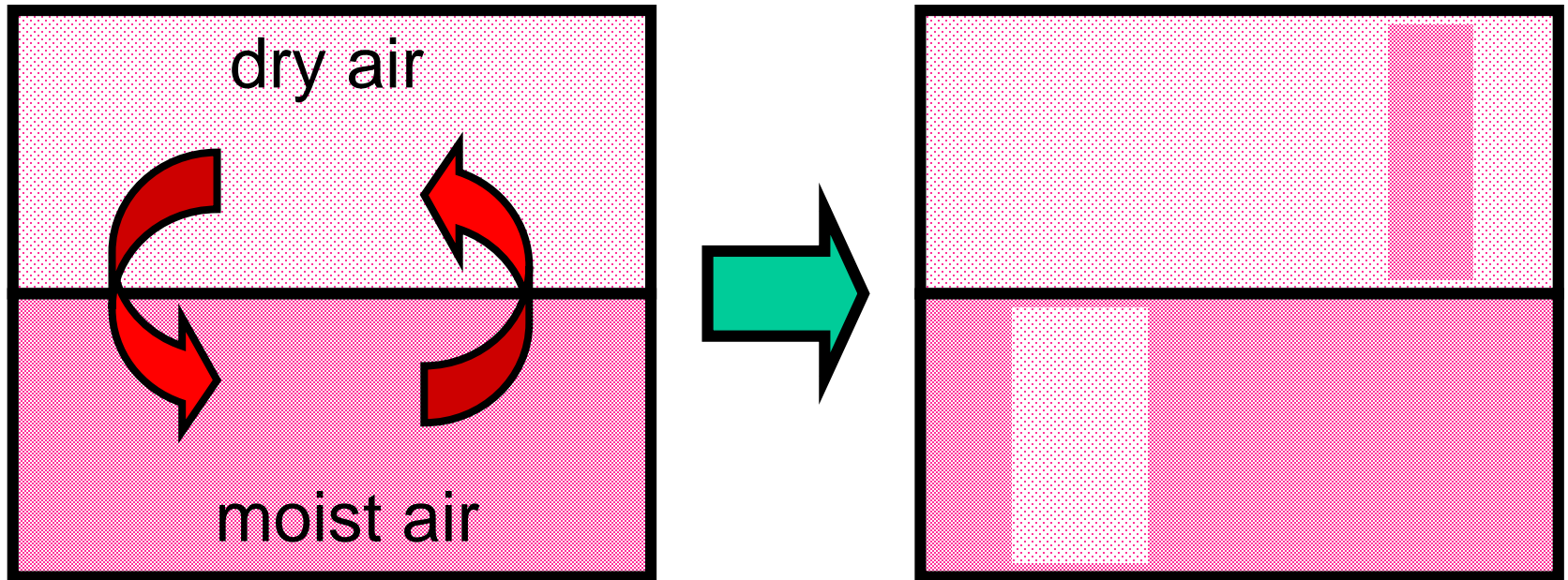


## Processes that can affect PDF moments



# Example: Turbulence

In presence of vertical gradient of total water, turbulent mixing can increase horizontal variability



