Numerical Weather Prediction
Parameterization of diabatic processes

Convection II: The mass flux approach and the IFS scheme

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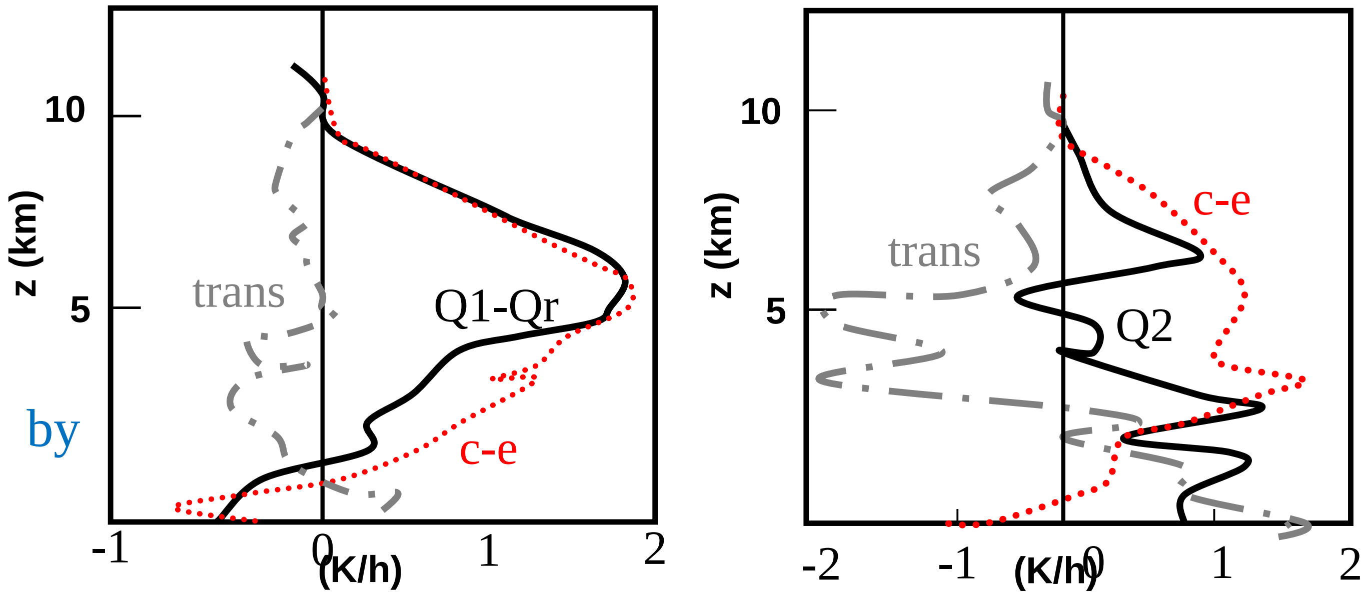
ECMWF

Task of convection parametrization: Q1 and Q2

To calculate the collective effects of an ensemble of convective clouds in a model column as a function of grid-scale variables. Hence parameterization needs to describe Condensation/Evaporation and Transport

$$Q_{1C} \equiv Q_1 - Q_R \equiv L(\bar{c} - \bar{e}) - \frac{\partial \overline{\omega' s'}}{\partial p}$$



Q1c is dominated by condensation term

but for Q2 the transport and condensation terms are equally important

Caniaux, Redelsperger, Lafore, JAS 1994

Types of convection schemes

- Schemes based on moisture budgets
  - Kuo, 1965, 1974, *J. Atmos. Sci.*
- Adjustment schemes
  - moist convective adjustement, Manabe, 1965, *Mon. Wea. Rev.*
  - penetrative adjustment scheme, Betts and Miller, 1986, *Quart. J. Roy. Met. Soc.*, Betts-Miller-Janic
- Mass-flux schemes (bulk+spectral)
  - entraining plume - spectral model, Arakawa and Schubert, 1974, Fraedrich (1973,1976), Neggers et al (2002), Cheinet (2004), *all J. Atmos. Sci.* ,
  - Entraining/detraining plume - bulk model, e.g., Bougeault, 1985, *Mon. Wea. Rev.*, Tiedtke, 1989, *Mon. Wea. Rev.*; Gregory and Rowntree, 1990, *Mon. Wea . Rev.*; Kain and Fritsch, 1990, *J. Atmos. Sci.*, Donner , 1993 *J. Atmos. Sci.*; Bechtold et al 2001, *Quart. J. Roy. Met. Soc.*; Park, *2014. J. Atmos. Sci.*
  - episodic mixing, Emanuel, 1991, *J. Atmos. Sci.*

The mass-flux approach

$$Q_{1C} \equiv L(\bar{c} - \bar{e}) - \frac{\partial \overline{\omega' s'}}{\partial p}$$

Condensation term

Eddy transport term

Aim: Look for a simple expression of the eddy transport term

$$\overline{\omega' \Phi'} = ?$$

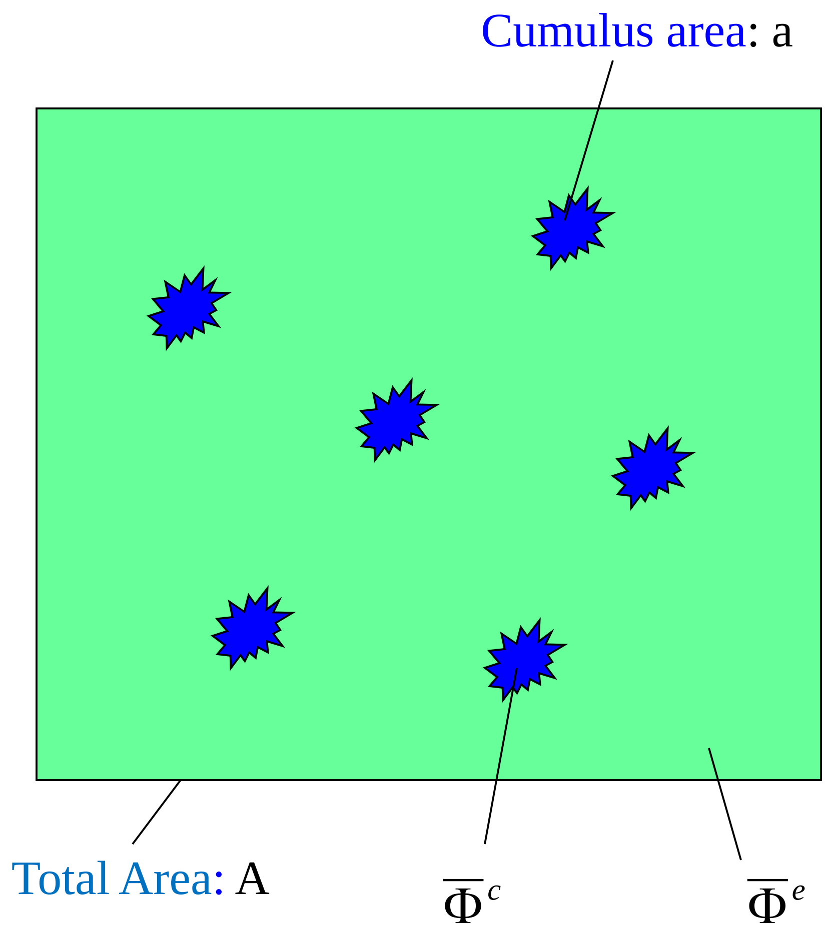
The mass-flux approach

Reminder:
$$\Phi = \overline{\Phi} + \Phi' \qquad \text{with} \qquad \overline{\Phi'} = 0$$

Hence
$$\begin{aligned}\overline{\omega\Phi} &= \overline{\left(\overline{\omega} + \omega'\right)\left(\overline{\Phi} + \Phi'\right)} \\ &= \overline{\overline{\omega}\,\overline{\Phi}} + \underset{\overset{\|}{0}}{\overline{\overline{\omega}\Phi'}} + \underset{\overset{\|}{0}}{\overline{\omega'\overline{\Phi}}} + \overline{\omega'\Phi'}\end{aligned}$$

and therefore
$$\overline{\omega\Phi} = \overline{\omega}\,\overline{\Phi} + \overline{\omega'\Phi'}$$
$$\overline{\omega'\Phi'} = \overline{\omega\Phi} - \overline{\omega}\,\overline{\Phi}$$

The mass-flux approach:
Cloud – Environment decomposition



Fractional coverage with cumulus elements:

$$\sigma = \frac{a}{A}$$

Define area average:

$$\overline{\Phi} = \sigma \overline{\Phi}^c + (1 - \sigma) \overline{\Phi}^e$$

The mass-flux approach:
Cloud-Environment decomposition

With the above:

$$\overline{\omega\Phi} = \sigma\overline{\omega\Phi}^c + (1-\sigma)\overline{\omega\Phi}^e$$

Average over cumulus elements Average over environment

$$\overline{\omega\Phi}^c = \overline{\omega}^c\overline{\Phi}^c + \cancel{\overline{\omega''\Phi''}^c}$$

and

$$\overline{\omega\Phi}^e = \overline{\omega}^e\overline{\Phi}^e + \cancel{\overline{\omega''\Phi''}^e}$$

Neglect subplume variations : (1) The top hat assumption

(see also Siebesma and Cuijpers, JAS 1995 for a discussion of the validity of the top-hat assumption)

The mass-flux approach:

$$\overline{\omega'\Phi'} = \overline{\omega\Phi} - \bar{\omega}\bar{\Phi} = \sigma\overline{\omega\Phi}^{c} + (1-\sigma)\overline{\omega\Phi}^{e} - \bar{\omega}\bar{\Phi}$$

Use Reynolds averaging again for cumulus elements and environment separately:

$$= \sigma\overline{\omega\Phi}^{c} + (1-\sigma)\overline{\omega\Phi}^{e} - (\sigma\overline{\omega}^{c} + (1-\sigma)\overline{\omega}^{e})\bar{\Phi}$$

(1) top hat approximation

$$= \sigma\bar{\omega}^{c}\bar{\Phi}^{c} + (1-\sigma)\bar{\omega}^{e}\bar{\Phi}^{e} - (\sigma\overline{\omega}^{c} + (1-\sigma)\overline{\omega}^{e})\bar{\Phi}$$

$$= \sigma\bar{\omega}^{c}(\bar{\Phi}^{c} - \bar{\Phi}) + (1-\sigma)\bar{\omega}^{e}(\bar{\Phi}^{e} - \bar{\Phi})$$

Either drop this term (small area approximation) or

further expanding, for your exercise!