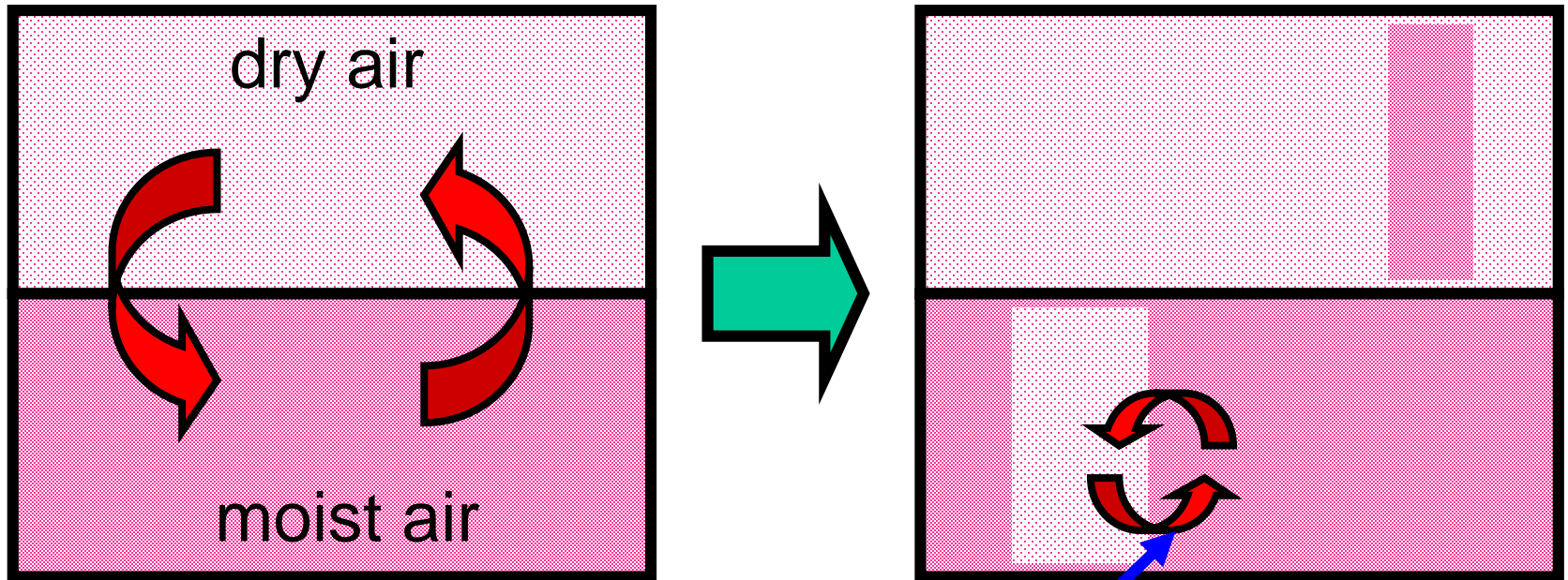
Rate of change  
of total water  
variance

$$\frac{d \overline{q_t'^2}}{dt} = -2 \overline{w' q_t'} \frac{d \overline{q_t}}{dz}$$



# Example: Turbulence

In presence of **vertical gradient** of total water, turbulent mixing can **increase horizontal variability**



while **subgrid mixing** in the **horizontal plane** naturally **reduces** the **horizontal variability**

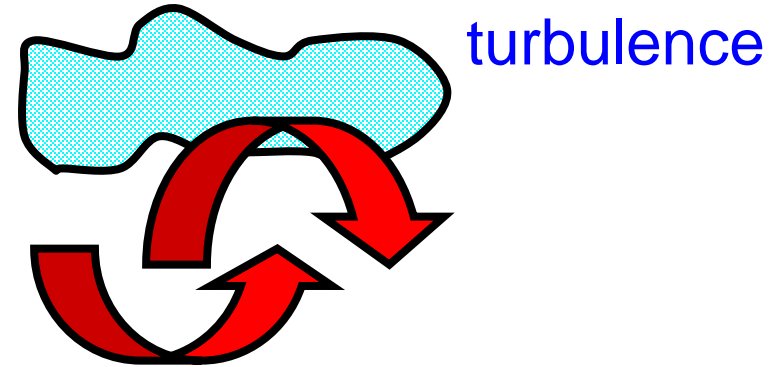
$$\frac{d\overline{q_t'^2}}{dt} = -\frac{q_t'^2}{\tau}$$