The mass-flux approach:

$$\overline{\omega'\Phi'} = \overline{\omega\Phi} - \bar{\omega}\bar{\Phi}$$

$$= \sigma(1-\sigma)\left(\bar{\omega}^c - \bar{\omega}^e\right)\left(\bar{\Phi}^c - \bar{\Phi}^e\right)$$

$$= \sigma\left(\bar{\omega}^c - \bar{\omega}^e\right)\left(\bar{\Phi}^c - \bar{\Phi}\right) = \sigma\bar{\omega}^c \underbrace{\left(1 - \frac{\bar{\omega}^e}{\bar{\omega}^c}\right)}_{f(dx)}\left(\bar{\Phi}^c - \bar{\Phi}\right)$$

(2) The small area approximation

$$\boldsymbol{\sigma} << 1 \Rightarrow (1-\boldsymbol{\sigma}) \approx 1; \qquad \bar{\boldsymbol{\omega}}^c >> \bar{\boldsymbol{\omega}}^e$$

$$\overline{\omega'\Phi'} = \sigma\bar{\omega}^c\left(\bar{\Phi}^c - \bar{\Phi}\right)$$

$$\boxed{-\overline{\omega'\Phi'} = gM_c\left(\bar{\Phi}^c - \bar{\Phi}\right)}$$

$$\boxed{M_c = \frac{-\sigma\bar{\omega}^c}{g} = \rho\sigma\bar{w}^c}$$

The mass-flux approach

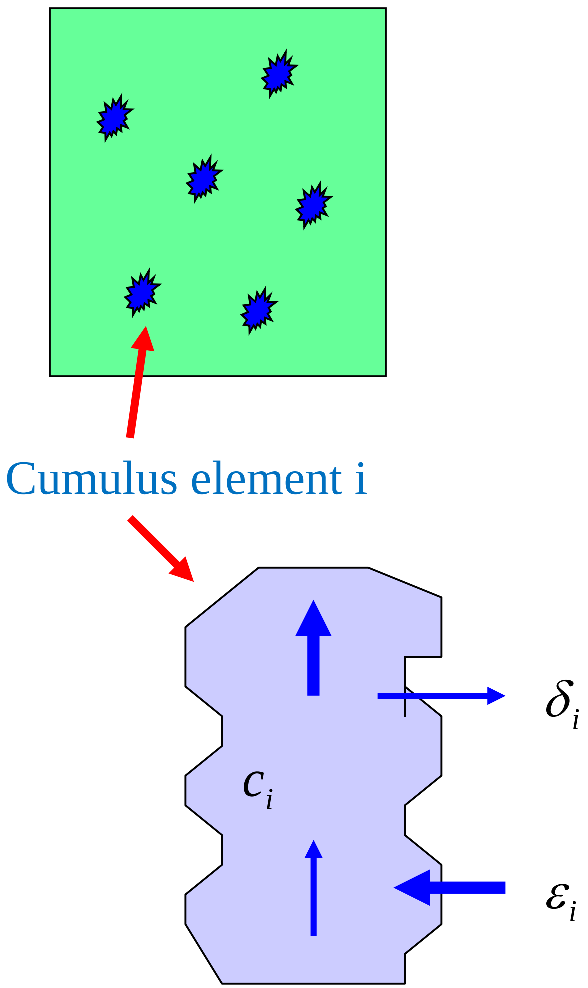
With the above we can rewrite:

$$Q_{1C} \equiv L(\bar{c} - \bar{e}) + g\,\frac{\partial\left[M_c(\bar{s}^c - \bar{s})\right]}{\partial p}$$

$$Q_2 \equiv L(\bar{c} - \bar{e}) - Lg\,\frac{\partial\left[M_c(\bar{q}^c - \bar{q})\right]}{\partial p}$$

To predict the influence of convection on the large-scale we now need to describe the convective mass-flux, the values (s, q, u, v) inside the convective elements and the condensation/evaporation term. This requires, as usual, a cloud model and a closure to determine the absolute (scaled) value of the mass flux.

Mass-flux entraining plume models



Entraining plume model

Continuity:

$$\frac{\partial \sigma_i}{\partial t} + D_i - E_i - g\frac{\partial M_i}{\partial p} = 0$$

Heat:

$$\frac{\partial(\sigma_i s_i)}{\partial t} + D_i s_i - E_i \bar{s} - g\frac{\partial(M_i s_i)}{\partial p} = Lc_i$$

Specific humidity:

$$\frac{\partial(\sigma_i q_i)}{\partial t} + D_i q_i - E_i \bar{q} - g\frac{\partial(M_i q_i)}{\partial p} = -c_i$$

Mass-flux entraining plume models
Simplifications

1. Steady state plumes, i.e., $\frac{\partial X}{\partial t} = 0$

most mass-flux convection parametrizations make that assumption, some (e.g. Gerard&Geleyn) are prognostic

2. Instead of spectral (Arakawa Schubert 1974) use one representative updraught=bulk scheme with entrainment/detrainment written as

$$\frac{1}{M}\frac{dM}{dz} = \varepsilon - \delta \Rightarrow -g\frac{\partial M_c}{\partial p} = E - D$$

ε,δ [m $^{-1}$] denote fractional entrainment/detrainment,
E,D [s $^{-1}$] entrainment/detrainment rates

Large-scale cumulus effects deduced from mass-flux models

$$-g\frac{\partial M_c}{\partial p} = E - D$$

$$-g\frac{\partial\left(M_c\bar{s}^c\right)}{\partial p} = E\bar{s} - D\bar{s}^c + Lc$$

$$\boxed{Q_{1C} \equiv L(c-e) + g\frac{\partial\left[M_c(\bar{s}^c - \bar{s})\right]}{\partial p}}$$

Flux form

Combine:

$$\boxed{Q_{1C} \equiv -gM_c\frac{\partial\bar{s}}{\partial p} + D(\bar{s}^c - \bar{s}) - Le}$$

Advective form

Large-scale cumulus effects deduced using mass-flux models: Interpretation

$$Q_{1C} \equiv -gM_c \frac{\partial \bar{s}}{\partial p} + D(\bar{s}^c - \bar{s}) - Le$$

Convection affects the large scales by

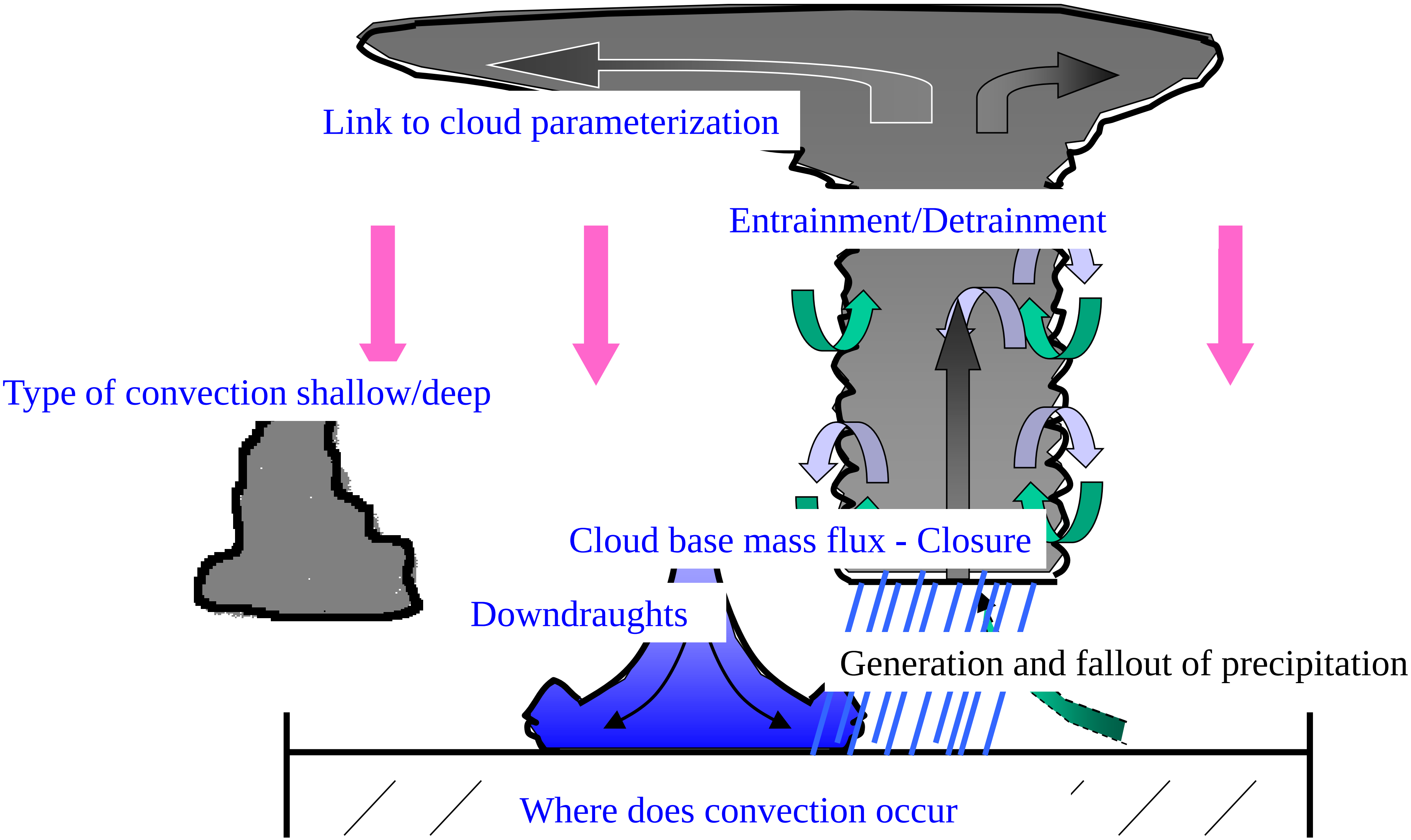
Heating through compensating subsidence between cumulus elements (term 1)

The detrainment of cloud air into the environment (term 2)

Evaporation of cloud and precipitation (term 3)

Note: In the **advective form** the condensation heating does not appear directly in Q_1. It is however the dominant term using the **flux form** and is a crucial part of the cloud model, where this heat is transformed in kinetic energy of the updrafts.

The IFS bulk mass flux scheme
What needs to be considered



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Basic Features

- Bulk mass-flux scheme
- Entraining/detraining plume cloud model
- 3 types of convection: deep, shallow and mid-level - mutually exclusive
- saturated downdraughts
- simple microphysics scheme
- closure dependent on type of convection
  - deep: CAPE adjustment
  - shallow: PBL equilibrium
- strong link to cloud parameterization - convection provides source for cloud condensate

Large-scale budget equations: Heat & moisture

$M=\rho w$; $M_u>0$; $M_d<0$

Heat (dry static energy):

Prec. evaporation in downdraughts

Freezing of condensate in updraughts

$$\left(\frac{\partial s}{\partial t}\right)_{cu} = g\frac{\partial}{\partial p}\left[\underbrace{M_u s_u + M_d s_d - (M_u + M_d)\bar{s}}_{\text{Mass-flux transport in up- and downdraughts}}\right] + L(c_u - e_d - e_{subcld}) - L_f(M - F)$$

condensation in updraughts

Prec. evaporation below cloud base

Melting of precipitation

Humidity:

$$\left(\frac{\partial q}{\partial t}\right)_{cu} = g\frac{\partial}{\partial p}\left[M_u q_u + M_d q_d - (M_u + M_d)\bar{q}\right] - \left(c_u - e_d - e_{subcld}\right)$$

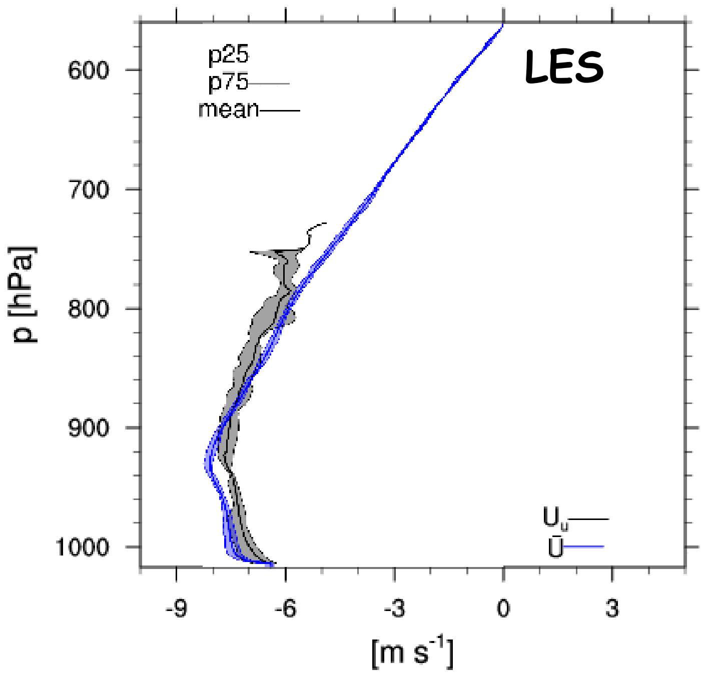
Detrained Condensate:

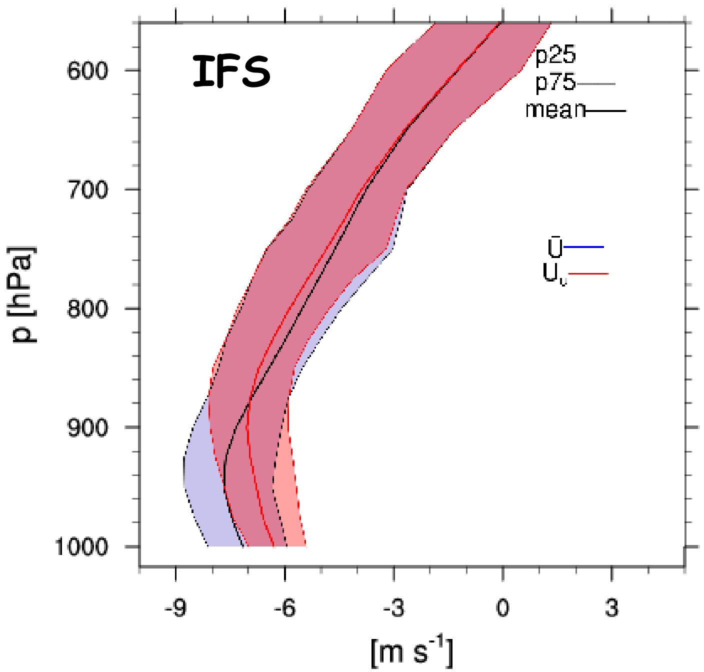
$$\left(\frac{\partial l}{\partial t}\right)_{cu} = -g\frac{\partial}{\partial p}\left[(M^u + M^d)\bar{l}\,\right] + D_u l_u$$

Large-scale budget equations: Momentum

$$\left(\frac{\partial u}{\partial t}\right)_{cu} = g\,\frac{\partial}{\partial p}\left[M_u u_u + M_d u_d - (M_u + M_d)\bar{u}\right]$$

$$\left(\frac{\partial v}{\partial t}\right)_{cu} = g\,\frac{\partial}{\partial p}\left[M_u v_u + M_d v_d - (M_u + M_d)\bar{v}\right]$$





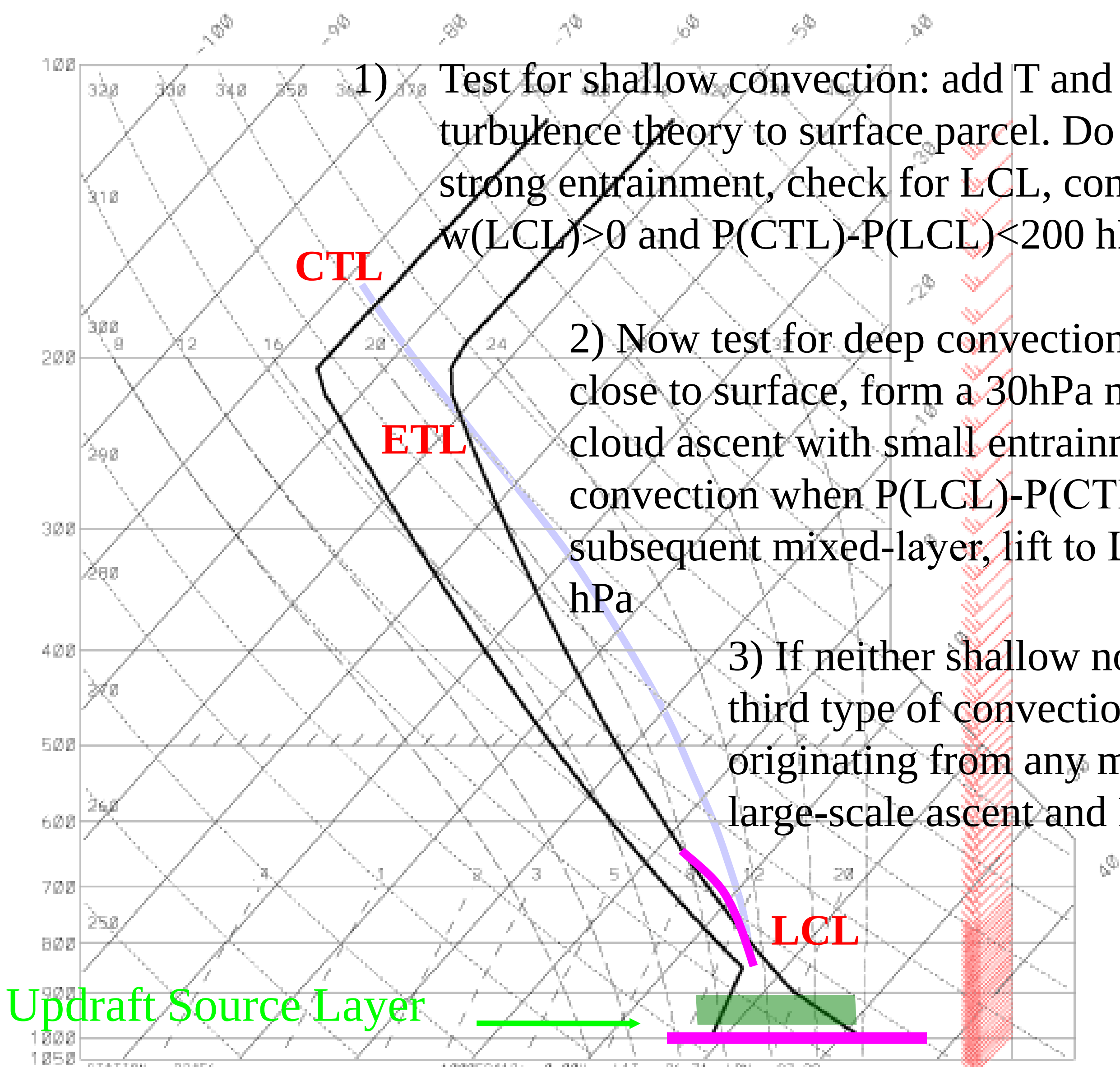
Shallow cumulus: convective momentum transport reduces on average the shear