# Building a statistical cloud scheme



## Predicting change of $q_t$ variance due to turbulence

If a process is fast compared to a GCM timestep, an equilibrium can be assumed, e.g. turbulence



$$\frac{d\overline{q_t'^2}}{dt} = \underbrace{-2\overline{w'q_t'}}_{\text{Source}} \frac{d\overline{q_t}}{dz} - \underbrace{\frac{q_t'^2}{\tau}}_{\text{Dissipation}} \xrightarrow{\text{Local equilibrium}} q_t'^2 = -\tau 2\overline{w'q_t'} \frac{d\overline{q_t}}{dz}$$

Example: Ricard and Royer, Ann Geophys, (93), Lohmann et al. J. Clim (99)

- **Disadvantage:**

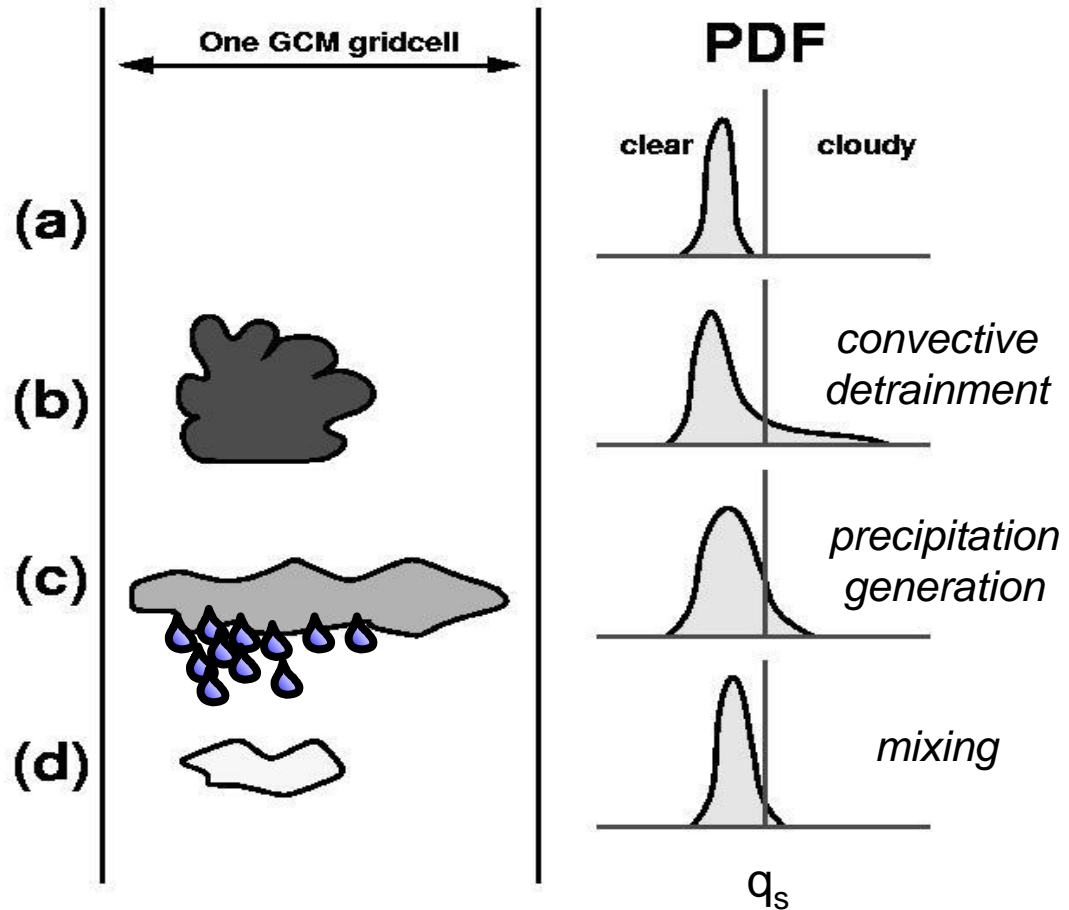
- Can give good estimate in boundary layer, but above, other processes will determine variability, that evolve on slower timescales

# Building a statistical cloud scheme



## Example: Tompkins (2002) prognostic PDF

- Tompkins (2002) prognostic statistical scheme (implemented in ECHAM5 climate GCM).
- Prognostic equations are introduced for variables representing the **mean**, **variance** and **skewness** of the total water PDF (Beta fn)
- Some of the sources and sinks are rather ad-hoc in their derivation!