Occurrence of convection:
make a first-guess parcel ascent

1) Test for shallow convection: add T and q perturbation based on turbulence theory to surface parcel. Do ascent with w-equation and strong entrainment, check for LCL, continue ascent until w<0. If w(LCL)>0 and P(CTL)-P(LCL)<200 hPa : shallow convection

2) Now test for deep convection with similar procedure. Start close to surface, form a 30hPa mixed-layer, lift to LCL, do cloud ascent with small entrainment+water fallout. Deep convection when P(LCL)-P(CTL)>200 hPa. If not …. test subsequent mixed-layer, lift to LCL etc. … and so on until 300 hPa

3) If neither shallow nor deep convection is found a third type of convection – “midlevel” – is activated, originating from any model level below 10 km if large-scale ascent and RH>80%.



Cloud model equations – updraughts
E and D are positive by definition

Mass (Continuity)

$$-g\frac{\partial M_u}{\partial p} = E_u - D_u$$

Heat

$$-g\frac{\partial M_u s_u}{\partial p} = E_u\bar{s} - D_u s_u + Lc_u$$

Humidity

$$-g\frac{\partial M_u q_u}{\partial p} = E_u\bar{q} - D_u q_u - c_u$$

Liquid+Ice

$$-g\frac{\partial M_u l_u}{\partial p} = -D_u l_u + c_u - G_{P,u}$$

Precip

$$-g\frac{\partial M_u r_u}{\partial p} = -D_u r_u + G_{P,u} - Sfout$$

Momentum

$$-g\frac{\partial M_u u_u}{\partial p} = E_u\bar{u} - D_u u_u$$

$$-g\frac{\partial M_u v_u}{\partial p} = E_u\bar{v} - D_u v_u$$

Kinetic Energy (vertical velocity) – use height coordinates

$$\frac{\partial K_u}{\partial z} = -\frac{E_u}{M_u}(1+\beta C_d)2K_u + \frac{1}{f(1+\gamma)}g\frac{T_{v,u}-\overline{T}_v}{\overline{T}_v}, \quad K_u = \frac{w_u^2}{2}$$

Downdraughts

1. Find level of free sinking (LFS)

   highest model level for which an equal saturated mixture of cloud and environmental air becomes negatively buoyant

2. Closure

$$M_{d,LFS} = -\alpha M_{u,b} \qquad \alpha = 0.3$$

Cloud model equations – downdraughts

E and D are defined positive

$$g\frac{\partial M_d}{\partial p} = E_d - D_d$$

Mass

$$g\frac{\partial M_d s_d}{\partial p} = E_d\bar{s} - D_d s_d + Le_d$$

Heat

$$g\frac{\partial M_d q_d}{\partial p} = E_d\bar{q} - D_d q_d + e_d$$

Humidity

$$\left.\begin{aligned} g\frac{\partial M_d u_d}{\partial p} &= E_d\bar{u} - D_d u_d \\ g\frac{\partial M_d v_d}{\partial p} &= E_d\bar{v} - D_d v_d \end{aligned}\right\}$$

Momentum

Entrainment/Detrainment (1)

$$-g\frac{\partial M_u}{\partial p} = E_u - D_u = \frac{M_u}{\rho}\left(\varepsilon - \delta\right) = \frac{M_u}{\rho}\left(\varepsilon_{turb} - \delta_{turb} - \delta_{org}\right)$$

ε and δ are generally given in units (m^{-1})

$$\varepsilon = \underbrace{c_1\left(1.3 - RH\right)}_{buoy>0} F_\varepsilon; \quad RH = \frac{\bar{q}}{\bar{q}_s}; \quad \delta_{turb} = c_2$$

$$c_1 = 1.75\times10^{-3}m^{-1}; c_2 = 0.75\times10^{-4}m^{-1};$$

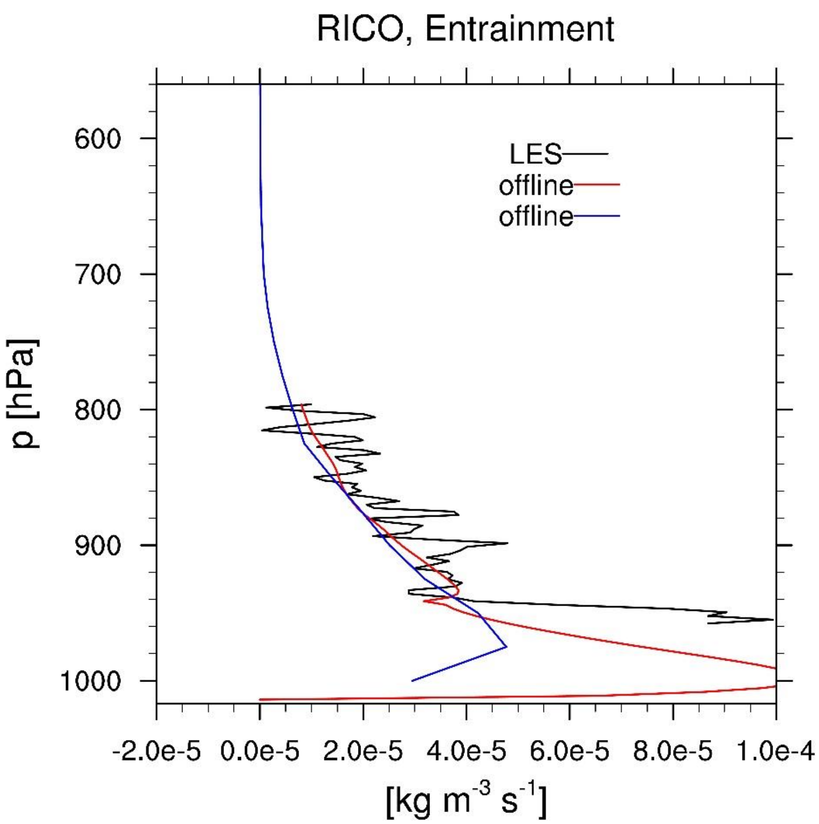
$$F_\varepsilon = \left(\frac{\bar{q}_s}{\bar{q}_{s\,base}}\right)^3$$

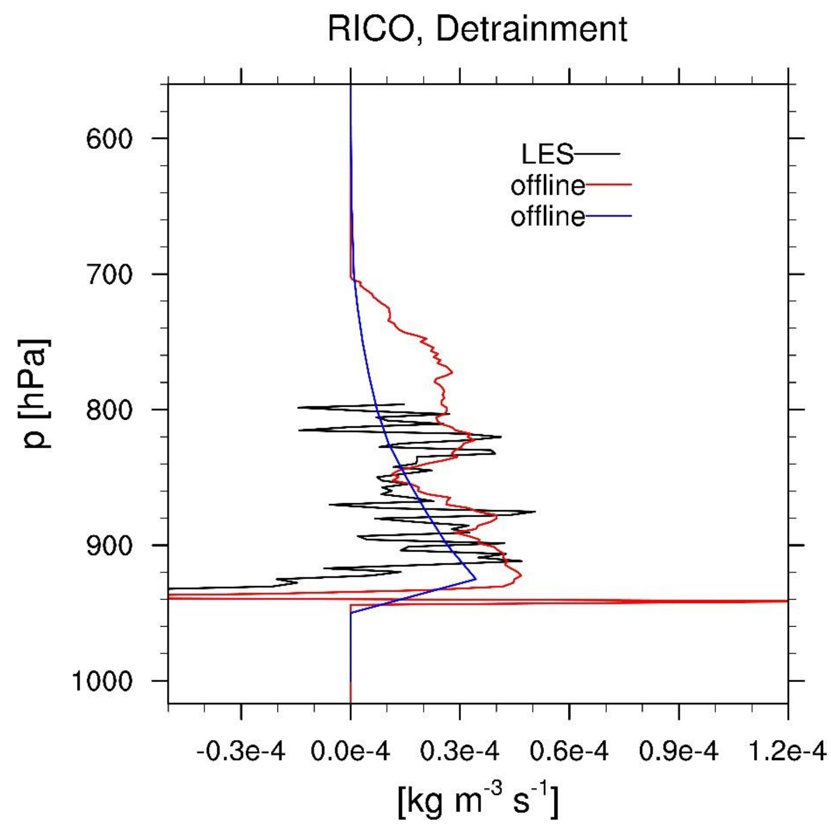
Constants

Scaling function to mimick a cloud ensemble

Entrainment/Detrainment

Entrainment formulation sooo simple: so how does it compare to LES ?