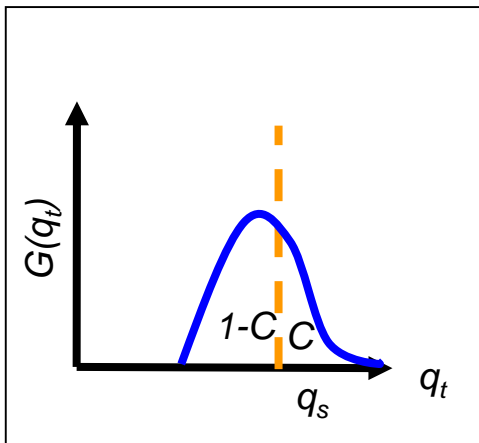


# The ECMWF Cloud Scheme

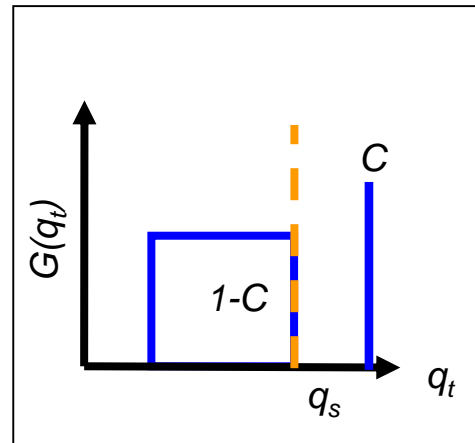


## Comparison with Tompkins prognostic PDF scheme

Tompkins (2002)



Tiedtke(1993) in ECMWF IFS



A bounded beta function with positive skewness.

Effectively 3 prognostic variables:

Mean  $q_t$   
Variance of PDF  
Skewness of PDF

A mixed 'uniform-delta' total water distribution is assumed for the condensation process.

3 prognostic variables:

Humidity,  $q_v$   
Cloud condensate,  $q_c$   
Cloud fraction,  $C$

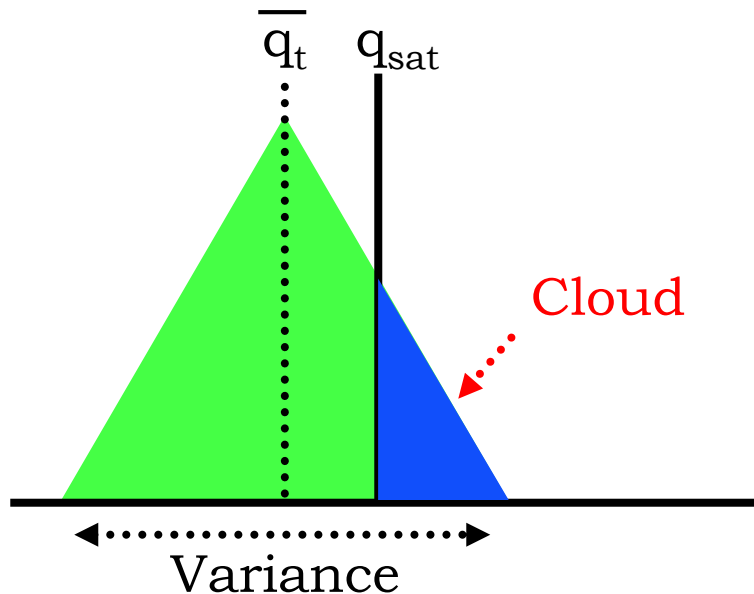
Same degrees of freedom ?

# Prognostic statistical PDF scheme: Which prognostic variables/equations?

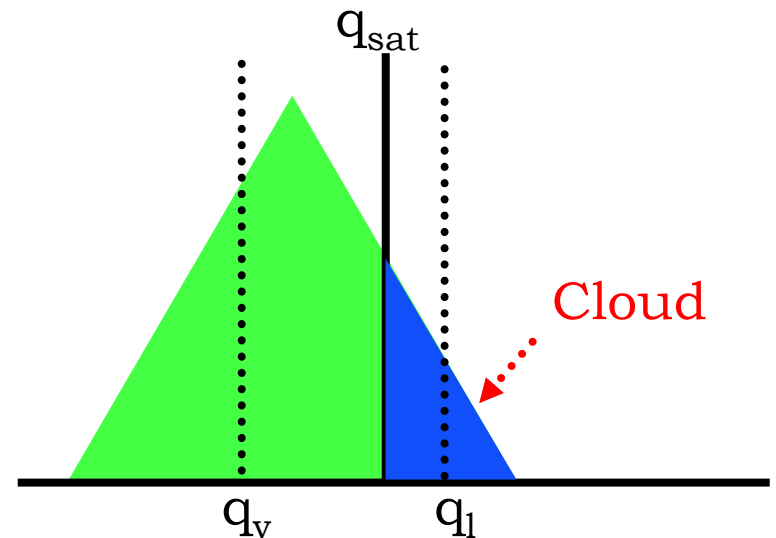


Take a 2 parameter distribution & partially cloudy conditions

- (1) Can specify distribution with
- (a) Mean
  - (b) Variance of total water



- (2) Can specify distribution with
- (a) Water vapour
  - (b) Cloud water mass mixing ratio