LES (black)   IFS   IFS formula with LES data

Schlemmer et al. 2017

Nota: entrainment for deep typically factor 2 larger than that for shallow

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Entrainment/Detrainment (3)

Organised Detrainment:

When updraught kinetic energy K decreases with height (negative buoyancy), compute mass flux at level z+Δz with following relation:

$$\frac{M_u(z)}{M_u(z+\Delta z)} \approx (1.6 - RH)\sqrt{\frac{K_u(z)}{K_u(z+\Delta z)}}; \quad \Rightarrow D_u = \frac{\Delta M_u}{\bar{\rho}\Delta z}$$

with $$K_u = \frac{w_u^2}{2}$$

Precipitation

Liquid+solid precipitation fluxes:

$$P^{rain}(p) = \int_{Ptop}^{P} (G^{rain} - e_{down}^{rain} - e_{subcld}^{rain} + Melt)dp / g$$

$$P^{snow}(p) = \int_{Ptop}^{P} (G^{snow} - e_{down}^{snow} - e_{subcld}^{snow} - Melt)dp / g$$

Where P^{rain} and P^{snow} are the fluxes of precip in form of rain and snow at pressure level p. G^{rain} and G^{snow} are the conversion rates from cloud water into rain and cloud ice into snow. Evaporation occurs in the downdraughts e_{down}, and below cloud base e_{subcld}, *Melt* denotes melting of snow.

Generation of precipitation in updraughts

$$\rho G_{P,u} = M_u \frac{c_0}{w_u} l_u \left[1 - e^{-\left(\frac{l_u}{l_{crit}}\right)^2} \right]$$

Simple representation of Bergeron process included in c_0 and l_{crit}

Precipitation

Fallout of precipitation from updraughts

$$\rho S_{fallout} = M_u \frac{V_{prec}}{w_u \Delta z} r_u$$

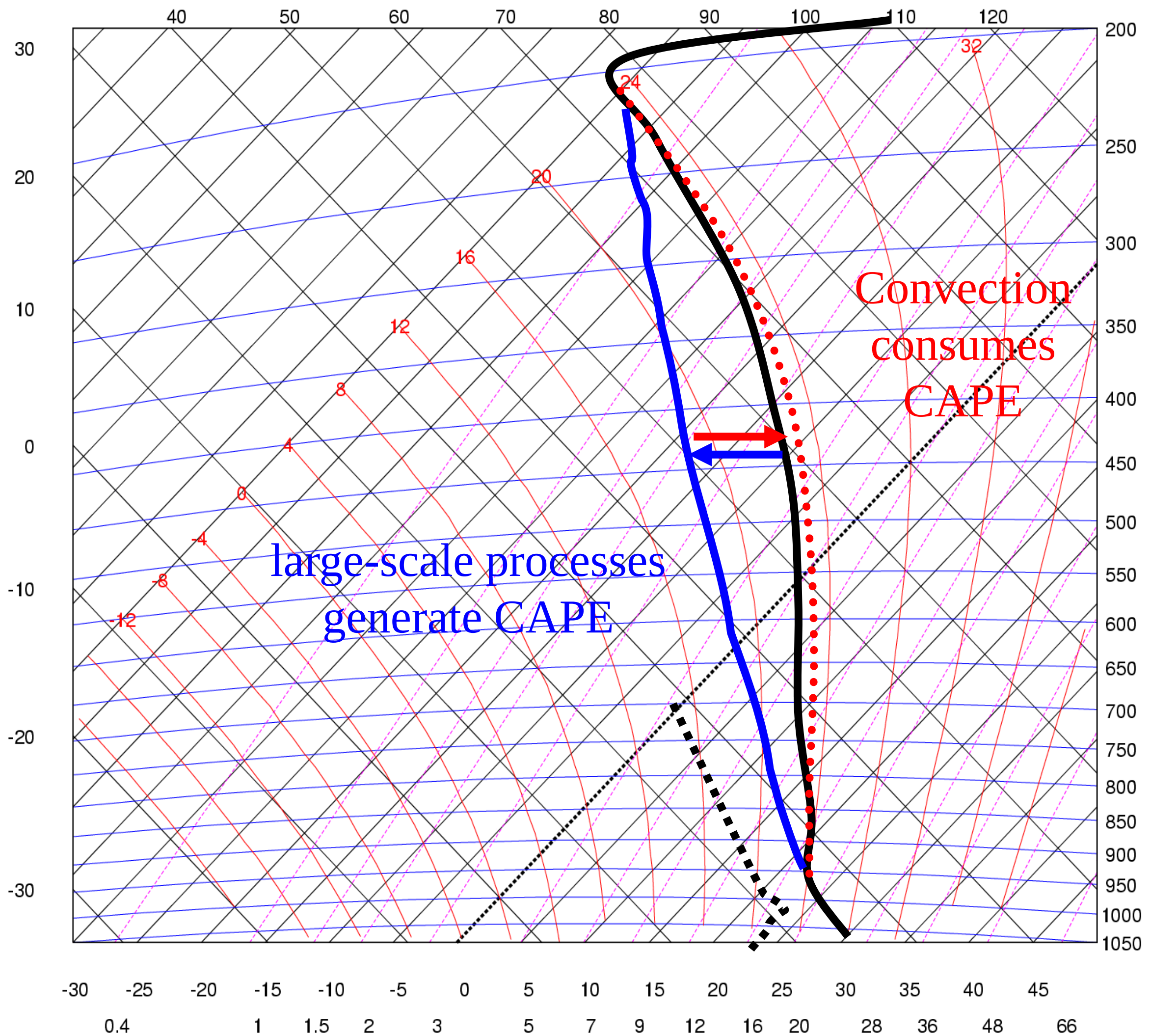
$$V_{prec,rain} = 5.32 r_u^{0.2} \qquad V_{prec,ice} = 2.66 r_u^{0.2}$$

Evaporation of precipitation

1. Precipitation evaporates to keep downdraughts saturated
2. Precipitation evaporates below cloud base

$$e_{subcld} = \sigma \alpha_1 \left(RH\, q_s - \overline{q} \right) \left(\frac{\sqrt{p/p_{surf}}}{\alpha_2} \frac{\overline{P}}{\sigma} \right)^{\alpha_3}, \quad \text{assume a cloud fraction } \sigma = 0.05$$

CAPE closure - the basic idea



Closure - Deep convection

$$CAPE = g \int_{cloud} \frac{T_{v,u} - \overline{T}_v}{\overline{T}_v} dz \approx g \int_{cloud} \frac{\theta_{e,u} - \overline{\theta}_{esat}}{\overline{\theta}_{esat}} dz$$

Use instead density scaling, time derivative then relates to mass flux:

$$PCAPE = -\int_{Pbase}^{Ptop} \frac{T_{v,u} - \overline{T}_v}{\overline{T}_v} dp$$

$$\frac{\partial PCAPE}{\partial t} \approx \underbrace{-\int_{Pbase}^{Ptop} \frac{1}{\overline{T}_v} \frac{\partial \overline{T}_v}{\partial t} dp}_{LS+Cu} \underbrace{- \int_{Pbase}^{Ptop} \frac{1}{\overline{T}_v} \frac{\partial T_{v,u}}{\partial t} dp + \left. \frac{T_{v,u} - \overline{T}_v}{\overline{T}_v} \right|_{base} \frac{\partial p_{base}}{\partial t}}_{BL+Cu} =$$

$$= \left. \frac{\partial PCAPE}{\partial t} \right|_{LS} + \left. \frac{\partial PCAPE}{\partial t} \right|_{BL} + \left. \frac{\partial PCAPE}{\partial t} \right|_{Cu=shal+deep}$$

this is a prognostic CAPE closure: now try to determine the different terms and try to achieve balance $\partial PCAPE / \partial t \ll \partial PCAPE / \partial t|_{cu}, \partial PCAPE / \partial t|_{LS}$

Closure - Deep convection

1
$$\left.\frac{\partial PCAPE}{\partial t}\right|_{cu,1} = -\frac{PCAPE - PCAPE_{BL}}{\tau}; \qquad \tau = \frac{H}{\overline{w}_u}$$

2
$$\left.\frac{\partial PCAPE}{\partial t}\right|_{cu,2} = \int_{Pbase}^{Ptop} \frac{1}{\overline{T}_v} \left.\frac{\partial \overline{T}_v}{\partial t}\right|_{cu} dp = -\int_{zbase}^{ztop} \frac{g}{\overline{T}_v} M \left(\frac{\partial \overline{T}_v}{\partial z} + \frac{g}{c_p}\right) dz$$

$$= -\frac{M_{u,b}}{M^*_{u,b}} \int_{zbase}^{ztop} \frac{g}{\overline{T}_v} M^* \left(\frac{\partial \overline{T}_v}{\partial z} + \frac{g}{c_p}\right) dz$$

Nota: all the trick is in the $PCAPE_{BL}$ term=PCAPE not available to deep convection but used for boundary-layer mixing (see Bechtold et al. 2014).

If $PCAPE_{BL}$=0 then wrong diurnal cycle over land!