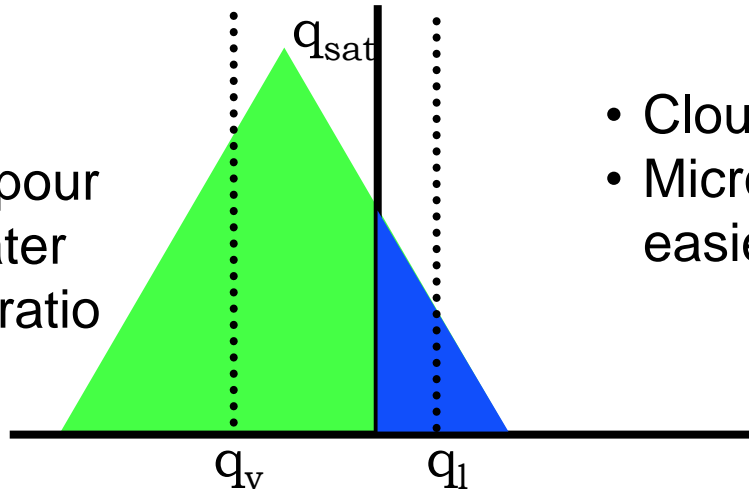


# Prognostic statistical scheme:

## (1) Water vapour and cloud water ?



(a) Water vapour  
(b) Cloud water  
mass mixing ratio

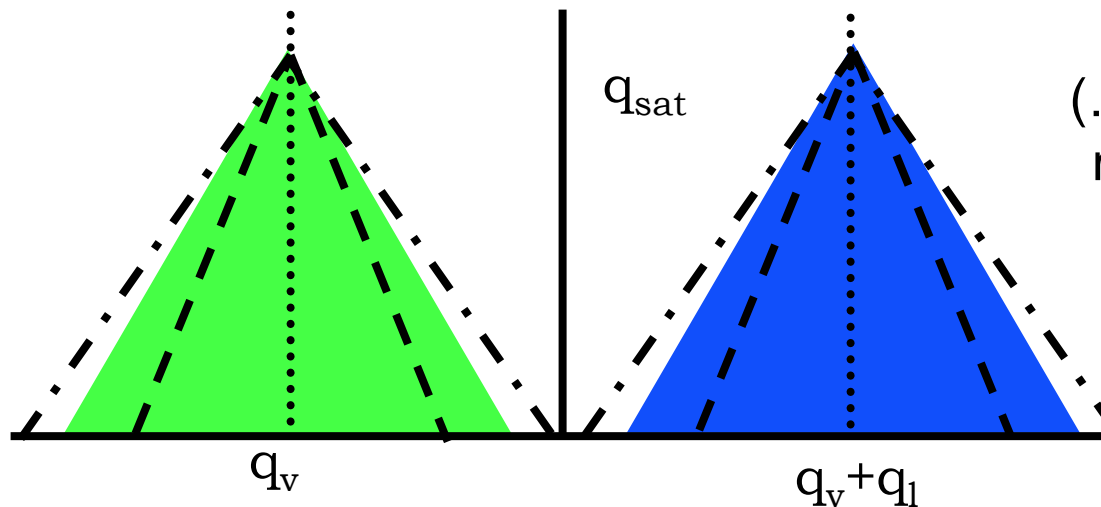


- Cloud water budget conserved.
- Microphysical sources and sinks easier to parametrize.

But problems arise in...

Clear sky  
conditions  
(turbulence)

Need to specify  
width,  $RH_{crit}$

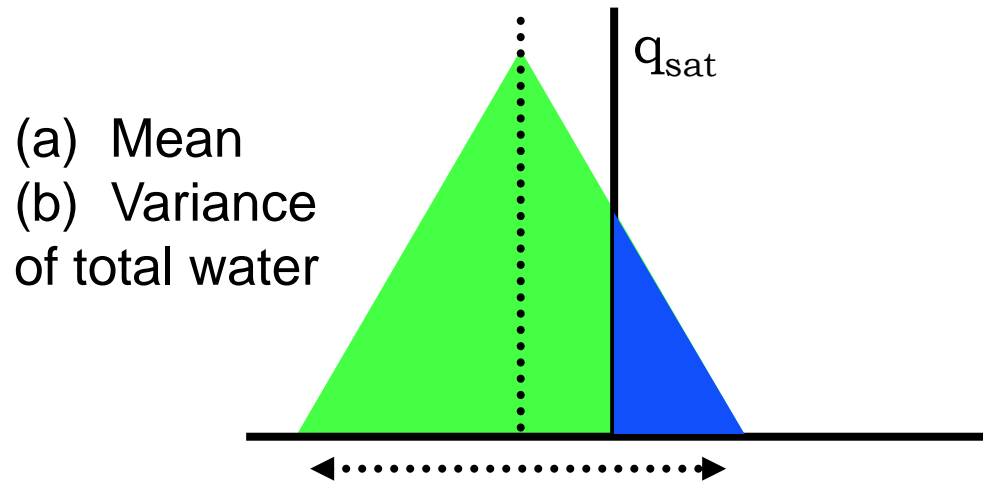


Overcast  
conditions  
(...convection +  
microphysics)

# Prognostic statistical scheme:

## (2) Total water mean and variance ?

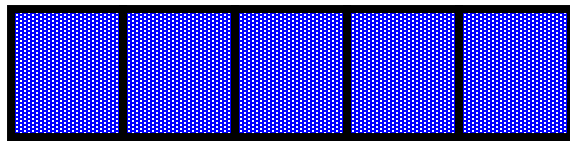
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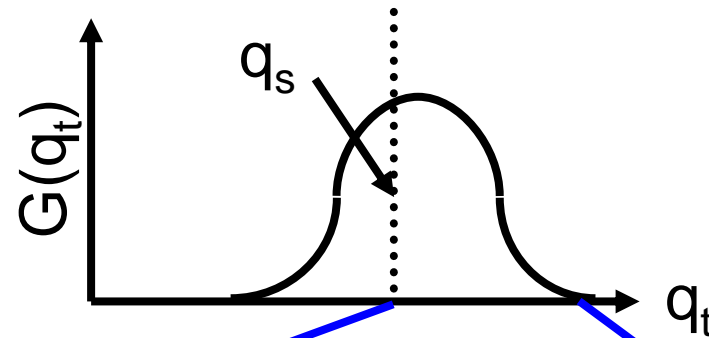
- “Cleaner solution”.
- But conservation of liquid water may be difficult (eg. advection)
- Parametrizing microphysics sources, sinks can be more difficult.