Closure - Deep convection

Solve now for the cloud base mass flux by equating 1 and 2

$$M_{u,b} = M^*_{u,b} \frac{PCAPE - PCAPE_{BL}}{\tau} \frac{1}{\int\limits_{cloud} M^* \frac{g}{\overline{T}_v} \frac{\partial \overline{T}_v}{\partial z} dz}; \qquad M_{u,b} \geq 0$$

$$PCAPE_{BL} = -\tau_{BL} \frac{1}{T^*} \int\limits_{psurf}^{pbase} \left. \frac{\partial \overline{T}_v}{\partial t} \right|_{BL} dp$$

$M^* = M_u + M_d$ Mass flux from the updraught/downdraught computation

$M^*_{u,b}$ initial updraught mass flux at base, set proportional to 0.1Δp

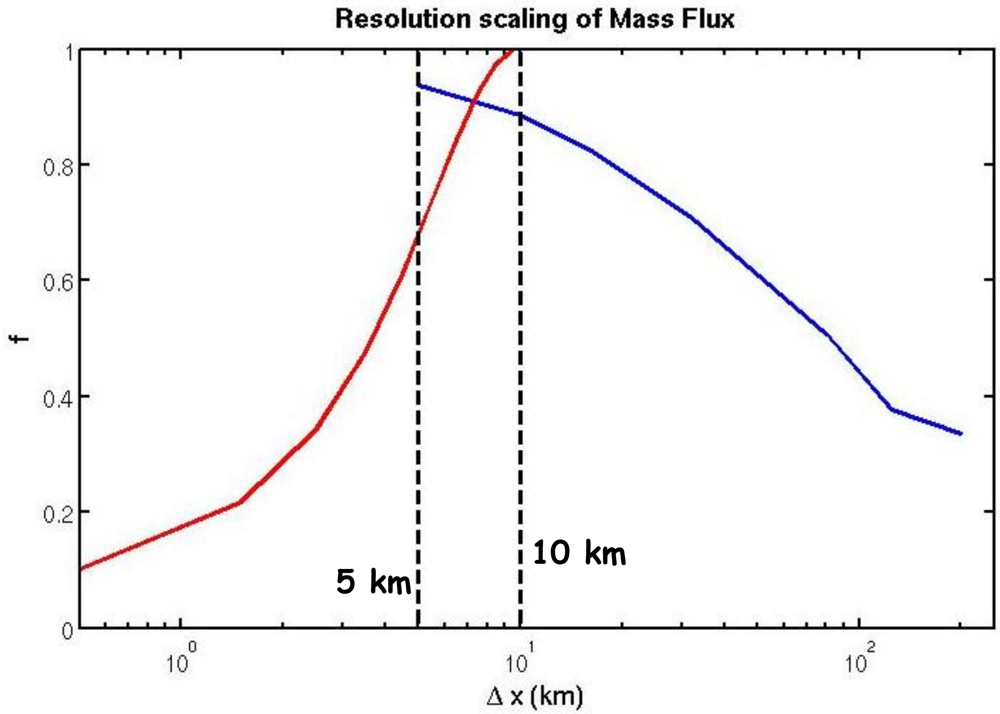
$PCAPE_{bl}$ contains the boundary-layer tendencies due to surface heat fluxes, radiation and advection

Resolution scaling

$$\overline{\omega'\Phi'} = \overline{\omega\Phi} - \bar{\omega}\bar{\Phi}$$

$$= \sigma\bar{\omega}^{c} \underbrace{\left(1 - \frac{\bar{\omega}^{e}}{\bar{\omega}^{c}}\right)}_{f(dx)} \left(\bar{\Phi}^{c} - \bar{\Phi}\right)$$

Developed in collaboration with Deutsche Wetterdienst and ICON model



Kwon and Hong, 2016 MWR independently developed very similar relations

Closure - Shallow convection

Based on PBL equilibrium : what goes in must go out - including downdraughts

$$\int_{psurf}^{pbase} \frac{\partial \bar{h}}{\partial t} dp = 0$$

$$\int_{0}^{cbase} \left[g \left. \frac{\partial \left(\overline{w'h'} \right)}{\partial p} \right|_{cu} + \left(\frac{\partial \bar{h}}{\partial t} \right)_{turb} + \left(\frac{\partial \bar{h}}{\partial t} \right)_{dyn} + \left(\frac{\partial \bar{h}}{\partial t} \right)_{rad} \right] dp = 0$$

$$\bar{\rho} \left(\overline{w'h'} \right)_{cu,b} = M_{u,b} \left(h_u - \varepsilon h_d - (1-\varepsilon)\bar{h} \right)_{base}; \quad \varepsilon = M_u / M_d;$$

Assume 0 convective flux at surface, then it follows for cloud base flux

$$\boxed{M_{u,b} = \frac{-\frac{1}{g} \int_{psurf}^{pbase} \left[\left(\frac{\partial \bar{h}}{\partial t} \right)_{turb} + \left(\frac{\partial \bar{h}}{\partial t} \right)_{dyn} + \left(\frac{\partial \bar{h}}{\partial t} \right)_{rad} \right] dp}{\left(h_u - \varepsilon h_d - (1-\varepsilon)\bar{h} \right)_{cbase}}}$$

Closure - Midlevel convection

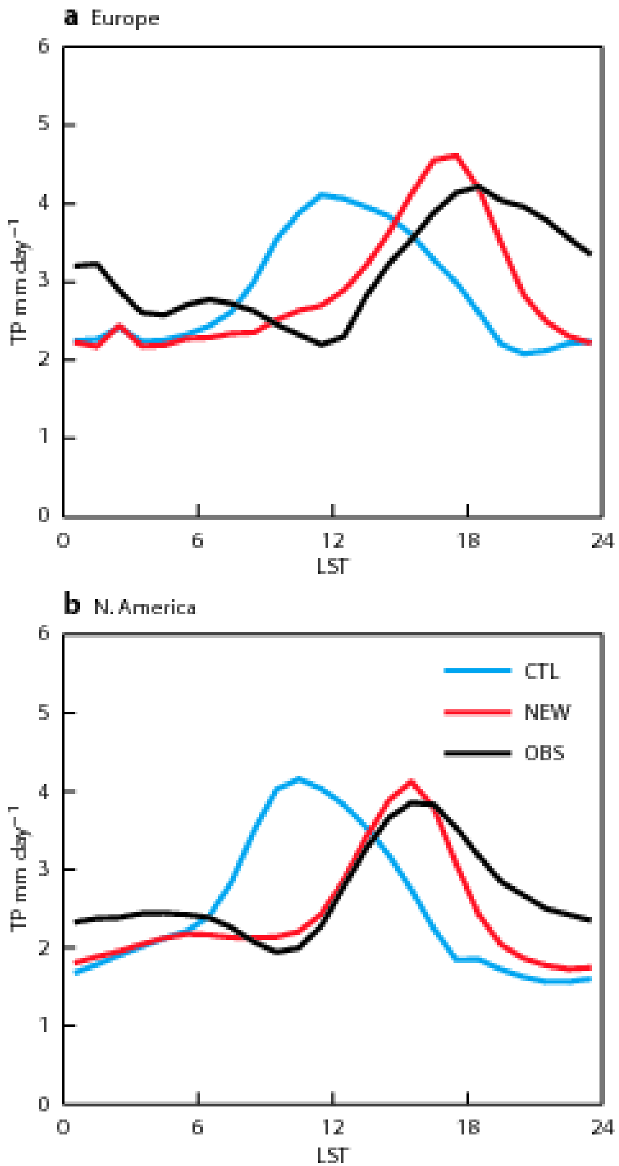
Roots of clouds originate outside PBL

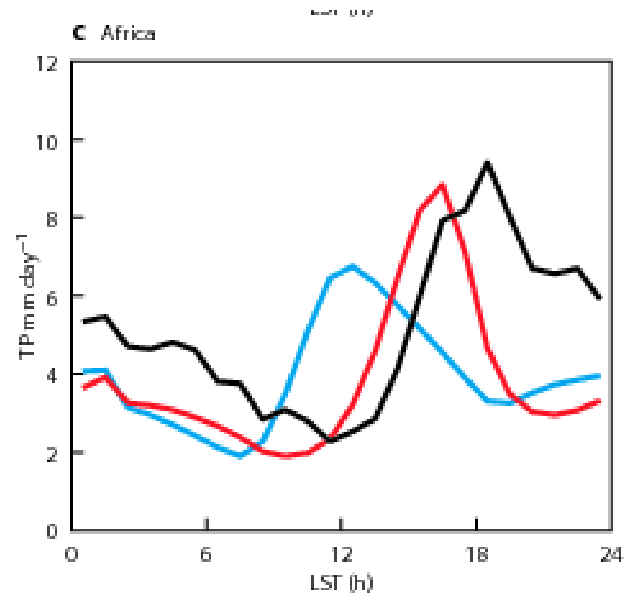
assume midlevel convection exists if there is large-scale ascent, RH>80% and there is a convectively unstable layer

Closure:

$$M_{u,b} = \rho \overline{w}_b$$

Impact of closure on diurnal cycle
JJA 2011-2012 against Radar





Obs radar

NEW=with PCAPEBbl term

Bechtold et al., 2014, J. Atmos. Sci.
ECMWF Newsletter No 136 Summer 2013

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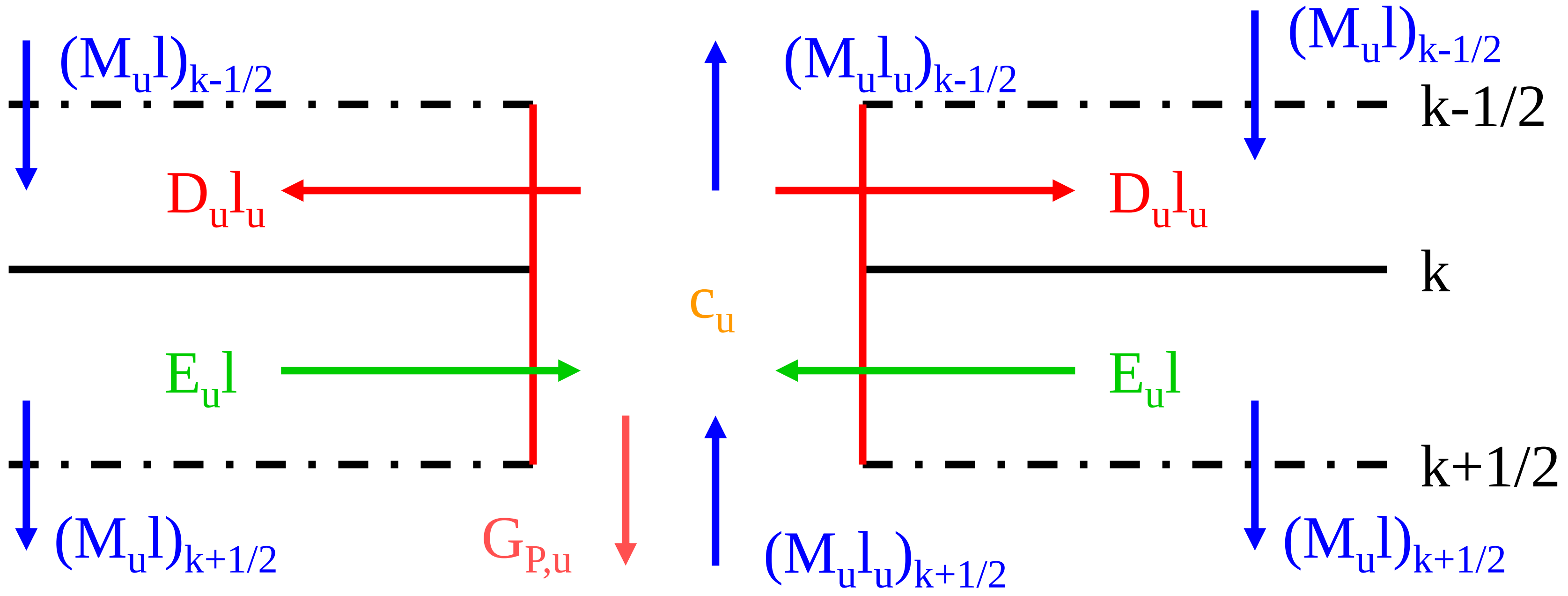
Vertical Discretisation

Fluxes on half-levels, state variable and tendencies on full levels



Numerics: solving Tendency advection equation explicit solution

$$\left.\frac{\partial\overline{\psi}}{\partial t}\right|_{conv} = g\frac{\partial}{\partial p}\left[M^u(\psi^u - \overline{\psi})\right] + S; \qquad \text{if } \psi = T,q \qquad S = \frac{\partial}{\partial p}\mathrm{Pr}$$

Use vertical discretisation with fluxes on half levels (*k+1/2*), and tendencies on full levels *k*, so that

$$\Delta p = P_{k+1/2} - P_{k-1/2}$$

$$\left.\frac{\partial\overline{\psi}_k}{\partial t}\right|_{conv} = \frac{g}{\Delta p}\left[M^u_{k+1/2}\psi^u_{k+1/2} - M^u_{k-1/2}\psi^u_{k-1/2} - M^u_{k+1/2}\boldsymbol{\overline{\psi}_{k+1/2}} + M^u_{k-1/2}\boldsymbol{\overline{\psi}_{k-1/2}}\right] + S_k$$

In order to obtain a better and more stable "upstream" solution ("compensating subsidence", use shifted half-level values to obtain:

$$\overline{\psi}_{k-1/2} = \overline{\psi}_{k-1}$$

$$\left.\frac{\partial\overline{\psi}_k}{\partial t}\right|_{conv} = \frac{g}{\Delta p}\left[M^u_{k+1/2}\psi^u_{k+1/2} - M^u_{k-1/2}\psi^u_{k-1/2} - M^u_{k+1/2}\boldsymbol{\overline{\psi}_{k}} + M^u_{k-1/2}\boldsymbol{\overline{\psi}_{k-1}}\right] + S$$

Numerics: implicit solution

$$\left.\frac{\partial\bar{\psi}}{\partial t}\right|_{conv} = g\frac{\partial}{\partial p}\left[M^u(\psi^u - \bar{\psi})\right] + S;$$

if $\psi = T,q$ $\quad S = \frac{\partial}{\partial p}\mathrm{Pr}$

Use temporal discretisation with $\bar{\psi}$ on RHS taken at future time $\bar{\psi}^{n+1}$ and not at current time $\bar{\psi}^{n}$

$$\Delta p = P_{k+1/2} - P_{k-1/2}$$

For "upstream" discretisation as before one obtains: $\bar{\psi}_{k\text{-}1/2} = \bar{\psi}_{k\text{-}1}$

$$\boxed{\bar{\psi}_k^{n+1}} - \bar{\psi}_k^n = g\frac{\Delta t}{\Delta p}\left[M^u_{k+1/2}\psi^u_{k+1/2} - M^u_{k-1/2}\psi^u_{k-1/2} - M^u_{k+1/2}\boxed{\bar{\psi}_k^{n+1}} + M^u_{k-1/2}\boxed{\bar{\psi}_{k\text{-}1}^{n+1}}\right] + \Delta t S_k^n$$

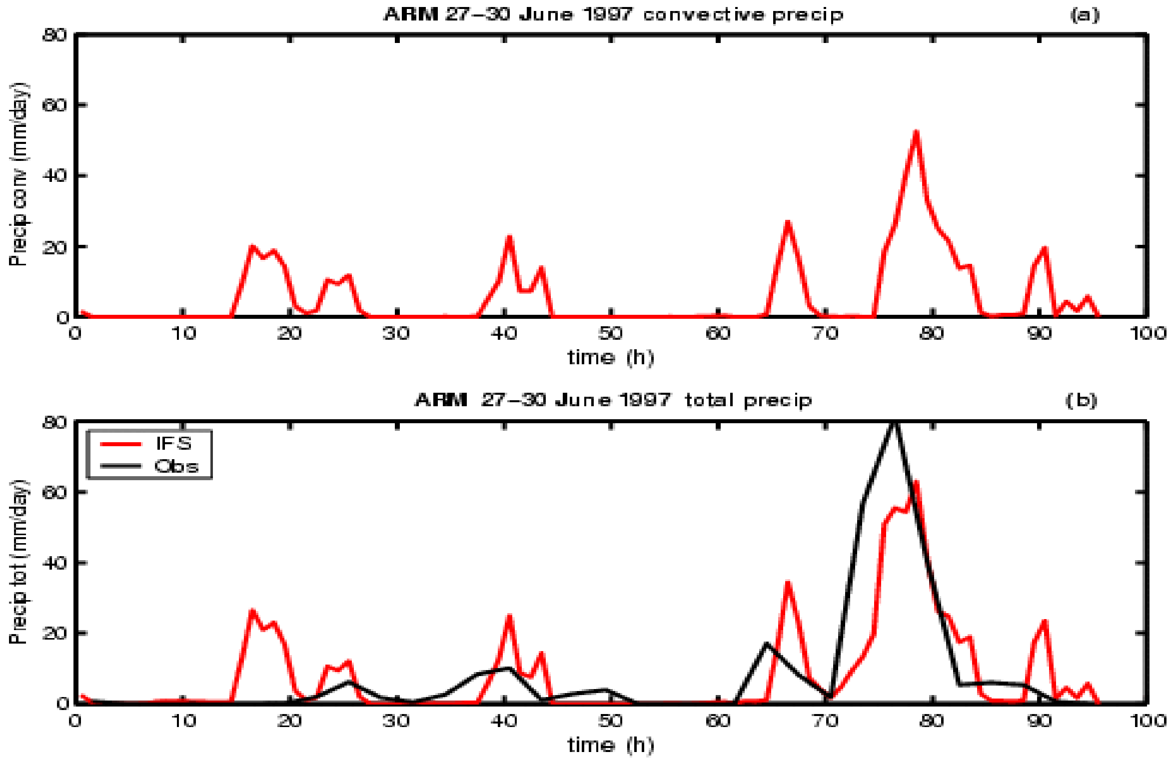
$$\left(1 + \frac{g\Delta t}{\Delta p}M^u_{k+1/2}\right)\boxed{\bar{\psi}_k^{n+1}} - \frac{g\Delta t}{\Delta p}M^u_{k-1/2}\boxed{\bar{\psi}_{k\text{-}1}^{n+1}} = \bar{\psi}_k^n + \frac{g\Delta t}{\Delta p}\left[M^u_{k+1/2}\psi^u_{k+1/2} - M^u_{k-1/2}\psi^u_{k-1/2}\right] + \Delta t S_k^n$$

$\rightarrow$ Only bi-diagonal linear system, and tendency is obtained as

$$\boxed{\left.\frac{\partial\bar{\psi}_k}{\partial t}\right|_{conv} = \frac{\bar{\psi}_k^{n+1} - \bar{\psi}_k^n}{\Delta t}}$$

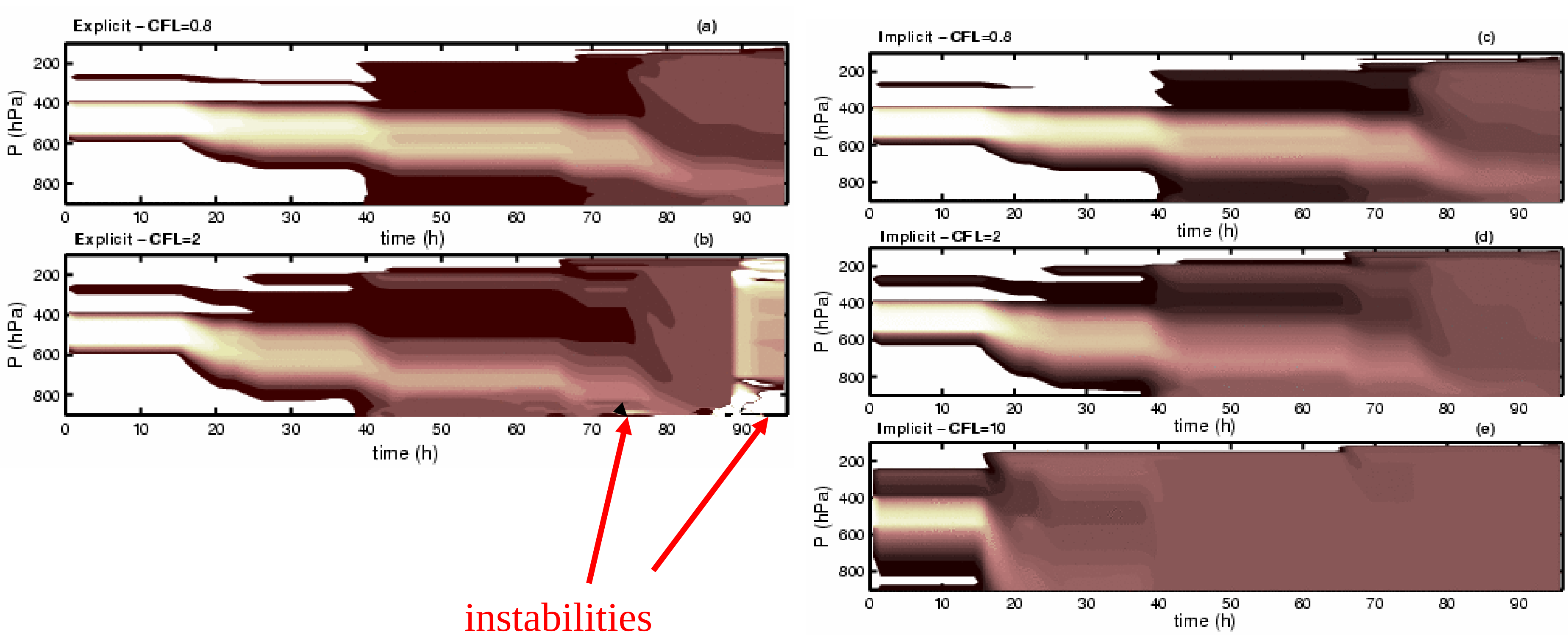
Tracer transport experiments
Single-column simulations (SCM)

Surface precipitation; continental convection during ARM



ECMWF

Tracer transport in SCM
Stability in implicit and explicit advection

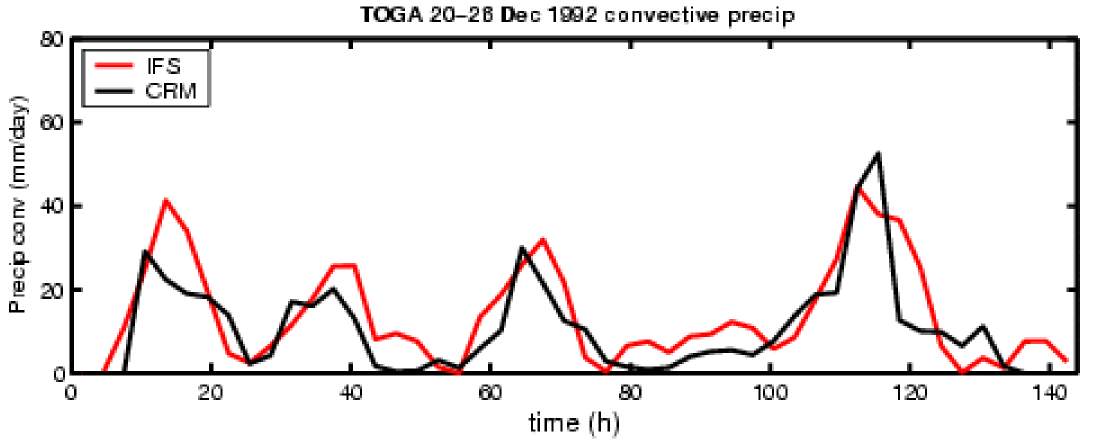


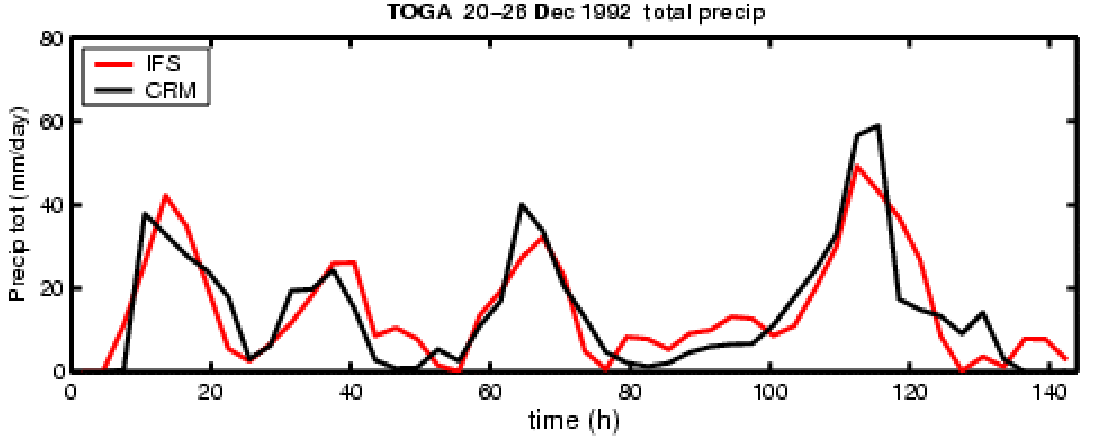
- Implicit solution is stable.
- If mass fluxes increases, mass flux scheme behaves like a diffusion scheme: well-mixed tracer in short time

ECMWF

Tracer transport experiments (2)
Single-column model against CRM

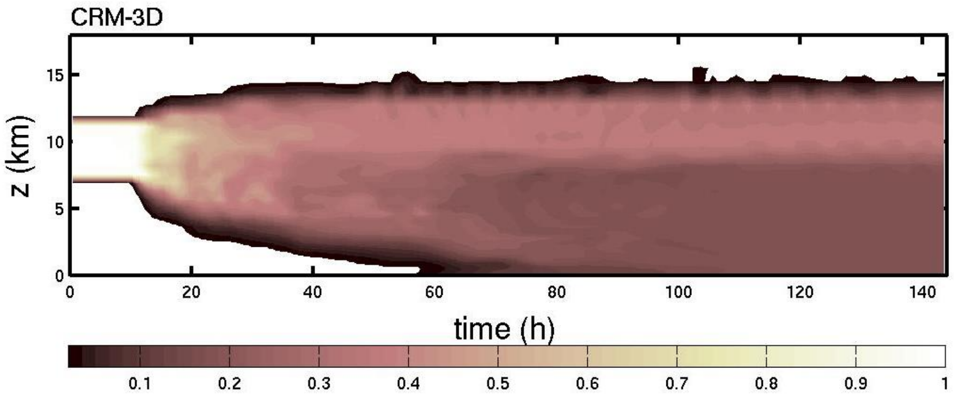
Surface precipitation; tropical oceanic convection during TOGA-COARE

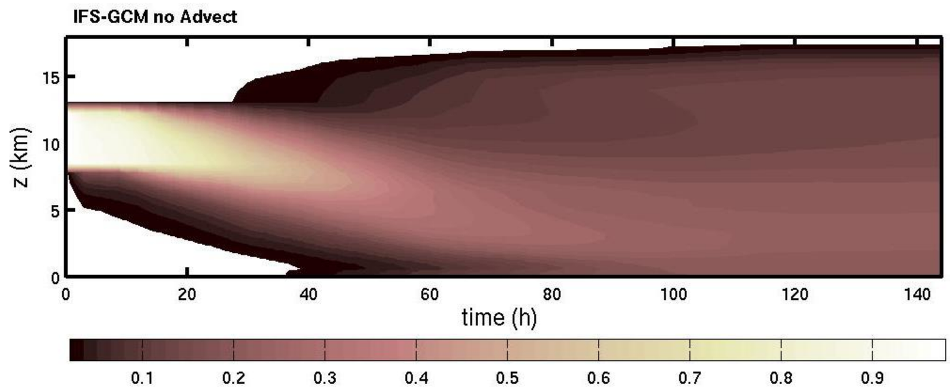


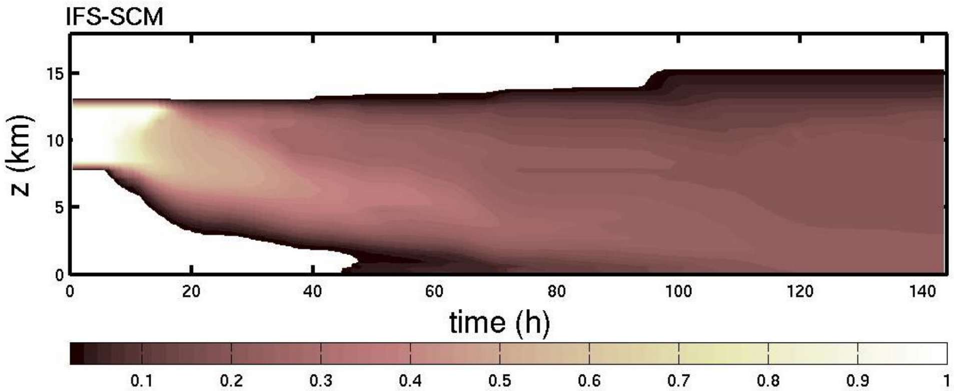


Tracer transport
SCM and global model against CRM

Mid-tropospheric Tracer







- Mid-tropospheric tracer is transported upward by convective draughts, but also slowly subsides due to cumulus induced environmental subsidence
- IFS SCM (convection parameterization) diffuses tracer somewhat more than CRM
- In GCM tropopause higher, normal, as forcing in other runs had errors in upper troposphere

ECMWF