# Cloud inhomogeneity in microphysics

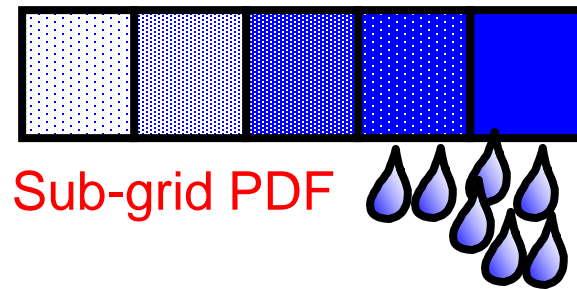
Many current microphysical schemes use the grid-mean or cloud fraction cloud mass (i.e: neglect in-cloud variability)



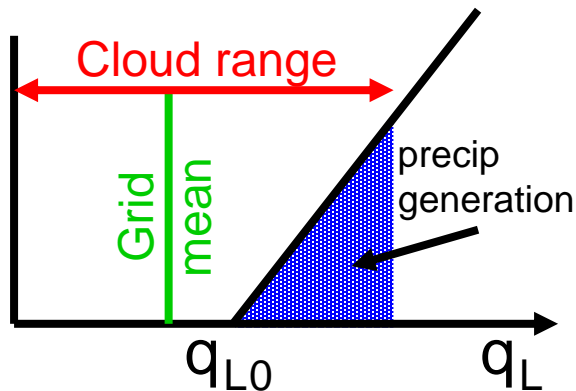
Homogeneous



cloud



Sub-grid PDF



For example, Kessler autoconversion scheme:  
Result is not equal in the two cases since microphysical processes are non-linear  
**In the homogeneous case the grid mean cloud is less than threshold and gives zero precipitation formation**

# Prognostic statistical PDF scheme: Knowing the PDF....

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- **Advantages**

- Information concerning subgrid fluctuations of humidity and cloud condensate is available (for all parametrizations)
- Use of underlying PDF means cloud variables (condensate, cloud fraction) are always self-consistent.

- **Challenges...**

- Deriving these sources and sinks rigorously is difficult, especially for higher order moments for more complex PDFs!
- Limited observations to define PDF
- If variance and skewness are used instead of cloud water and humidity, conservation of the latter is not ensured.
- Is a fixed PDF shape, even with variable moments, able to represent the wide range of variability in the atmosphere?
- How do we treat the ice phase, supersaturation, mixed-phase cloud, sedimentation? These are important questions!

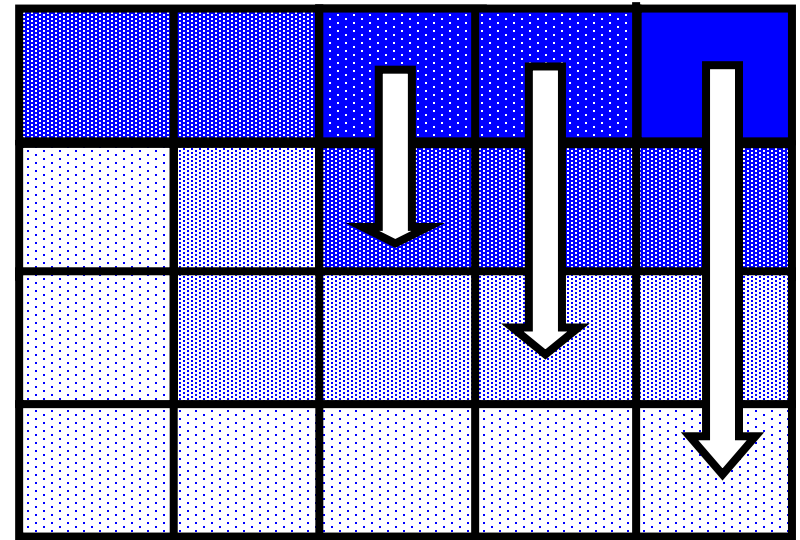
# Subgrid heterogeneity:



## How do we treat subgrid heterogeneity and sedimentation ?

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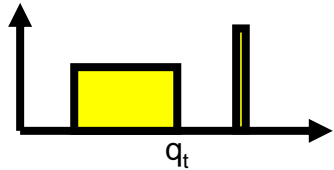
- Analytically, can quickly get intractable
- Subcolumn approach, as for radiation? but computationally expensive.
- Memory of subgrid precipitation fluxes?



# Sub-grid cloud parametrization



## Current status in GCMs...?



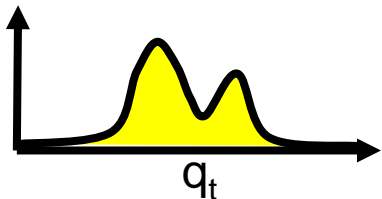
Uniform-delta:  
Tiedtke (1993)

- The ECMWF global NWP model has prognostic water vapour, cloud water and cloud fraction. With a uniform function for heterogeneity in the clear air and a delta function (homogeneous) in-cloud.

- The UK Met Office global NWP model (PC2 scheme) also has prognostic water vapour, cloud water and cloud fraction.



- Many other operational global NWP/climate models have diagnostic sub-grid cloud schemes, e.g. NCEP GFS: Sundquist et al. (1989)



Double-Gaussian  
(CLUBB)

- Research is ongoing for statistical schemes with prognostic PDF moments (e.g. Tompkins scheme tested in ECHAM, CLUBB tested in CAM).

# Summary



## Representing subgrid scale heterogeneity

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- Representing sub-gridscale heterogeneity in GCMs is important for cloud formation, microphysical processes, radiation etc.
- Many different approaches have been tried, with varying degrees of complexity to represent the variability observed in the atmosphere.
- More degrees of freedom allow greater flexibility to represent the real atmosphere, but we need to have enough knowledge/information to understand and constrain the problem (form of pdf/sources/sinks)!
- Cloud, convection and BL turbulence are all part of the subgrid heterogeneity – active research into unified schemes.
- Statistical prognostic PDF schemes have many advantages but challenges remain for clouds other than warm-phase boundary layer cloud!
- However, we should continue to strive for a **consistent representation of this heterogeneity** for all processes in the model.



# References

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- Larson, V. E., R. Wood, P. R. Field, J.-C. Golaz, T. H. Vonder Haar, and W. R. Cotton, (2001). Small-Scale and Mesoscale Variability of Scalars in Cloudy Boundary Layers: One-Dimensional Probability Density Functions. *J. Atmos. Sci.*, **58**, 1978-1994
- Sundqvist, H. Berge, E., Kristjansson, J. E., 1989: Condensation and cloud parametrization studies with a mesoscale numerical weather prediction model. *Mon. Wea. Rev.*, **177**, 1641-1657.
- Tompkins, A. M., (2002). A prognostic parametrization for the subgrid-scale variability of water vapor and clouds in large-scale models and its use to diagnose cloud cover. *J. Atmos. Sci.*, **59**, 1917-1942.
- Wood, R., Field, P. R., (2000). Relationships between total water, condensed water and cloud fraction in stratiform clouds examined using aircraft data. *J. Atmos. Sci.*, **57**, 1888-1905.
- Xu, K. M., and D. A. Randall, (1996). A semi-empirical cloudiness parameterization for use in climate models. *J. Atmos. Sci.*, **53**, 3084-3102.